Industrial Organization

Final Exam Spring 2025 – Solution Topics

1. False.

Firms will collude if and only if (i) their interaction is infinitely repeated, and (ii) the present value of profits under collusion exceeds the present value of profits from deviation.

An interest rate (r) close to one implies a discount factor $\delta = \frac{1}{1+r} \approx \frac{1}{2}$. In a situation where $\delta \approx \frac{1}{2}$ but the present value of collusive profits is lower than deviation profits, collusion is not sustainable.

To illustrate, suppose that n firms compete à la Bertrand. In this case, firms will collude if:

$$\frac{\pi^M}{n} + \frac{\pi^M}{n}\delta + \frac{\pi^M}{n}\delta^2 + \cdots > \pi^M + 0\delta + 0\delta^2 + \cdots \leftrightarrow (\ldots) \leftrightarrow \delta > 1 - \frac{1}{n}$$

If $n = 3$, collusion requires $\delta > \frac{2}{3}$. Since $\delta \approx \frac{1}{2}$, this condition is not met, and firms will not collude.

Therefore, when the interest rate is very close to one, it is not necessarily true that firms will collude.

2. False.

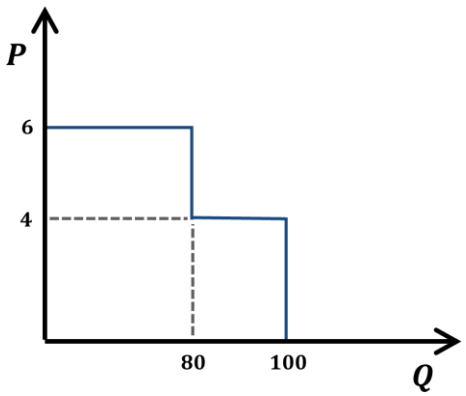
Firms will collude if and only if (i) their interaction is infinitely repeated, and (ii) the present value of profits under collusion exceeds the present value of profits from deviation.

If either condition fails to hold, sustained cooperation is not feasible. For example, in a market with an infinite number of firms, collusion is highly unlikely due to coordination and enforcement difficulties. However, this does not invalidate the theory of tacit collusion; it merely indicates that the necessary conditions for collusion are not present in such a market.

3.

(i)

$$\boldsymbol{Q(P) = \begin{cases} 0 \; if \; P > 6 \\ 80 \; if 4 < P \leq 6 \\ 100 \; if \; P \leq 4 \end{cases}}$$



(ii)

If firm L sets a price above 4, firm F faces two strategic options:

- **Option 1: set a price of 4**
    - By pricing at 4, firm F attracts both poor and rich consumers. Specifically, it captures all poor consumers and half of the rich consumers. The resulting profit is: $\pi_F = (4-2) \times 60 = 120$.
- **Option 2: set a price above 4 and target rich consumers**
    - In this case, the optimal strategy is to charge a price of 6, thereby attracting only half of the rich consumers (regardless of firm L's price). The profit in this scenario is: $\pi_F = (6-2) \times 40 = 160$

Since targeting rich consumers yields a higher profit, firm F should optimally charge a price of 6 whenever firm L sets a price above 4.

(iii)

If firm L prices equal or below 4, firm F faces two strategic options:

- **Option 1: Set a price above 4 and target only rich consumers**
    - As established earlier, setting a price of 6 allows firm F to capture half of the rich consumers, yielding a profit of: $\pi_F = (4-2) \times 60 = 120$
- **Option 2: Undercut firm L to attract all poor consumers**
    - In this case, firm F would undercut firm L's price to capture all 20 poor consumers. The profit under this strategy is: $\pi_F = (P_L - 2) \times 60$

Firm F will only prefer option 2 over option 1 if it yields a higher profit:

$$(P_L - 2) \times 60 > 160 \leftrightarrow 60P_L - 120 > 160 \leftrightarrow P_L > 4.6$$

Since firm L is assumed to set a price equal to or below 4, this inequality is never satisfied. Therefore, option 1 always yields a higher profit.

Conclusion: If firm L sets a price equal to or below 4, firm F should optimally set a price of 6 and target the rich consumers.

(iv)

As previously established, targeting only rich consumers and thus charging a price of 6 is a dominant strategy for firm F. Thus, firm F should charge a price of 6, irrespective of the pricing decision made by firm L.

(v)

Given that firm L anticipates that firm F will charge a price of 6, it faces two strategic options:

- **Option 1: Set a price of 4 to attract both poor and rich consumers**
    - By setting a price of 4, firm L captures all poor consumers and half of the rich consumers. The resulting profit is: $\pi_L = (4-2) \times 60 = 120$.
- **Option 2: set a price above 4 and target rich consumers**
    - In this case, the optimal strategy is to set a price of 6, which allows firm L to attract only half of the rich consumers. The corresponding profit is: $\pi_L = (6-2) \times 40 = 160$

Since the profit under option 2 is higher, firm L should optimally charge a price of 6.

(vi)

As previously established, both firms will choose to target rich consumers and set a price of 6. In equilibrium, each firm captures half of the rich segment, resulting in a profit of $\pi = (6-2) \times 40 = 160$.

(vii)

No, because both * leader and the follower — earn the same profit of 160.

(viii)

As previously established, setting a price of 6 and targeting rich consumers constitutes a dominant strategy — that is, it yields a higher payoff regardless of the rival's pricing decision. Consequently, if firms were to set prices simultaneously rather than sequentially, the equilibrium outcome would remain unchanged: both firms would still choose to solely target the rich consumers (i.e., non-active searchers) and thus charge a price of 6.

4.

(i)

The demand faced by the domestic firms is:

$$D'(P) = D(P) - I(P)$$

$$D'(P) = \begin{cases} 8 - P, & if\ P \geq 2 \\ 10 - P, & if\ P < 2 \end{cases}$$

(ii)

The optimal collusion quantity corresponds to the monopolist quantity:

$$\pi = P(Q)Q - cQ \Leftrightarrow \pi = (8-Q)Q - 2Q$$

$$\max_Q \pi^M = (8-Q)Q - 2Q$$

$$FOC: \frac{d\pi^M}{dQ} = 0 \Leftrightarrow 8 - 2Q - 2 = 0 \Leftrightarrow \boldsymbol{Q^* = 3}$$

Therefore, the optimal collusion quantity is 3, split between the two domestic firms (i.e., 1.5 each).

(iii)

The two domestic firms will collude if the present value profits under collusion exceeds the present value of profits from deviation:

$$\frac{\pi^M}{2}+\frac{\pi^M}{2}\delta+\frac{\pi^M}{2}\delta^2+\cdots>\pi^{Deviation}+\pi^{Cournot}\delta+\pi^{Cournot}\delta^2+\cdots$$

$$\frac{9}{2}+\frac{9}{2}\delta+\frac{9}{2}\delta^2+\cdots>5{,}0525+4\delta+4\delta^2+\cdots+\cdots\leftrightarrow(\ldots)\leftrightarrow\boldsymbol{\delta\geq 0{,}53}$$

- $\boldsymbol{\pi^M=(5-2)*3=9}$
- $\boldsymbol{\pi^{Cournot}}$

$$\max_{q1}\pi_1=(8-q1-q2)q1-2q1$$

$$\pi_1=P(q_1,q_2)q_1-c_1q_1=(8-q_1-q_2)q_1-2q_1$$

$$FOC\colon\frac{d\pi_1}{dq_1}=0\Leftrightarrow 8-2q_1-q_2-2=0\Leftrightarrow\boldsymbol{q_1=3-\frac{1}{2}q_2}\ \ (\boldsymbol{BR_1})$$

$since\ c_1=c_2, by\ symmetry\rightarrow\ \boldsymbol{q_1=q_2=2\ \wedge P^*=4\wedge\pi=(4-2)\times 2=4}$

- $\boldsymbol{\pi^{Deviation}}$

If Firm 1 deviates, it will select the quantity that constitutes its best response given that Firm 2 continues to produce the collusive quantity ($q_2\ =\ 1{,}5$).

$$\boldsymbol{q_1=3-\frac{1}{2}\times 1{,}5=2{,}25\wedge q_2=1{,}5\wedge P=4{,}25}$$

$\boldsymbol{\pi^{Deviation}=(4{,}25-2)\times 2{,}25=5{,}0625}$

(iv)

Each domestic firm will gain $\boldsymbol{\frac{\pi^M}{2}=4{,}5.}$

The importer will gain $3\times 2=6$.

(v)

Under this cartel agreement, each firm will produce $q_i=\frac{4}{3}$ and gain $\pi_i=(6-2)\times\frac{4}{3}=\frac{16}{3}\approx 5{,}3$.

Comparing the 2 situations:

Scenario	Importer's Quantity	Price	Importer's Profit
A – Domestic collusion only	2	5	6
B – Equitable cartel	4⁄3	6	5,33

No, the importer will not accept joining the cartel.

By staying outside the agreement while the domestic firms collude, the importer earns **higher profits (6)** than it would as part of an equitable cartel (approx. **5,33**). Joining requires it to cut output, which outweighs the benefit of a higher price. Therefore, it has no incentive to accept the proposal.

(vi)

Yes, the importer free-rides and imposes a negative externality on the domestic firms. By remaining outside the collusive agreement, the importer maintains a higher output level and benefits from the elevated market price resulting from the domestic firms' collusion. As a result, the foreign firm earns a profit of 6, compared to 5.33 under an equitable cartel. This behavior reduces the profits of the domestic firms relative to the fully collusive outcome. Specifically, each domestic firm earns 4.5 instead of 5.33, incurring a loss of 0.83 due to the importer's free riding.