Advanced Microeconomics

Spring 2025
Final Exam

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1. You have a total of 120 minutes (2 hours) to solve the exam.
2. The use of calculators is not allowed.
3. If you need additional space to answer a question, you can use the back *of the same page*.

Read each question carefully. Good luck!

I (4.5 points)

In a pure exchange economy there are two goods (quantities x and y) and two agents, Anne (A) and Beth (B). Anne owns eight units of the x-good and none of the y-good. Beth owns none of the x-good, and three units of the y-good.
Their preferences are described by the utility functions $U_A = x_A y_A$ and $U_B = y_B + \ln x_B$.

a. (1.5 points) Find the expression for the contract curve. Explain your steps and reasoning clearly.

$MRS_A = MRS_B$
$\Leftrightarrow \frac{y_A}{x_A} = \frac{1}{x_B}$
$\Leftrightarrow y_A = \frac{x_A}{x_B}$
$\Leftrightarrow y_A = \frac{x_A}{8 - x_A}$

Expression is only valid for x_A between 0 and 6, because y_A only takes values between 0 and 6 and $y_A = 0 \Rightarrow x_A = 0$, $y_A = 3 \Rightarrow x_A = 6$

Contract curve:

$$\begin{cases} y_A = \frac{x_A}{8 - x_A}, x_A \in [0,6[\\ y_A = 3, x_A \in [6,8] \end{cases}$$

Grading: 0.4 for $MRS_A = MRS_B$, 0.4 for the expressions of the marginal rates of substitution, 0.1 for market clearing condition, 0.2 for the 1st part of the contract curve, 0.2 for writing the domain of the 1st part of the contract curve, 0.2 for the full contract curve.

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b. (2 points) Find the equilibrium price ratio and allocation in this economy.

Find demands:

$$Max\ U_A = x_A \times y_A$$
$$s.t. x_A \times P_x + y_A \times P_y = 8P_x$$
$$\Leftrightarrow \begin{cases} \dfrac{y_A}{x_A} = \dfrac{P_x}{P_y} \\ y_A \times P_y = 8P_x - x_A P_x \end{cases}$$
$$\Leftrightarrow \begin{cases} y_A = x_A \times \dfrac{P_x}{P_y} \\ - \end{cases}$$
$$\Leftrightarrow \begin{cases} - \\ x_A P_x = 8P_x - x_A P_x \end{cases}$$
$$\Leftrightarrow \begin{cases} - \\ 2x_A P_x = 8P_x \end{cases}$$
$$\Leftrightarrow \begin{cases} - \\ x_A = 4 \end{cases}$$
$$\Leftrightarrow \begin{cases} y_A = 4\dfrac{P_x}{P_y} \\ x_A = 4 \end{cases}$$

$$Max\ U_B = y_B + \ln(x_B)$$
$$s.t. x_B \times P_x + y_B \times P_y = 3P_y$$
$$\Leftrightarrow \begin{cases} \dfrac{1}{x_B} = \dfrac{P_x}{P_y} \\ x_B \times P_x = 3P_y - y_B P_y \end{cases}$$
$$\Leftrightarrow \begin{cases} x_B = \dfrac{P_y}{P_x} \\ - \end{cases}$$
$$\Leftrightarrow \begin{cases} - \\ P_y = 3P_y - y_B P_y \end{cases}$$
$$\Leftrightarrow \begin{cases} - \\ 2x_A P_x = 8P_x \end{cases}$$
$$\Leftrightarrow \begin{cases} x_B = \dfrac{P_y}{P_x} \\ y_B = 2 \end{cases}$$

Market clearing conditions:

$$x_A + x_B = 8 \Leftrightarrow x_B = 8 - 4 = 4 \Rightarrow \frac{P_y}{P_x} = 4 \Leftrightarrow \frac{P_x}{P_y} = \frac{1}{4}$$
$$y_A + y_B = 3 \Leftrightarrow y_A = 3 - 2 = 1$$

Equilibrium: $(x_A, y_A, x_B, y_B, \frac{P_x}{P_y}) = (4,1,4,2,\frac{1}{4})$

Grading: 1 for the demands (0.5 for each), 0.4 for market clearing condition(s), 0.3 for equilibrium price ratio, 0.3 for equilibrium allocation.

c. (1 point) Can the allocation $x_A = 7$, $y_A = 3$ be achieved in equilibrium? Explain why or why not.

Yes. It is efficient (included in the contract curve), and agents' preferences are monotonic and convex, so we can apply the 2nd welfare theorem, which states that all efficient points can be achieved in equilibrium by redistributing the initial endowment, provided that agents' preferences are monotonic and convex.

Grading: 0.3 for recognizing that the allocation is efficient because it is on the contract curve, 0.3 for naming the welfare theorem that applies, 0.3 for stating the conditions to apply the theorem, 0.1 for concluding.

II (2 points)

A game has two players but player 2 can be of type A (with probability 0<p<1) or type B (and player 1 does not know player 2's type). The payoff matrices are:

Type A

1\2	L	R
U	(0,2)	(2,1)
D	(1,1)	(2,0)

Type B

1\2	L	R
U	(0,1)	(2,2)
D	(1,0)	(2,1)

Find all the *pure-strategy* Bayes-Nash equilibria.

If player 1 plays U, player 2 chooses L if type A (2 > 1) and chooses R if type B (1 < 2).

Then, player 2's best response to U is (L,R). In this case, player 1's expected payoffs are:

- From playing U: $p * 0 + (1 - p) * 2 = 2 - 2p$
- From playing D: $p * 1 + (1 - p) * 2 = 2 - p$

Since $0 < p < 1$, $2 - p > 2 - 2p$ for any value of p, and the best response of player 1 to (L,R) is D. In this way, there is no BNE where player 1 plays U.

If player 1 plays D, player 2 chooses L if type A (1 > 0) and chooses R if type B (0 < 1).

Then, player 2's best response to U is (L,R). Once again, player 1's expected payoffs are:

- From playing U: $p * 0 + (1 - p) * 2 = 2 - 2p$
- From playing D: $p * 1 + (1 - p) * 2 = 2 - p$

Since $0 < p < 1$, $2 - p > 2 - 2p$ for any value of p, and the best response of player 1 to (L,R) is D. In this way, there is a BNE characterized by [D, (L,R)].

Grading: 0.4 for 2's best response to U, 0.5 for 1's best response to (L,R), 0.1 for concluding no BNE exists where 1 plays U; 0.4 for 2's best response to D, 0.5 for 1's best response to (L,R), 0.1 for concluding that a BNE where 1 plays D and characterizing it.

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III (4.5 points)

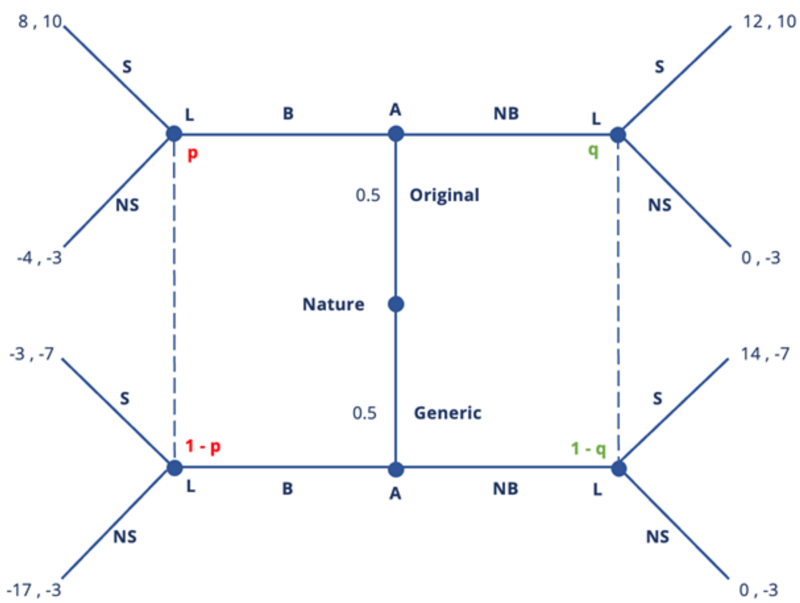
Lena (L) is a music producer looking to sign a new artist (A) to represent her country in the next Euromelody Song Contest. She cannot audition artists in person due to strict time constraints, but she knows that there are two types of aspiring performers: original (O) and generic (G), and that only 50% of the performers are original artists.

Lena must decide whether to sign (S) or not to sign (SN) an artist. Since Lena wants to maintain her reputation for unique and high-quality acts, she strongly prefers original performers. Signing a generic artist would hurt her brand, giving her a disutility of 7, while signing an original artist would greatly boost her reputation giving her a utility of 10. If she does not sign any artist, it would give her a disutility of 3 due to lost exposure and sponsorships.

On the other hand, the artists also get different payoffs according to their type. A generic artist gains 14 if they are signed due to commercial exposure, while an original artist gains only 12 if they are signed (less commercial exposure). Not being signed gives 0 utility to either type.

Artists can still attempt to signal their originality by giving an interview to Blitz, a very selective music publication. Denote by B the choice to give the interview (and by NB the choice not to give the interview). For original artists, it will be easier (but still costly) to achieve an interview, resulting in a disutility of 4. For generic artists, being interviewed would require bribing the journalists, incurring a disutility of 17. Lene observes whether there was an interview before making the decision to sign the artist.

a) (1.5 points) Represent this strategic interaction as a game in extensive form.



Grading: 0.9 for the structure, 0.6 for the payoffs.

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b) (2 points) Can the interview in Blitz produce an information signal in this game? Explain why, or why not.

If player A chooses (B,NB), player L's beliefs will be:

$$p = P(Original|B) = \frac{P(B|Original) * P(Original)}{P(B)} = \frac{1 * 0.5}{0.5} = 1$$

$$q = P(Original|BN) = \frac{P(NB|Original) * P(Original)}{P(NB)} = \frac{0 * 0.5}{0.5} = 0$$

Player L's best response given the previous beliefs (i.e., $p = 1$ and $q = 0$):

- After observing B:
  - S: $1 * 10 + 0 * (-7) = 10$
  - NS: $1 * (-3) + 0 * (-3) = -3$

- After observing NB:
  - S: $0 * 10 + 1 * (-7) = -7$
  - NS: $0 * (-3) + 1 * (-3) = -3$

Then, player L's best response is (S,NS).

Player A's best response to (S,NS) is to play B if the artist is original (8 > 0), and to play NB if the player is generic (-3 < 0).

Since there is a separating PBE, [(B,NB) , (S,NS) , $p = 1$, $q = 0$], the interview in Blitz can produce an informative signal in this game.

Grading: 0.1 for recognizing the strategy as (B,NB), 0.2 for each of Player L's beliefs, 0.25 for an application of Bayes rule, 0.15 for Player L's best response after observing B, 0.15 for Player L's best response after observing NB, 0.15 for original artist's best response, 0.15 for the generic artist's best response, 0.15 for concluding that there is PBE, 0.25 for describing the PBE, 0.25 for concluding that giving the interview constitutes an informative signal.

c) (1 point) Is there a PBE where both types choose to give the interview?

If player A chooses (B,B), player L's beliefs will be:

$$p = P(Original|B) = \frac{P(B|Original) * P(Original)}{P(B)} = \frac{1 * 0.5}{1} = 0.5$$

q is free, i.e. $q \in [0,1]$

Player L's best response given the previous beliefs (i.e., $p = 0.5$ and $q \in [0,1]$):

- After observing B:
  - S: $0.5 * 10 + 0.5 * (-7) = 1.5$

- NS: $0.5 * (-3) + 0.5 * (-3) = -3$

- After observing NB:
  - S: $q * 10 + (1 - q) * (-7) = 17q - 7$
  - NS: $q * (-3) + (1 - q) * (-3) = -3$

Then, player L's best response is (S,NS) if $q \leq \frac{4}{17}$, and (S,S) if $q \geq \frac{4}{17}$.

Player A's best response to (S,NS) is to play B if the artist is original (8 > 0), and to play NB if the player is generic (-3 < 0).

Player A's best response to (S,S) is to play NB if the artist is original (12 > 8), and to play NB if the player is generic (14 > -3).

There is no PBE where both types choose to give the interview.

Grading: 0.1 for recognizing the strategy as (B,B), 0.15 for each of Player L's beliefs, 0.2 for Player's L best response if $q \leq \frac{4}{17}$ or if $q \geq \frac{4}{17}$, 0.2 for the best response of Player A to (S,NS) and concluding there is no PBE, 0.2 for the best response of Player A to (S,S) and concluding there is no PBE.

IV (4.5 points)

Director Emma just hired John for her upcoming movie. John can either slack off during rehearsals ($e = 0$) or put a lot of effort into it ($e = 1$).
John's utility is given by the function $U(w, e) = \sqrt{w-2} - 2e$, where w denotes the wage he receives in millions of dollars. Assume that John has a reservation utility of 1.

Emma only cares about the net profit resulting from the movie, given by $\pi = x - w$, where x denotes the revenue in millions of dollars generated by the movie. It can be either a blockbuster, generating 100 million dollars in revenue, or a flop, generating 40 million dollars in revenue. The probability of each outcome is contingent on the level of effort chosen by John, given by:

Movie Performance	$e = 0$	$e = 1$
Blockbuster	1/2	3/4
Flop	1/2	1/4

a. (1.5 points) What would be John's certainty equivalent for a lottery that gives 18 with probability ½ and 6 with probability ½? Based on this, what can you conclude about John's attitude towards risk?

$$E[U_{Lot}] = \frac{1}{2}\left(\sqrt{18-2}\right) + \frac{1}{2}\left(\sqrt{6-2}\right) = 3$$

$$U(CE) = E[U_{Lot}] \Leftrightarrow \sqrt{CE-2} = 3 \Leftrightarrow CE = 11$$

$$E[Lot] = \frac{1}{2} * 18 + \frac{1}{2} * 6 = 12$$

Since $CE < E[Lot]$, we can conclude that John is a risk-averse agent.

Grading: 0.25 for the expected utility of the lottery, 0.75 for the CE, 0.25 for the expected value of the lottery (or the risk premium), 0.25 for the conclusion.

b. (1.5 points) What effort level would Emma want to implement if she could observe it?

If $e = 0$:

$$U_J(w_L, e = 0) = \sqrt{w_L - 2} - 0 = \sqrt{w_L - 2}$$

In order for John to accept this contract, the level of utility must be above his reservation level:

$\sqrt{w_L - 2} \geq 1 \Leftrightarrow w_L \geq 3 \rightarrow$ To minimize costs, Emma will set w_L as low as possible, i.e., $w_L = 3$.

$$E[\pi_E | e = 0] = \frac{1}{2}(100) + \frac{1}{2}(40) - 3 = 67$$

If $e = 1$:

$$U_J(w_H, e = 1) = \sqrt{w_H - 2} - 2$$

In order for John to accept this contract, the level of utility must be above his reservation level:

$\sqrt{w_H - 2} - 2 \geq 1 \Leftrightarrow w_H \geq 11 \rightarrow$ To minimize costs, Emma will set w_H as low as possible, i.e., $w_H = 11$.

$$E[\pi_E|e = 1] = \frac{3}{4}(100) + \frac{1}{4}(40) - 11 = 74$$

Since $E[\pi_E|e = 1] > E[\pi_E|e = 0]$, Emma would want to implement high effort.

Grading: 0.6 for low effort wage and expected profit, 0.6 for high effort wage and expected profit, 0.3 for the conclusion.

c. (1.5 points) Assuming that Emma cannot observe John's effort level, find the contract she would offer if she wanted to implement high effort.

If Emma cannot observe John's effort level, she will set one wage for each type of movie:

$w_B \rightarrow$ Wage if the film is a blockbuster
$w_F \rightarrow$ Wage if the film is a flop

In this setting:

$$E\left[U_J\middle|e = 1\right] = \tfrac{3}{4}\left(\sqrt{w_B - 2} - 2\right) + \tfrac{1}{4}\left(\sqrt{w_F - 2} - 2\right) = \tfrac{3}{4}\sqrt{w_B - 2} + \tfrac{1}{4}\sqrt{w_F - 2} - 2$$

$$E\left[U_J\middle|e = 0\right] = \tfrac{1}{2}\sqrt{w_B - 2} + \tfrac{1}{2}\sqrt{w_F - 2}$$

If Emma would want John to implement high effort, then:

$$E\left[U_J\middle|e = 1\right] \geq E\left[U_J\middle|e = 0\right] \Leftrightarrow \tfrac{3}{4}\sqrt{w_B - 2} + \tfrac{1}{4}\sqrt{w_F - 2} - 2 \geq \tfrac{1}{2}\sqrt{w_B - 2} + \tfrac{1}{2}\sqrt{w_F - 2} \Leftrightarrow \tfrac{1}{4}\sqrt{w_B - 2} - \tfrac{1}{4}\sqrt{w_F - 2} \geq 2$$

To minimize costs, Emma would set w_F as low as possible, i.e., $w_F = 2$.

Then:

$$\frac{1}{4}\sqrt{w_B - 2} \geq 2 \Leftrightarrow w_B \geq 66$$

To minimize costs, Emma would set w_B as low as possible, i.e., $w_B = 66$.

If Emma wanted to implement high effort, she would offer $w_B = 66$ and $w_F = 2$.

Grading: 0.3 for the expression of $E\left[U_J\middle|e = 1\right]$, 0.3 for the expression of $E\left[U_J\middle|e = 0\right]$, 0.5 for the result of $E\left[U_J\middle|e = 1\right] > E\left[U_J\middle|e = 0\right]$, 0.2 for the value of w_F, 0.2 for the value of w_B.

V (4.5 points)

Valentina owns the exclusive rights to operate a ride-hailing service in a large, congested city. Due to strict municipal regulations, only Valentina's platform is authorized to connect drivers to paying passengers. Independent drivers must obtain a contract from her company in order to legally operate and earn income.

Each driver generates €20 per hour of driving (h) in revenue for Valentina's platform, and in return, Valentina pays them a *total* transfer t for their work. However, drivers differ in their efficiency and fuel/maintenance costs, which depend on their driving habits and vehicle condition. Each driver's cost of providing *h* hours of service is given by $C(h) = \gamma h^2$, where either $\gamma_L=1$ or $\gamma_H=2$. Drivers are risk neutral, and their reservation utility is zero (due to high unemployment and the lack of other legal earning opportunities).

a) (1.5 points) Assume Valentina can observe a driver's type. Derive and explain the contracts (h, t) she could offer to each type.

> Valentina solves max $20h - t$ *s.t.* $t - \gamma h^2 \geq 0$.
> At the optimum we will have $t = \gamma h^2$.
> Therefore, Valentina's simplified problem is max $20h - \gamma h^2$ yielding the FOC $20 = 2\gamma h$ and therefore $h = 10/\gamma$ and $t = 100/\gamma$.
> The optimal contracts will be ($h_L = 10$, $t_L = 100$) and ($h_H = 5$, $t_H = 50$).
>
> *Grading: 0.5 for setting up the problem, 0.5 for solving it, 0.5 for conclusion.*

b) (3 points) Now assume she cannot observe types, but she believes that 2/3 of drivers are of type H. Find the menu of contracts that make drivers accept and reveal their type. Which type would get an informational rent?

> Valentina now solves max $(1/3).(20h_L - t_L) + (2/3).(20h_H - t_H)$
> *s.t.*
> (IC_L) $t_L - h_L^2 \geq t_H - h_H^2$
> (IC_H) $t_H - 2h_L^2 \geq t_L - 2h_L^2$
> (IR_L) $t_L - h_L^2 \geq 0$
> (IR_H) $t_H - 2h_H^2 \geq 0$
>
> We can show that IC_L and IR_H will bind (and imply IR_L)
> Therefore at the optimum $t_H = 2h_H^2$ and $t_L = h_L^2 + h_H^2$
> Replacing in the objective function we have:
> max max $(1/3).(20h_L - h_L^2 - h_H^2) + (2/3).(20h_H - 2h_H^2)$ yielding $h_L = 10$ and $h_H = 4$.
> Moreover, $t_L = 116$ and $t_H = 32$.
>
> There is an informational rent of 16 for type L.
>
> *Grading: 1 for setting up the problem correctly, 0.75 for indicating binding constraints (including 0.5 for the proofs), 1 for solving the problem and 0.25 for the indication of the informational rent.*

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