Microeconomics II

Spring 2025

Final Exam

Paulo P. Côrte-Real

You have a total of 120 minutes (2 hours) to solve the exam.
Identify each sheet with your Student Number and Name.
Good luck!

I (5 points)

Air Shangri-La is the only airline allowed to fly between the islands of Shangri-La and Nirvana. There are two types of passengers, tourist and business. Business travelers are willing to pay more than tourists. The airline, however, cannot tell directly whether a ticket purchaser is a tourist or a business traveler. The two types do differ, though, in how much they are willing to pay to avoid having to purchase their tickets in advance. (Passengers do not like to commit themselves in advance to traveling at a particular time).

More specifically, the utility levels of each of the two types net of the price of the ticket, P, for any given amount of time W prior to the flight that the ticket is purchased are given by:

Business: $v - \theta_B P - W$
Tourist: $v - \theta_T P - W$
Where $\theta_T > \theta_B > 0$

(Note that for any given level of W, the business traveler is willing to pay more for her ticket. Also, the business traveler is willing to pay more for any given reduction in W).

The proportion of travelers who are tourists is λ. Assume that the cost of transporting each passenger is c.

Assume in a) to d) that Air Shangri-La wants to carry both types of passengers.

a) Draw the indifference curves of the two types in the (P,W)-space. Draw the airline's isoprofit curves. Now formulate mathematically the optimal (profit-maximizing) price discrimination problem that Air Shangri-La would want to solve (Hint: Impose nonnegativity of prices as a constraint since, if it charged a negative price, it would sell an infinite amount of tickets at this price).
b) Show that in the optimal solution, tourists are indifferent between buying a ticket and not going at all.
c) Show that in the optimal solution, business travelers never buy their ticket prior to the flight and are just indifferent between doing this and buying when tourists buy.
d) Describe fully the optimal price discrimination scheme under the assumption that they sell to both types. How does it depend on the underlying parameters λ, θ_T, θ_B and c?
e) Under what circumstances will Air Shangri-La choose to serve only business travelers?

II (5 points)

A firm is hiring a worker whose labor disutility k ∈ $[\underline{k},\overline{k}]$ is exogenous and observable to the individual but not to the firm. The cumulative density function (*F*) of k is common knowledge and the associated probability density function *f* takes strictly positive values in $[\underline{k},\overline{k}]$. Assume that *F/f* is strictly increasing in *k*.

The firm cannot force participation and must offer type-contingent contracts specifying an extra payment (bonus) *t* and a workload *l*. Let *w* denote the fixed wage rate; the worker's income will be *w.l + t*. The worker's utility is quasilinear in income: *u*(*t, l, k*) = *t + wl – k.h*(*l*), where *h* is strictly increasing and strictly convex.

a. Assuming that the goal of the firm is to minimize the bonus, design the optimal type-contingent contract.
b. Compare the answer in a. with the first-best scenario.

III (5.5 points)

A seller wants to auction an object online. Knowing that she will face three risk-neutral buyers with valuations that are independently drawn from the uniform distribution on [0; 2], and knowing that the website uses second-price sealed-bid auctions only, the seller is considering the possibility of placing a secret bid as well.

(a) What is the value of the secret bid p_s that maximizes the seller's expected revenue? For that value for p_s, is the mechanism optimal? Is it efficient?
(b) Suppose that the website decides to switch to first-price sealed-bid auctions. Will her expected revenue be the same? Would your answer change if the buyers were risk-averse? (no additional calculations are required for this question.)

IV (4.5 points)

Let X be the set of alternatives with |X|> 2 and let each agent have any strict preference ordering over the
alternatives.

a) Show that a social choice function that is strategy-proof must satisfy the Pareto property.
b) Consider the social welfare functional that assigns to any profile of individual preference orderings (R_1, . . . , R_n) the social preference relation R which is defined as follows:
Let x and y be two alternatives.
Then, x R y $\Longleftrightarrow$ *x* R_i *y for at least one individual i* ∈ *{1, . . . , n}.*
Does this violate any of the Arrow theorem assumptions?