Problem Set 2

1. MWG 14.B.6

2. Assume you are in love and are about to get into a relationship with the person you are in love with. You know you will derive benefits from your partner's displays of affection x_O according to the function $B_I(x_O)$ but your displays of affection will cost you $C_I(x_I)$ and give your partner benefits $B_O(x_I)$. Assume that $B_I'>0$ and $B_I''<0$ and $B_O'>0$ and $B_O''<0$, and $C_I'>0$ and $C_I''>0$.

   Your partner's feelings can be true (θ_T) with probability p or untrue (θ_U) with probability 1-p. The costs $C_O(x_O, \theta_i)$ of displaying affection are lower if he/she actually feels that affection i.e. for all $x_O>0$, $C_O(x_O, \theta_T)< C_O(x_O, \theta_U)$ and $C_O'(x_O, \theta_T) < C_O'(x_O, \theta_U)$.
   [Assume also that every function mentioned above yields 0 for x equal to 0).

   Your future partner is very easygoing and will enter a relationship as long as the net benefit is non-negative, regardless of type. You are definitely willing to enter the relationship: you are in love and think your reservation utility is infinitely negative, but you want to make the most of it by proposing a certain type of relationship (defined by the two levels of displays of affection).

   a) Assume your partner's feelings are observable. Determine (and represent in a graph) the contracts you will offer for each type.
   b) More realistically, you cannot observe your partner's feelings. Design (and represent in a graph) the menu of relationship contracts you will propose. Which type gets an informational rent? For which type will there be efficiency?

3. The government would like to reduce pollution and is announcing a subsidy program to reward firms that invest in "clean" technologies. If x stands for the firm's pollution reduction, the corresponding cost is $\theta . x^2$, where θ is firm-specific and $\theta \in [1,2]$.
   The subsidy program specifies investment level x and a transfer t that the firm will receive. Firm participation in the subsidy program is voluntary (and therefore, the subsidy needs to at least cover the costs).
   Both the government and the firm are risk-neutral. A firm's utility from accepting the subsidy scheme is $t - \theta . x^2$. Since the government would like to reduce pollution, the government's objective function includes the term $2x$. On the other hand, the government prefers to pay the lowest possible subsidy to the firm and a payment of t implies a disutility to the government of αt, where $\alpha \in [0,1]$.. Therefore, the government's objective function is $2x - \alpha t$.

   a. Calculate the first-best optimal levels $x^*(\theta)$ and $t^*(\theta)$.
   b. Assume that the government does not observe θ but knows that θ follows a uniform distribution on [1; 2], i.e., f (θ) = 1, F (θ) = $\theta - 1$. Find the optimal subsidy policy for the government $\{x(\theta), t(\theta) | \theta \in [1,2]\}$.
   c. It may be preferable to exclude firms with very high costs from the subsidy program. The government can design a menu $\{x(\theta), t(\theta) | \theta \in [1, \overline{\theta}]\}$, where $\overline{\theta}$ is a parameter between 1 and 2. Calculate the optimal menu, fixing $\overline{\theta}$. Prove that, in this example, it is optimal for the government to design a menu intended for all firm types (i.e., $\overline{\theta}$ = 2).

4. FT 7.2

5. Each of N agents have a project that needs funding. The value each agent places on funding is $\mu_i \sim F$ on $[0,1]$. The Ministry of Science wants to fund the most worthwhile project, but cannot observe the realizations of μ_i.
   Agents write proposals that are time consuming: an agent who spends time t_i on a proposal gains utility $u_i(\mu_i) = P_i\mu_i - t_i$, where the project is funded with probability P_i. The Ministry of Science can only observe the time t_i each agent spends writing their proposal. Assume that the Ministry is worried about the upcoming election and the aim is to maximise the sum of the utilities of the agents.

   a. Specify the problem and characterise the agent's utility under incentive compatibility in terms of an integral equation and a monotonicity constraint.
   b. Suppose $\frac{1-F(x)}{f(x)}$ is strictly decreasing in x. Show that the Ministry of Science's optimal policy is to allocate the grant randomly (Hint: start by deriving the optimal policy ignoring the monotonicity constraint).

6. Consider a sealed bid *all-pay* auction with I symmetric buyers. Each buyer's valuation θ_i is independently drawn from the interval [0; 1] according to a uniform density i.e. $\theta_i \sim U\ [0; 1]$. The mechanism is as follows: Every buyer submits a non-negative bid, the highest bidder receives the good, and every buyer pays the seller the amount of her bid regardless of whether she wins.
   a) Show that it is a symmetric Bayesian Nash equilibrium for each bidder to bid according to $b(\theta_i) = \frac{I-1}{I}\theta_i^I$.
   b) Assume that this is the unique Bayesian Nash equilibrium of the game. Argue, using the revenue equivalence theorem, that the auction yields the same expected revenue as the sealed-bid second price auction.
   c) Assume now that I = 3 and that θ_1 is uniformly distributed on [0,10], and θ_2 and θ_3 are uniformly distributed on [0,1]. Design an optimal auction. Explain how you might implement it by allowing asymmetric reserve bids in one of the standard auctions (sealed bid first-price, Vickrey…).
   d) Suppose instead that it is known that bidder 1 has the highest valuation: say θ_1 is uniformly distributed on [1,11], and θ_2 and θ_3 are still uniformly distributed on [0,1]. What does the optimal auction look like in this case?