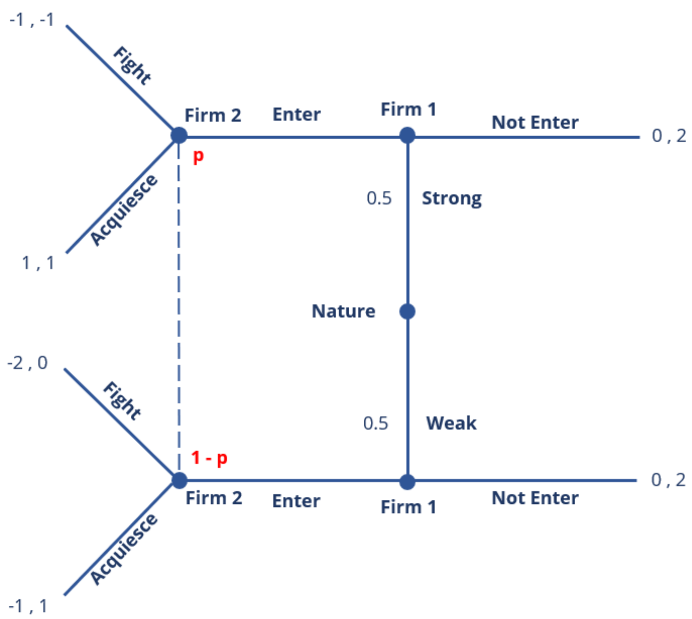
1. Consider the classical entry game. The entrant (Firm 1) may be Strong or Weak with probabilities $p = 0.5$ each. If F1 decides not to enter, the Entrant and Incumbent (Firm 2) get (0,2) as payoffs. If Firm 1 decides to enter, then Firm 2 can Fight or Acquiesce. If F2 decides to fight, and F1 is Strong, then the payoffs are (-1,-1), while if F2 decides to acquiesce they get (1,1). On the other hand, if F1 is weak, then fighting gives payoffs (-2,0) and acquiescing leads to (-1,1).

Find the PBE

Before we starting to solve the exercise, it might be useful to draw the game tree associated with this question:



And it will also be useful if we list the strategies of each firm:

- **Firm 1's strategies:** (Enter, Enter), (Enter, Not Enter), (Not Enter, Enter), (Not Enter, Not Enter) [1]
- **Firm 2's strategies:** Fight, Acquiesce

Is there a PBE where Firm 1 plays (Enter, Not Enter)?

In this case, the belief p of Firm 2 will be:

$$p = P(Strong|Enter) = \frac{P(Enter|Strong) \cdot P(Strong)}{P(Enter)} = \frac{1 \cdot 0.5}{0.5} = 1$$

With the belief that $p = 1$, the expected payoffs of Firm 2 if Firm 1 plays (Enter, Not Enter) will be:

After observing Enter:

- **Fight:** $1 \cdot (-1) + 0 \cdot 0 = -1$

[1] The first action of each strategy represents what Firm 1 plays if it is Strong, while the second action of each strategy represents what Firm 1 plays if it Weak.

- **Acquiesce:** $1 \cdot 1 + 0 \cdot 1 = 1$

Since $1 > -1$, Firm 2 chooses to play Acquiesce. So, the best response of Firm 2 to (Enter, Not Enter), given the belief that $p = 1$, is Acquiesce.

Going back to Firm 1, we can see that if Firm 2 plays Acquiesce then Firm 1 will choose Enter if Strong (1>0) and Not Enter if Weak (0>-1). So we can conclude that Firm 1 will wish to play (Enter, Not Enter) if Firm 2 is playing Acquiesce.

Therefore, we conclude that [(Enter, Not Enter), Acquiesce, $p = 1$] is a (separating) PBE.

Is there a PBE where Firm 1 plays (Not Enter, Enter)?

In this case, the belief p of Firm 2 will be:

$$p = P(Strong|Enter) = \frac{P(Enter|Strong) \cdot P(Strong)}{P(Enter)} = \frac{0 \cdot 0.5}{0.5} = 0$$

With the belief that $p = 0$, the expected payoffs of Firm 2 if Firm 1 plays (Not Enter, Enter) will be:

After observing Enter:

- **Fight:** $0 \cdot (-1) + 1 \cdot 0 = 0$
- **Acquiesce:** $0 \cdot 1 + 1 \cdot 1 = 1$

Since $1 > 0$, Firm 2 chooses to play Acquiesce. So, the best response of Firm 2 to (Not Enter, Enter), given the belief that $p = 0$, is Acquiesce.

Going back to Firm 1, we can see that if Firm 2 plays Acquiesce then Firm 1 will choose Enter if Strong (1>0) and Not Enter if Weak (0>-1). So we can conclude that Firm 1 will wish to play (Enter, Not Enter) if Firm 2 is playing Acquiesce.

Therefore, we conclude that there is no (separating) PBE where Firm 1 plays (Not Enter, Enter).

Is there a PBE where Firm 1 plays (Enter, Enter)?

In this case, the belief p of Firm 2 will be:

$$p = P(Strong|Enter) = \frac{P(Enter|Strong) \cdot P(Strong)}{P(Enter)} = \frac{1 \cdot 0.5}{1} = 0.5$$

With the belief that $p = 0.5$, the expected payoffs of Firm 2 if Firm 1 plays (Enter, Enter) will be:

After observing Enter:

- **Fight:** $0.5 \cdot (-1) + 0.5 \cdot 0 = -0.5$
- **Acquiesce:** $0.5 \cdot 1 + 0.5 \cdot 1 = 1$

Since $1 > 0.5$, Firm 2 chooses to play Acquiesce. So, the best response of Firm 2 to (Enter, Enter), given the belief that $p = 0.5$, is Acquiesce.

Going back to Firm 1, we can see that if Firm 2 plays Acquiesce then Firm 1 will choose Enter if Strong (1>0) and Not Enter if Weak (0>-1). So we can conclude that Firm 1 will wish to play (Enter, Not Enter) if Firm 2 is playing Acquiesce.

Therefore, we conclude that there is no (pooling) PBE where Firm 1 plays (Enter, Enter).

Is there a PBE where Firm 1 plays (Not Enter, Not Enter)?

The definition of the belief p stays the same:

$$p = P(Strong|Enter)$$

However, given that now no type of Firm 1 chooses Enter, $P(Enter) = 0$ and the information set associated with p (which would be the one where Firm 2 observes Enter being chosen) is off the equilibrium path. Therefore, we are unable to use the Bayes' Rule and p is free. In other words, p can take any value between 0 and 1.

With the belief that $p \in [0, 1]$, the expected payoffs of Firm 2 if Firm 1 plays (Not Enter, Not Enter) will be:

After observing Enter:

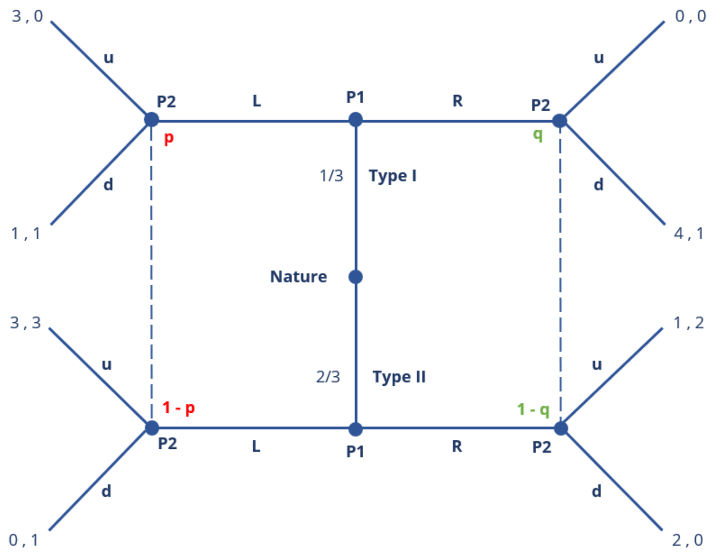
- **Fight:** $p \cdot (-1) + (1 - p) \cdot 0 = -p$
- **Acquiesce:** $p \cdot 1 + (1 - p) \cdot 1 = 1$

Now we can not that given $p \in [0, 1]$, it will always be the case that $1 > -p$, therefore the expected payoff of choosing Acquiesce is always greater than that of choosing Fight. Hence, Firm 2 chooses Acquiesce. Therefore, for a given belief p, Firm 2's best response to (Not Enter, Not Enter) is Acquiesce.

Going back to Firm 1, we can see that if Firm 2 plays Acquiesce then Firm 1 will choose Enter if Strong (1>0) and Not Enter if Weak (0>-1). So we can conclude that Firm 1 will wish to play (Enter, Not Enter) if Firm 2 is playing Acquiesce.

Therefore, we conclude that there is no (pooling) PBE where Firm 1 plays (Not Enter, Not Enter).

2. [PARTIAL SOLUTION, ONLY FOR STRATEGY (R,R)] Check if separating strategies [(L,R), (R,L)] and pooling strategies [(L,L), (R,R)] lead to PBE in the following game, and describe the equilibria:



Is there a PBE where Player 1 plays RR?

In this case, the beliefs p and q of Firm 2 will be:

$$p = P(TypeI|L) = \frac{P(L|TypeI) \cdot P(TypeI)}{P(L)}$$

However, given that now no type of Player 1 chooses L, $P(L) = 0$ and the information set associated with p is off the equilibrium path. Therefore, we are unable to use the Bayes' Rule and p is free. In other words, p can take any value between 0 and 1.

$$q = P(TypeI|R) = \frac{P(R|TypeI) \cdot P(TypeI)}{P(R)} = \frac{1 \cdot \frac{1}{3}}{1} = \frac{1}{3}$$

With the belief that $p \in [0, 1]$ and $q = \frac{1}{3}$, the expected payoffs of Firm 2 if Firm 1 plays (Not Enter, Not Enter) will be:

After observing L:

- u: $p \cdot 0 + (1 - p) \cdot 3 = 3 - 3p$
- d: $p \cdot 1 + (1 - p) \cdot 1 = 1$

So, Player 2 chooses u if $3 - 3p > 1 \leftrightarrow p < \frac{2}{3}$, and chooses d if $3 - 3p < 1 \leftrightarrow p > \frac{2}{3}$.

After observing R:

- u: $\frac{1}{3} \cdot 0 + \frac{2}{3} \cdot 2 = \frac{4}{3}$
- d: $\frac{1}{3} \cdot 1 + \frac{2}{3} \cdot 0 = \frac{1}{3}$

Since $\frac{4}{3} > \frac{1}{3}$, after observing R, player 2 will choose to play u.

Therefore, for a given belief p and for $q = \frac{1}{3}$, Player 2's best response to (R,R) is:

- uu if $p < \frac{2}{3}$
- du if $p < \frac{2}{3}$

Going back to Player 1, we can see that if Player 2 plays uu, then Player 1 will choose L if Type I (3>0) and L if Type II (3>1). So we can conclude that Player 1 will wish to play (L,L) if Firm 2 is playing uu.

Similarly, we can see that if Player 2 plays du, then Player 1 will choose L if Type I (1>0) and R if Type II (1>0). So we can conclude that Player 1 will wish to play (L,R) if Firm 2 is playing du.

Therefore, we conclude that there is no (pooling) PBE where Player 1 plays (R,R).