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Public Economics

Spring 2025
Midterm Exam

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You have a total of 80 minutes (2 hours) to solve the exam.
Read each question carefully. If you need additional space to write, please use the back of the same page. Good luck!

I (6 points)

Discuss the following propositions (max. 10 lines for each).

a. (2 points) In a representative democracy, lobbying improves efficiency in the provision of a public good.

In a representative democracy, elected officials may or may not represent the median voter's choice with respect to public good provision. Moreover, the median voter choice may or may not coincide with the efficient level due to concentration of costs and benefits. Lobbying might help convey information on intensity of preferences but it might improve or hinder efficiency, since it is generally promoted by a small number of economic agents.

Grading: 0.5 for the discussion of representative democracy issues and the connection between the median voter and efficiency and 1.5 for the discussion of the (positive or negative) impact of lobbying.

b. (2 points) In an economy with two agents and two goods, equal opportunities and free trade lead to an efficient and envy-free allocation.

True. If preferences are well-behaved, a walrasian equilibrium from equal division exists. The resulting allocation is efficient (1^{st} welfare theorem) and must be envy-free, since all agents maximize utility subject to the same budget constraint that includes equal division (i.e. they have equal opportunities).

Grading: 0.5 points for the connection of equal opportunities with equal division as the starting point, 0.5 for the mention of the walrasian equilibrium from equal division, 1 for the explanation of the efficiency and no-envy implication and for the conclusion.

c. (2 points) In order to address the danger of a nuclear leakage with uncertainty about the costs of reduction, switching from price intervention to quantity intervention would generate a Pareto improvement.

The marginal damage curve will be very steep for an externality such as a nuclear leakage. If there was no uncertainty about marginal costs of reduction, both price and quantity intervention would achieve efficiency. If, however, uncertainty is present, price intervention involves a greater deadweight loss than quantity intervention when the marginal damage curve is very steep. Still, moving from inefficiency to efficiency does not guarantee a Pareto improvement.

Grading: 0.5 points for the reasoning with respect to the slope of the marginal damage curve; 1 for the comparison of price and quantity intervention; 0.5 for the conclusion on Pareto improvement.

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II (4.5 points)

An economy has two agents and two goods. Utilities are given by $U_A = x_A + 4y_A$ and $U_B = x_B y_B$. The total amount of X in the economy is 16. The total amount of Y is 4.

a. (1.5 points) Using an Edgeworth box, find the set of Pareto efficient points.

Setting $MRS^a = MRS^b$, we get that efficient allocations will be such that: $y_a = \frac{x_a}{4}$

Grading: 1 point for identification and justification of efficient allocations. 0.5 graphical representation

b. (1.25 points) Find and draw the Utility Possibility Frontier for this economy.

From the set of efficient allocations and resource constraints, we have that $U_b = \frac{1}{4} * (16 - \frac{U_a}{2})^2$, or $U_B = \left(8 - \frac{U_A}{4}\right)^2$, for $U_A \in [0,32]$.

Grading: 1 point for calculation of UPF; 0.25 for drawing a convex UPF for the correct levels of U_A.

c. (1.75 points) Find the Utilitarian choice for this economy [if you have not solved b., assume that the UPF is given by $U_B = \left(8 - \frac{U_A}{4}\right)^2$]. Is it compatible with egalitarian equivalence?

We want to maximize $U_A + U_B$ s.t. $U_b = \left(8 - \frac{U_a}{4}\right)^2$. As UPF is convex, we obtain a minimum from the FOC, instead of a maximum.
The choice will be a corner solution:

$U_a = 0 : U_b = 64$
$U_b = 0 : U_a = 32$

Utilitarian allocation: $(x_a, y_a) = (0,0)$, $(x_b, y_b) = (16,4)$.
Leading to: $U_a = 0, U_b = 64$

This allocation is not egalitarian-equivalent: the indifference curve for agent B through (16,4) does not intersect with the indifference curve for agent 1 through (0,0), and therefore there is no reference bundle.

Grading: 0.5 for the formulation, 0.5 for the solution, 0.75 for the conclusion on egalitarian-equivalence (including 0.25 for the concept).

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III (4 points)

A new music festival venue was just established in *Music City*. Letting x denote the number of monthly music festival events, revenue from ticket sales is given by $200x$ and the total cost of organizing the festivals is x^2.
The owner of a hotel located next to the festival venue is now concerned about the effect of the noise from the festivals on the number of guests. Letting y denote the number of hotel rooms booked per month, the revenue for the hotel business is $250y - x * y$ and the total cost is y^2.

a. (1 point) Find the number of monthly music festivals and booked hotel rooms that will result from the market.

Festival solves:

$$\max_x \pi_F = 200x - x^2$$

FOC: $200 - 2x = 0 \Leftrightarrow x = 100$

Hotel solves:

$$\max_y \pi_H = 250y - xy - y^2$$

FOC: $250 - x - 2y = 0 \Leftrightarrow y = \frac{250-x}{2} \Leftrightarrow y = 75$

Grading: 0.5 for setting up the maximization problem, 0.5 for solution.

b. (1.75 points) Find the efficient number of monthly music festivals and booked hotel rooms.

We need to maximize total welfare by simultaneously maximizing the sum of profits:

$$\max_{x,y} \pi_F + \pi_H = 200x - x^2 + 250y - xy - y^{2\sim}$$

FOC:

$$\begin{cases} \frac{\partial \pi}{\partial x} = 0 \\ \frac{\partial \pi}{\partial y} = 0 \end{cases} \Leftrightarrow \begin{cases} 200 - y - 2x = 0 \\ 250 - x - 2y = 0 \end{cases} \Leftrightarrow \begin{cases} x^* = 50 \\ y^* = 100 \end{cases}$$

Grading: 1 for setting up the maximization problem, 0.75 for solution.

c. (1.25 points) Suppose that the local government of *Music City* wants to impose a Pigouvian tax on the music festival organizer. What should that tax be?

Pigouvian tax should equal the size of the externality at the social optimum.
The externality is caused by the festival to the hotel, per each music festival (x):

$$MEC = \frac{\partial \pi_H}{\partial x} = y$$

Thus, the Pigouvian tax is $t = MEC(y^* = 100) = 100$
Grading: 0.5 for understanding how the Pigouvian tax works, 0.75 for finding the MEC and solving.

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IV (5.5 points)

A small community is deciding how much to spend on maintaining a public library, which is a pure public good. There are three groups of residents, divided according to their demand functions for library spending. Individual demand functions in each group are given by:

- Group 1: P=9−0.3G
- Group 2: P=7−0.2G
- Group 3: P=5−0.1G

Each group has 10 individuals. The marginal cost of providing the public good is 150.

a. (1.5 points) Find the efficient level of public good provision.
Samuelson condition:
$\sum_i MB_i = MC$
$\Leftrightarrow 10(9 - 0.3\mathrm{G}) + 10(7 - 0.2\mathrm{G}) + 10(5 - 0.1\mathrm{G}) = 150$
$\Leftrightarrow \mathrm{G}^* = 10$

Grading: 0.5 for the Samuelson condition and 1 for the solution

b. (1 point) If the local government wanted to achieve a unanimous choice of the level you found in a., what tax-prices should it charge?

The tax that allows unanimity to be reach and for the public good to be fully funded is Lindahl taxation, where each agent is tax according to the marginal benefit it derives from the consumption of the public good at the socially optimal level:

$t_a = MB_a(G^* = 10) = 6$
$t_B = MB_b(G^* = 10) = 5$
$t_c = MB_c(G^* = 10) = 4$
Notice that $10 * t_a + 10 * t_b + 10 * t_c = 150$, meaning the public good is fully funded.

Grading: 0.25 for stating what Lindahl taxes are, 0.25 for finding each of the unitary taxes.

(1.5 points) Suppose the library is funded through an equal per-person tax. What level of library spending would defeat any other amount in a referendum? How does it compare to the one you found in a.?

An equal per-person tax that is able to fund the library would be: $t = \frac{MC}{30} = \frac{150}{30} = 5$
Since preferences over the public good are well-behaved and single-peaked, we can apply the Median Voter Theorem: the outcome of a referendum will be the median voter's choice, which in this case corresponds to Group 2.
At this tax level, the library spending chosen by Group 2 is: $7 - 0.2G = 5 \Leftrightarrow \mathrm{G}^* = 10$, which is the efficient level.

Grading: 0.25 for finding the tax rate, 0.25 for applying the median voter theorem, 1 for finding the spending level and stating it corresponds to the efficient level

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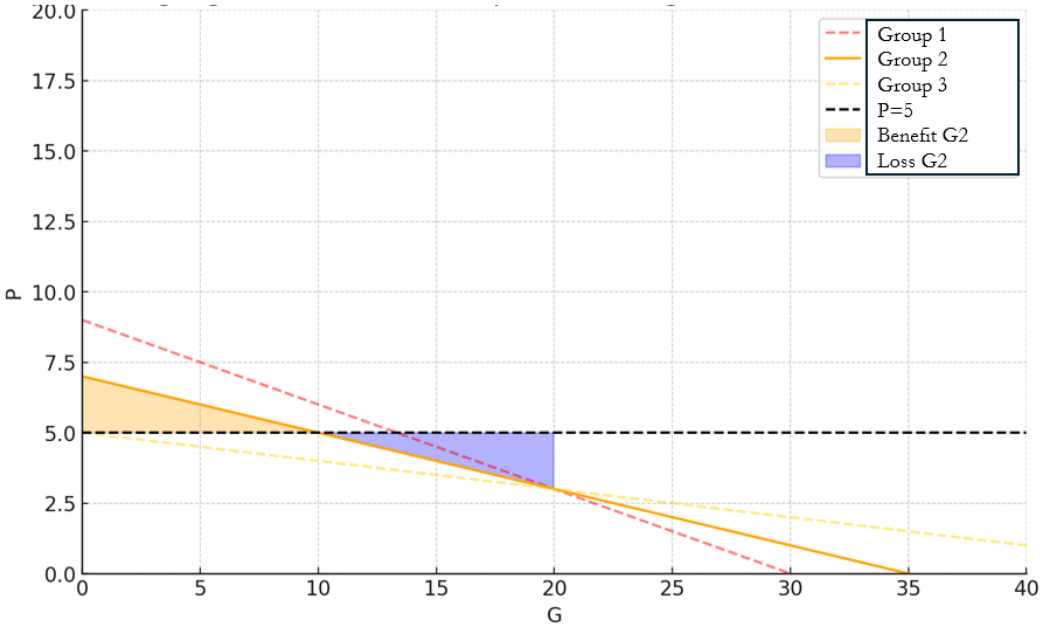
c. (1.5 points) Suppose that we still have equal per-person taxes, but the government official in charge of the library wants to maximize its budget and decides on the level of library spending. However, a majority of the community can hold a vote to decide to *eliminate* the library. What level of library spending will be chosen?

Local government will want to maximize the budget, which is given by 150G.
Since a majority can hold a vote to decide to eliminate the library, the government is constrained: it can only increase its budget up to the point at which the majority of people are at least as well off as in the case where no library is maintained.
Applying the Median Voter Theorem, this implies making sure that people in Group 2 are not worse off than in the case of no library.

Graphically, we can see that for people in Group 2, the optimal decision is G=10, at which point $MB_{priv} = MC_{priv}$ and their surplus is represented by the area in orange below. In order to determine the point at which people in Group 2 are indifferent to the case of no library, we increase G up until the surplus is 0 again, that is, the point where Benefit = Loss. Given the linearity of the MB curve, this occurs precisely at twice the optimal level of spending: $G^* = 2 * 10 = 20$



Alternatively, integrating the marginal benefit and marginal cost curve, we have that:

Government solves:

$$\max_G Budget_{Gov} = 150\text{G}\ s.t. 7G - 0.1G^2 \geq 5G$$

The government will set the highest level of library spending subject to $G \leq 20$, that is, $G^* = 20$.

Grading: 0.5 for formulation of the problem, 1 for the correct level of library spending.

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