Advanced Microeconomics

Midterm Exam - Spring 2025

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1. You have a total of 80 minutes (1 hour and 20 minutes) to solve the exam.
2. The use of calculators is not allowed.
3. If you need additional space to answer a question, use can use the back of the *same page*.

Read each question carefully. Good luck!

I. (5 points) In the pure exchange economy of Oz, there are two agents, Glinda (G) and Elphaba (E), with preferences over two goods given by:

$$U_G = x_G^{\frac{1}{3}} \cdot y_G^{\frac{1}{3}} \quad \text{and} \quad U_E = x_E$$

The initial endowment allocation is $(\omega_x^G, \omega_y^G, \omega_x^E, \omega_y^E)$=(1,1,1,3).

a) (2.5 points) Find the Walrasian equilibrium for this economy.

Demand for Glinda:

$U_G = x_G^{\frac{1}{3}} \cdot y_G^{\frac{1}{3}} \quad \text{and} \quad \omega_x^G = \omega_y^G = 1$

$\rightarrow$ **Income:** $p_x\omega_x^G + p_y\omega_y^G = p_x + p_y$

$\rightarrow$ **Share Rule:**

- $x_G^* = \frac{\alpha}{\alpha+\beta} \cdot \frac{Income}{p_x} \Leftrightarrow x_G^* = \frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{3}} \cdot \frac{p_x+p_y}{p_x} \Leftrightarrow x_G^* = \frac{1}{2}\frac{p_x+p_y}{p_x}$
- $y_G^* = \frac{\alpha}{\alpha+\beta} \cdot \frac{Income}{p_y} \Leftrightarrow y_G^* = \frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{3}} \cdot \frac{p_x+p_y}{p_y} \Leftrightarrow y_G^* = \frac{1}{2}\frac{p_x+p_y}{p_y}$

Demand for Elphaba:

$U_E = x_E \quad \text{and} \quad \omega_x^E = 1, \omega_y^E = 3$

$\rightarrow$ **Income:** $p_x\omega_x^E + p_y\omega_y^E = p_x + 3p_y$

$\rightarrow$ Elphaba only values x, so she will spend all her income on x:

- $x_E^* = \frac{p_x+3p_y}{p_x}$
- $y_E^* = 0$

Market clearing conditions:

$X = 2$ and $Y = 4$

$x_E + x_G = 2 \Leftrightarrow \frac{p_x+3p_y}{p_x} + \frac{p_x+p_y}{p_x} = 2 \Leftrightarrow (\frac{p_x}{p_y})^* = 7$

OR

$y_E + y_G = 4 \Leftrightarrow 0 + \frac{1}{2}\frac{p_x+p_y}{p_y} = 4 \Leftrightarrow (\frac{p_x}{p_y})^* = 7$

Equilibrium allocations:

- $x_G^* = \frac{1}{2}\frac{p_x+p_y}{p_x} = \frac{4}{7}$
- $y_G^* = \frac{1}{2}\frac{p_x+p_y}{p_y} = 4$
- $x_E^* = \frac{p_x+3p_y}{p_x} = \frac{10}{7}$
- $y_E^* = 0$

Grading:

- 0.75 points for the demand for Glinda.
- 0.75 points for the demand for Elphaba.
- 0.75 points for the equilibrium price ratio.
- 0.25 points for the equilibrium allocations.

b) (1.25 points) Is the equilibrium allocation in the core? Explain why or why not.

The core is the intersection between the mutual advantages set and the set of (Pareto) efficient points/contract curve.

Given that agents' preferences are (weakly) monotonic, we can apply the 1st Welfare Theorem, that states that equilibrium allocations are (Pareto) efficient. Thus the equilibrium is efficient.

The mutual advantages set includes all allocations that are Pareto superior to the initial endowment (i.e., that yield the agents a higher utility).

For Glinda: $U_{initial} = 1^{\frac{1}{3}} + 1^{\frac{1}{3}} < U_{equilibrium} = (\frac{4}{7})^{\frac{1}{3}} + 4^{\frac{1}{3}}$.

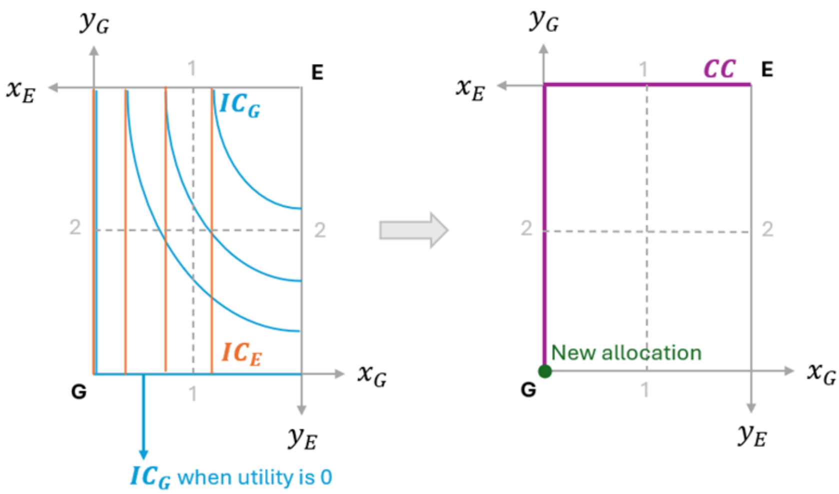
For Elphaba: $U_{initial} = 1 < U_{equilibrium} = \frac{10}{7}$.

So the equilibrium is in the mutual advantages set.

Grading:

- 0.45 points for defining what is the core.
- 0.4 points for explaining why the equilibrium is efficient. Note: You would also get full points if you found the contract curve and mentioned that the equilibrium is in that contract curve.
- 0.4 points for explaining why the equilibrium is in the mutual advantages set. Note: You would also get full points if you explained that to find the equilibrium you maximized agents' utility having as the only restriction the budget constraint, so if the initial endowment (which is, by definition, affordable) yielded a greater utility, it would have been the equilibrium solution.

c) (1.25 points) Would the allocation that gives nothing to Glinda and everything to Elphaba be Pareto efficient? Explain using an Edgeworth box.



Yes, it is efficient because it is on the contract curve.

Grading:

- 0.25 points for stating that being efficient implies being on the contract curve.
- 0.8 points for drawing the contract curve in the Edgeworth Box.
- 0.2 points for representing the given allocation in the Edgeworth box and concluding.

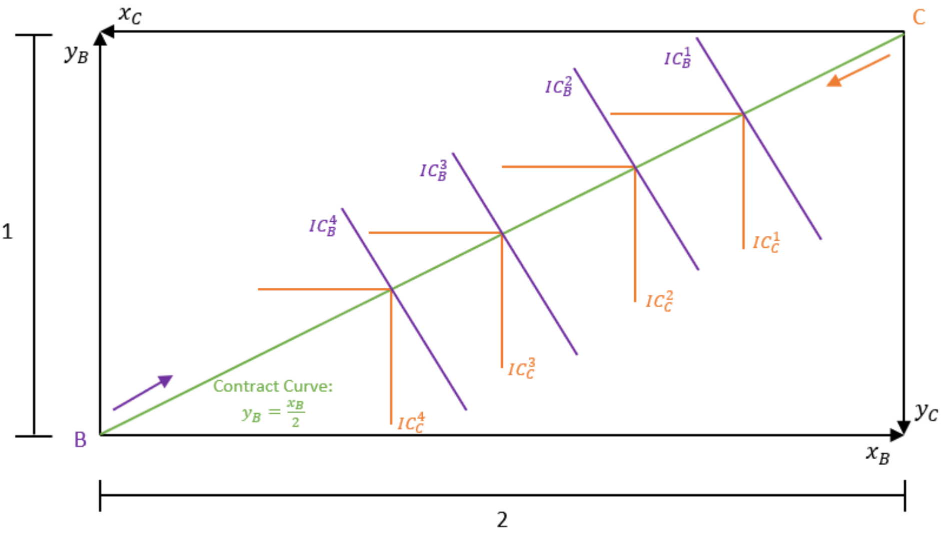
II. (5 points) Consider an economy in So Kamui with two agents, Belinda (B) and Chelsea (C), and two goods, medicine (good x) and fruit (good y). Belinda has preferences represented by $U_B(x_B, y_B) = 2x_B + y_B$, and Chelsea has preferences represented by $U_C = \min\{x_C, 2y_C\}$. There is a total of two units of good x and one unit of good y in the economy.

a) (2 points) Draw the contract curve for this economy.

Agent B: $U_B = 2x_B + y_B \rightarrow$ Perfect Substitutes
Agent C: $U_C = \min\{x_C, 2y_C\} \rightarrow$ Perfect Complements

Tangencies can only occur at the kink points of the indifference curves of the agent that regards the two goods as complements. This means that we have tangencies at:
$x_C = 2y_C \Leftrightarrow 2x_B = 2(1 - y_B) \Leftrightarrow 2 - x_B = 2 - 2y_B \Leftrightarrow 2y_B = x_B \Leftrightarrow y_B = \frac{x_B}{2}$

Contract Curve: $y_B = \frac{x_B}{2}$



Grading:

- 1 point for the correct graphical representation of the contract curve.
- 1 point for the justification (including drawing the indifference curves for each agent).

b) (1.5 points) Find the utility possibility frontier.

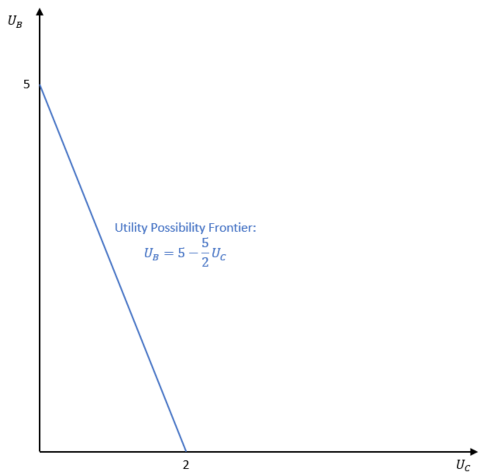
Put the contract curve expression found before in the utility of one of the agents:

$U_B = 2x_B + y_B \Leftrightarrow U_B = 2x_B + \frac{x_B}{2} \Leftrightarrow U_B = \frac{5}{2}x_B \Leftrightarrow x_B = \frac{2}{5}U_B$

Then:

$U_C = \min\{x_C, 2y_C\} \Leftrightarrow U_C = x_C = 2y_C \Leftrightarrow U_C = 2 - x_B \Leftrightarrow U_C = 2 - \frac{2}{5}U_B \Leftrightarrow U_B = 5 - \frac{5}{2}U_C$

Utility Possibility Frontier: $U_B = 5 - \frac{5}{2}U_C$



Grading:

- 0.5 points for using the contract curve.
- 0.5 points for using the feasibility condition.
- 0.5 points for finding the correct utility possibility frontier.

c) (1.5 points) Find the Rawlsian choice for this economy (if you haven't replied to b., use the UPF defined by $U_B = 5 - \frac{5}{2}U_C$).

According to the Rawlsian point of view of society: $W = \min\{U_B, U_C\}$

So, we solve:

$$\max_{U_B, U_C} W = \min\{U_B, U_C\}$$

s.t.

$$U_B = 5 - \frac{5}{2}U_C$$

Which simplifies to:

$$\max_{U_C} W = \min\{5 - \frac{5}{2}U_C, U_C\} \Leftrightarrow 5 - \frac{5}{2}U_C = U_C \Leftrightarrow \frac{5}{2}U_C + U_C = 5 \Leftrightarrow \frac{7}{2}U_C = 5 \Leftrightarrow U_C = \frac{10}{7}$$

And we have that: $U_B = 5 - \frac{5}{2} \cdot \frac{10}{7} = \frac{10}{7}$

To find individual quantities:
$U_C = \min\{x_C, 2y_c\} \Leftrightarrow x_C = \frac{10}{7}$ and $2y_C = \frac{10}{7} \Leftrightarrow y_C = \frac{5}{7}$

From the feasibility conditions:
$x_B = 2 - \frac{10}{7} = \frac{4}{7}$ and $y_B = 1 - \frac{5}{7} = \frac{2}{7}$

Grading:

- 0.5 points for setting up the maximization problem.
- 0.5 points for finding the correct levels of utility.
- 0.5 points for finding the correct individual quantities.

III. (2.5 points) Dim Towns does not have street lighting and is currently having a discussion on the number of streetlamps that should be installed. It is knows that the cost of installing one additional streetlamp is 20€.

Dim Town has 150 inhabitants, divided equally into three groups. Different groups have different preferences regarding streetlamps (L) and money (m). For group A, the individual utility function is $u_i(L,m_i) = 2m_i + 2\ln(L)$. For group B, the individual utility function is $u_i(L,m_i) = 2m_i + 4\ln(L)$. For group C, the individual utility function is $u_i(L,m_i) = 2m_i + 6\ln(L)$. All groups have the same size.

a) (1.5 points) Find the Pareto efficient level of the public good.

By the Samuelson condition, we know that:

$$\sum MRS = MC$$

Computing the MRS for each group ($n_A = n_B = n_C = 50$):

Group A:

- Individual $MRS_a = \frac{\frac{\partial u_i}{\partial L}}{\frac{\partial u_i}{\partial m}} = \frac{\frac{2}{L}}{2} = \frac{1}{L}$
- Group $MRS_a = n_A \cdot Individual\ MRS_a = 50 \cdot \frac{1}{L} = \frac{50}{L}$

Group B:

- Individual $MRS_b = \frac{\frac{\partial u_i}{\partial L}}{\frac{\partial u_i}{\partial m}} = \frac{\frac{4}{L}}{2} = \frac{2}{L}$
- Group $MRS_b = n_B \cdot Individual\ MRS_b = 50 \cdot \frac{2}{L} = \frac{100}{L}$

Group C:

- Individual $MRS_c = \frac{\frac{\partial u_i}{\partial L}}{\frac{\partial u_i}{\partial m}} = \frac{\frac{6}{L}}{2} = \frac{3}{L}$
- Group $MRS_c = n_C \cdot Individual\ MRS_c = 50 \cdot \frac{3}{L} = \frac{150}{L}$

$MC = 20$, then:

$$\sum MRS = MC \Leftrightarrow \frac{50}{L} + \frac{100}{L} + \frac{150}{L} = 20 \Leftrightarrow \frac{300}{L} = 20 \Leftrightarrow L^* = 15$$

Grading:

- 0.4 points for stating the Samuelson condition.
- 0.9 points for computing the MRS for the three groups (0.3 points each).
- 0.1 points for computing the marginal cost.
- 0.1 points for solving the Samuelson condition.

b) (1 point) How would you tax the agents if you wanted to achieve a unanimous choice of the level you found in a)?

I would use the Lindahl taxation, by taxing each agent according to his MRS evaluated at the optimal level.

$$\tau_a = \frac{1}{L}(L=15) \Leftrightarrow \tau_a = \frac{1}{15} \;\;|\;\; \tau_b = \frac{2}{L}(L=15) \Leftrightarrow \tau_b = \frac{2}{15} \;\;|\;\; \tau_c = \frac{3}{L}(L=15) \Leftrightarrow \tau_c = \frac{3}{15} = \frac{1}{5}$$

I would charge $\frac{1}{15}$ per lamp to each agent in group A, $\frac{2}{15}$ per lamp to each agent in group B, and $\frac{1}{5}$ per lamp to each agent in group C.

Grading:

- 0.4 points for describing Lindahl taxation. Note: If you got the solution right, you would get full points in this criterium even if you did not describe Lindahl taxation.

- 0.6 points for computing the tax to be paid (0.2 points per group).

IV. (2.5 points) Two firms, 1 and 2, are considering adopting a new technology. Each firm must decide simultaneously whether to Adopt (A) or Not Adopt (N). If both firms adopt the technology, they benefit from network effects, leading to higher profits. However, if one firm adopts while the other does not, the adopting firm incurs a high cost without sufficient benefits. The following payoff matrix describes their strategic interaction game.

1/2	A	N
A	2,2	-3,0
N	0,-3	0,0

Compute all the Nash Equilibria of the simultaneous game.

Nash Equilibria in **Pure Strategies**:

1/2	A	N
A	2,2	-3,0
N	0,-3	0,0

If Firm 1 plays A, Firm 2's best response is to play A (2>0). If Firm 1 plays N, Firm 2's best response is to play N (0>-3).
If Firm 2 plays A, Firm 1's best response is to play A (2>0). If Firm 2 plays N, Firm 1's best response is to play N (0>-3).

Nash Equilibria in Pure Strategies: (A,A); (N,N)

Nash Equilibria in **Mixed Strategies**:

		q	1-q
	1/2	A	N
p	A	2,2	-3,0
1-p	N	0,-3	0,0

If Firm 2 plays A with probability q and N with probability 1-q, then Firm 1's expected payoffs will be:

- **A:** $q \cdot 2 + (1-q) \cdot (-3) = 2q - 3 + 3q = 5q - 3$
- **N:** $q \cdot 0 + (1-q) \cdot 0 = 0$

Firm 1's best response is:

- **A** if $5q - 3 > 0 \Leftrightarrow q > \frac{3}{5}$
- **N** if $5q - 3 < 0 \Leftrightarrow q < \frac{3}{5}$
- **A, N or a combination of both** if $5q - 3 = 0 \Leftrightarrow q = \frac{3}{5}$

Then, Firm 1's best response function is given by:

$$p = \begin{cases} 0, & \text{if } q < \frac{3}{5} \\ [0,1], & \text{if } q = \frac{3}{5} \\ 1, & \text{if } q > \frac{3}{5} \end{cases}$$

If Firm 1 plays A with probability p and N with probability 1-p, then Firm 2's expected payoffs will be:

- **A:** $p \cdot 2 + (1-p) \cdot (-3) = 2p - 3 + 3p = 5p - 3$
- **N:** $p \cdot 0 + (1-p) \cdot 0 = 0$

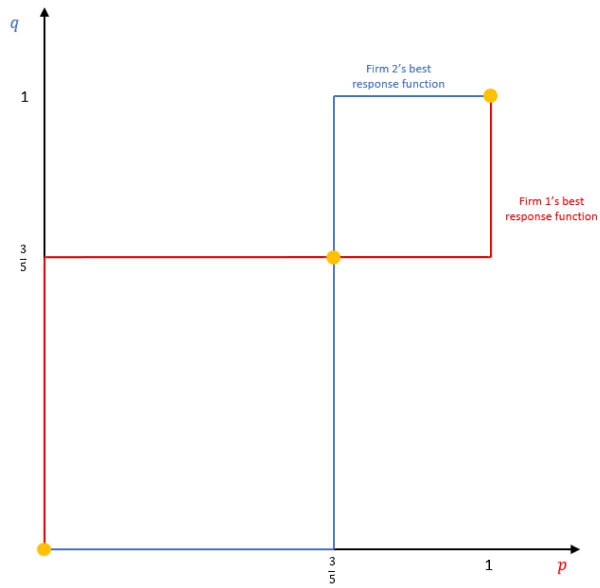
Firm 2's best response is:

- **A** if $5p - 3 > 0 \Leftrightarrow p > \frac{3}{5}$
- **N** if $5p - 3 < 0 \Leftrightarrow p < \frac{3}{5}$
- **A, N or a combination of both** if $5p - 3 = 0 \Leftrightarrow p = \frac{3}{5}$

Then, Firm 2's best response function is given by:

$$q = \begin{cases} 0, & \text{if } p < \frac{3}{5} \\ [0,1], & \text{if } p = \frac{3}{5} \\ 1, & \text{if } p > \frac{3}{5} \end{cases}$$

Representing both best response functions graphically, we have:

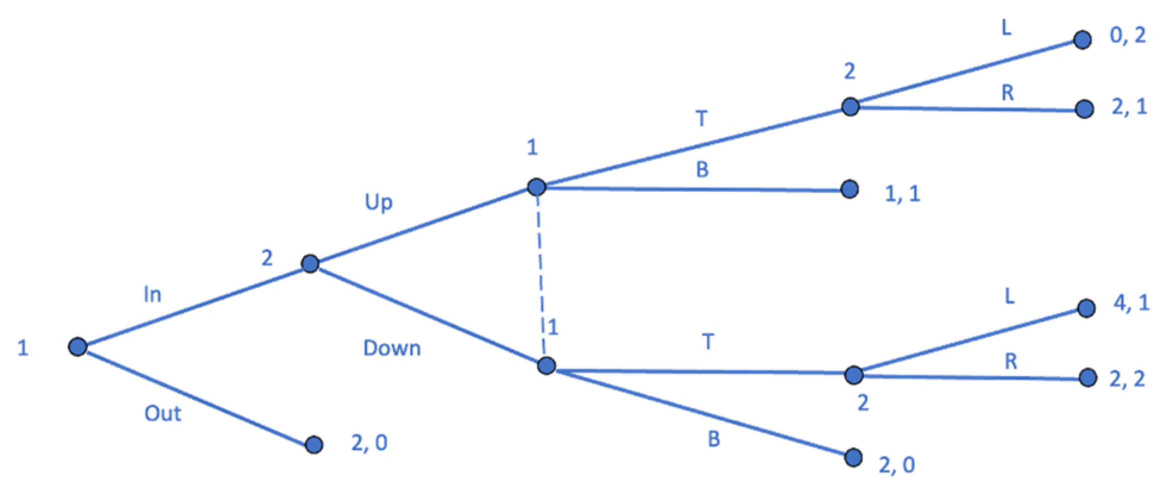


Nash Equilibria in Mixed Strategies: $(p = 0, q = 0)$; $(p = \frac{3}{5}, q = \frac{3}{5})$; $(p = 1, q = 1)$ or (N,N); $(p = \frac{3}{5}, q = \frac{3}{5})$; (A,A)

Grading:

- 0.75 points for the best response function of Firm 1.
- 0.75 points for the best response function of Firm 2.
- 1 point for the correct conclusion.
- (1 point in case only pure-strategy equilibria are calculated)

V. (5 points) Consider the following game in extensive form:



a) (1 points) How many pure strategies does each player have in this game?

Player 1 has 4 pure strategies and player 2 has 8 pure strategies.

Grading:

- 0.5 points for the number of strategies of player 1.
- 0.5 points for the number of strategies of player 2.

b) (1 points) How many subgames are there in this game?

4 subgames.

Grading:

- 1 point for the right number.
- 0.25 points or 0.5 points if wrong number but correct identification of some subgames.

c) (3 points) Find the Subgame-Perfect Nash Equilibrium of this game.

In the subgame after 2 plays Up and 1 plays T, 2 will play **L** (2>1).
In the subgame after 2 plays Down and 1 plays T, 2 will play **R** (2>1).
In the subgame after 1 plays In,

		q	1-q
	1/2	Up, L, R	Down, L, R
p	T	0,2	2,2
1-p	B	1,1	2,0

1 will set p=0 for q>0 and q=0 for p$\in [0, 1]$. 2 will set q=1 for p<1 and q$\in [0, 1]$ for p=1. There are two NE of the subgame: [T,(Down,L,R)] and [B,(Up,L,R)].

If the NE is [T,(Down,L,R)], then 1 will play In or Out (or any combination of the two) at the beginning. Let x denote the probability of playing In. The SPNE are therefore [($x \in [0, 1]$,T),(Down,L,R)] - including the pure-strategy equilibria [(In,T),(Down,L,R)] and [(Out,T),(Down,L,R)].

If the NE is [B,(Up,L,R)], 1 will play Out and the SPNE is [(Out,B),(Up,L,R)].

Grading:

- 0.5 points for each of the first subgames.
- 1 point for the subgame after 1 plays In.
- 0.5 points for the analysis of the beginning of the game and conclusion in each of the two cases.