Microeconomics II

Spring 2025

Midterm Exam

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You have a total of 120 minutes (2 hours) to solve the exam.
Identify each sheet with your Student Number and Name.
Good luck!

I (4.5 points)

In the context of a private-ownership economy, assume that preferences are continuous and monotonic.

a. Show that, in this context, Weak Pareto Efficiency (no allocation can strictly improve all agents' well-being) implies Strong Pareto Efficiency (no allocation can improve at least one agent without making someone else worse off).

(similar to the proof done in class)

b. Show that a walrasian equilibrium satisfies Weak Pareto Efficiency.

Let the walrasian equilibrium be denoted by $\left[p, (x^h)_{h\in H}, (y^j)_{j\in J}\right]$.
Assume, towards a contradiction, that it the allocation does satisfy Weak Pareto efficiency. Then, there is an allocation and production plan $(x^{h\prime})_{h\in H}, (y^{j\prime})_{j\in J}$ such that for all h we have $u^h(x^{h\prime}) > u^h(x^h)$ and such that $\sum_h x^{h'} = \sum_h \omega^h + \sum_j y^{j'}$.

Due to utility maximization, it must be that $p \cdot x^{h'} > p \cdot x^h$ for all $h \in S$.
But then, summing over $h \in S$, $p \cdot \sum x^{h'} > p \cdot \sum \omega^h + p \cdot \sum_j y^j$ and , $p \cdot \left(\sum_h \omega^h + \sum_j y^{j'}\right) > p \cdot \sum \omega^h$ i.e. $p \cdot \sum_j y^{j'} > 0 = p \cdot \sum_j y^j$, contradicting profit maximization in equilibrium.

II (6.5 points)

Consider a pure exchange economy with two goods, x and y, and two consumers, 1 and 2. The respective endowments are: $\omega_1 = (0,2)$, $\omega_2 = (2,0)$.
The preferences of consumer 1 are lexicographic, with priority for y over x. The preferences of consumer 2 are represented by the utility function $U_2(x_2, y_2) = x_2 + \sqrt{y_2}$ for $x_2 \geq 0$ and $y_2 \geq 0$.

Answer the following questions in a clearly drawn Edgeworth box:

(a) If this economy has Pareto optimal allocations, find them and show them clearly on the diagram. If there are none, state and explain that.

The set of Pareto efficient points will be $x_1 \in [0,2], y_1 = 2$ and $y_1 \in [0,2], x_1 = 0$.

(b) If this economy has equilibrium allocations, find them and show them clearly on the diagram. If there are none, state and explain that.

There is no equilibrium allocation.

Individual demands for 2 are $x_2 = 2 - \frac{p}{4}, y_2 = \frac{p^2}{4}$.

1 will consume all wealth on good y if prices are positive. If the price of x is 0, then 1 will want to consume 2 units of y and infinite amounts of x (and if the price of y is 0, then 1 will want to consume infinite amounts of y).

Assuming positive prices, the market clearing condition for y would yield $4+\frac{p^2}{4}=4$ and $p = 0$. If $p = 0$, there is only a quasiequilibrium: 1 would demand an infinite amount of x.

(c) Explain whether we can ensure that any allocation in the core can be achieved as a price equilibrium with transfers.

The only allocation in the core is the endowment: it is the only Pareto efficient point that satisfies mutual advantages. Therefore, even though the initial endowment may satisfy the expenditure minimization problem, it does not satisfy the utility maximization problem; therefore, a quasiequilibrium with transfers need not be an equilibrium with transfers. Note that the assumptions of the second welfare theorem are not met.

III (4 points)

Consider an economy with two consumers where there is only one consumption good and two equally likely states of nature at date 1.

Consumer 1's expected utility is $U_1(x_1) = \frac{1}{2}\ln(x_{11}) + \frac{1}{2}\ln(x_{12})$ and her endowment is (0,5). Consumer 2 has an expected utility of $U_2(x_2) = \frac{1}{2}\sqrt{x_{21}} + \frac{1}{2}\sqrt{x_{22}}$ and also has an endowment of (5,0).

a. Find the competitive equilibrium for this economy if there are markets for contingent goods that open at date 0.

Normalizing $p_2 = 1$, individual demands for 1 are $x_{11} = \frac{5}{2p}, x_{12} = \frac{5}{2}$.

Individual demands for 2 are $x_{21} = \frac{5}{1-p}, x_{22} = \frac{5p^2}{1+p}$.

Market clearing leads to $p = 1$ and allocation $x_{11} = x_{12} = x_{21} = x_{22} = \frac{5}{2}$

b. Using the result in a., characterize the rational expectations equilibrium with Arrow securities.

Setting $q_1 = q_2 = 1$, and $z_1(s_1) = z_2(s_2) = \frac{5}{2}$ and $z_1(s_2) = z_2(s_1) = -\frac{5}{2}$, we can make the allocation in a) affordable at the prices found in a) for the different states.

IV (5 points)

A group of 200 car owners are looking to sell their car. Each of the cars has a different quality θ, which is observed privately by its owner only. The quality θ is distributed uniformly on the interval [0; 1]. Let p denote the price on the car market. Car owners are willing to sell only if $p \geq \theta$. There are 100 type A buyers, who value a car of expected quality θ at 2θ, and 100 type B buyers who value such a car at $\frac{3}{2}\theta$.

(a) Find the average quality of the cars supplied as a function of the price. Find the supply of cars.

The total number of cars in the market is 200. The fraction of these cars that are supplied corresponds to the probability that $\theta \leq p$; given the uniform distribution on [0,1] this fraction is p. Therefore, the supply function is given by S(p)=200p, 0 ≤ p ≤ 1.

The average quality of supplied cars will be $E(\theta|\theta \leq p) = p/2$

(b) Specify the demand for cars.

All type A buyers will want to buy a car, since their WTP is $2E(\theta) = p \geq p$.

No type B buyer will want to buy a car, since their WTP is $\frac{3}{2}E(\theta) = \frac{3}{4}p < p$.

Demand is therefore 100, for 0 ≤ p ≤ 1

(c) Is there any price at which trade can take place? How many cars are traded and what is their average quality?

Market equilibrium will have $p = \frac{1}{2}$. 100 cars will be traded and the average quality is $\frac{1}{4}$.

(d) Will we have efficiency in this market? If so, explain why. If not, explain how it could be achieved.

No, adverse selection: it would be efficient to trade all cars. Signaling is a possible solution.