Annuity and perpetuity with growth:
Annuity with growth:

$$PV_t = \frac{CF_{t+1}}{r-g}\left[1 - \left(\frac{1+g}{1+r}\right)^n\right]$$

Perpetuity with growth:

$$PV_t = \frac{CF_{t+1}}{r-g}$$

where PV_t represents the present value at time t, r is the discount rate, g is the rate of growth, n is the number of periods for the annuity, and CF_t is the cash flow at time t.

Changing rate frequencies:

$$1 + PR_n = \left(1 + \frac{APR}{n}\right)$$

$$1 + r_n = (1 + EAR)^{\frac{1}{n}}$$

where PR_n is the proportional rate with a frequency implying n periods within a year, EAR is the effective annual rate, r_n is the effective rate at a frequency higher than annual, n is the times a year we observe the new frequency, and APR is the annual proportional rate.

Stock-related formulas:

$$ROE_t = \frac{NetIncome_t}{BV_{t-1}}$$

$$BV_t = BV_{t-1} + Inv_t$$

$$Div_t = p \times EPS_t$$

$$Inv_t = (1-p) \times EPS_t$$

$$g = (1-p) \times ROE_t$$

$$P_t = \frac{P_{t+1} + Div_t}{1+r}$$

$$P_t = \frac{Div_{t+1}}{r-g} \text{ (Gordon Growth Model)}$$

$$r = DividendYield + CapitalGains = \frac{Div_{t+1}}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

$$\text{Present value of growth opportunities (PVGO)} = P_t - \frac{EPS_t}{r}$$

where ROE_t is the return on equity from $t-1$ to t, r is the discount rate, BV_t is the book value of equity at time t, Inv_t is investment at time t, p is the payout ratio, g is the rate of growth of earnings per share or dividends, P_t is the price at time t, Div_t is the amount of dividends paid at time t, $EPS_t = \frac{NetIncome}{Numberofshare}$ is earnings per share.

Bond-related formulas:

$$P = \sum_{t=1}^{T} \frac{c \times FV}{(1+y)^t}$$

$$D = \frac{1}{P}\left(\sum_{t=1}^{T} t \times \frac{c \times FV}{(1+y)^t} + \frac{T \times FV}{(1+y)^t}\right)$$

$$ModifiedDuration = \frac{D}{1+y}$$

$$\Delta P \approx -D\Delta y$$

$$\frac{\Delta P}{P_t} \approx -\frac{D}{1+y}\frac{\Delta y}{y}$$

where c is the coupon rate, FV is the face value, y is the yield, and T is the maturity. D is Macaulay duration and Δ represents an increment.

Portfolios: Consider N assets indexed by i, then the expected return of the portfolio with a weight w_i in asset i is:

$$\mathbb{E}(r_p) = \sum_{i=1}^{N} w_i \mathbb{E}(r_i)$$

The variance of the portfolio return is

$$V(r_p) = \sum_{i=1}^{N} w_i^2 V(r_i) + 2\sum_{i=1}^{N}\sum_{i<j} w_i w_j cov(r_i, r_j)$$

The asset beta is given by:

$$\beta_i = \frac{cov(r_i, r_m)}{V(r_m)}$$

and the beta of the portfolio is

$$\beta_p = \sum_{i=1}^{N} w_i \beta_i$$

The correlation of assets i and j is given by:

$$\rho_{ij} = \frac{cov(r_i, r_j)}{\sqrt{V(r_i)V(r_j)}}$$

$\mathbb{E}$ indicates the expectation operator, V the variance operator, and cov the covariance. β_i corresponds to the CAPM β of asset i. r_m is the market portfolio return.

Basic statistics: If we have a sample $\{x_1, ..., x_N\}$, we can define the following sample moments:

$$\text{Sample Mean} \equiv \overline{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

$$\text{Sample Variance} \equiv \hat{\sigma}^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \overline{x})^2$$

$$\text{Sample Standard Deviation } = \sqrt{\hat{\sigma}^2}$$

If we have a sample $\{(x_1, y_1), ..., (x_N, y_N)\}$, we can define the following sample moments:

$$\text{Sample Covariance} \equiv cov(x, y) = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \overline{x})(y_i - \overline{y})$$