Public Economics

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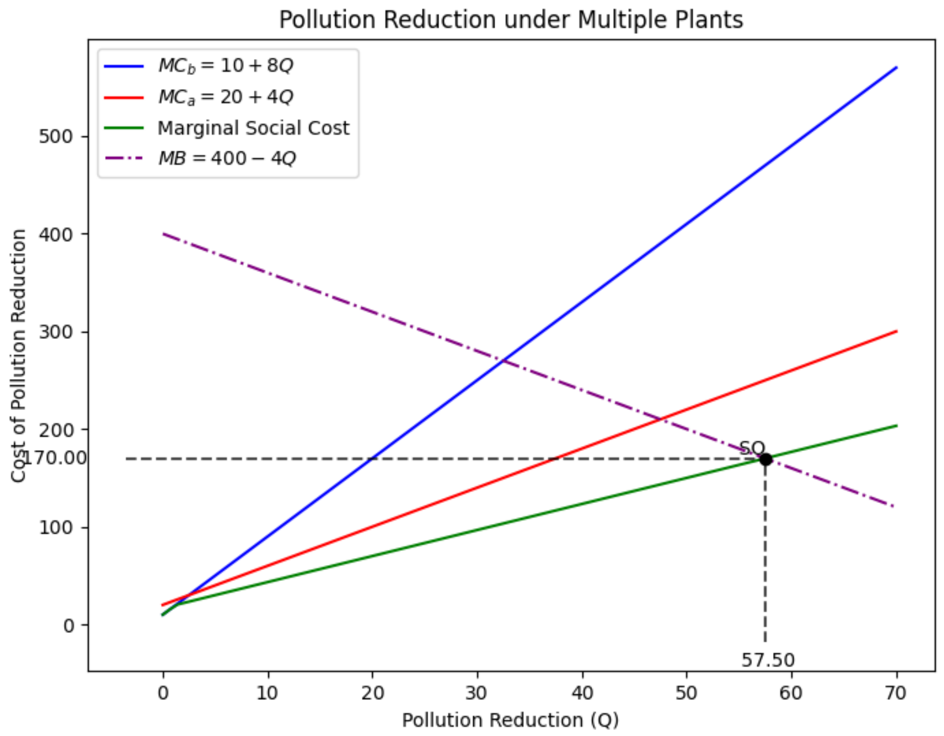
3- Externalities and Public Goods

3.1) Externalities (Chapter 5-6 Gruber)

PS 4: Externalities

Ex.2) Two firms are ordered by the federal government to reduce their pollution levels. Firm A's marginal costs associated with pollution reduction are $\boldsymbol{MC = 20 + 4Q.}$ Firm B's marginal costs associated with pollution reduction are $\boldsymbol{MC = 10 + 8Q.}$ The marginal benefit of pollution reduction are $\boldsymbol{MB = 400 - 4Q}$



- Efficient pollution reduction implies that $\boldsymbol{MC_a = MC_b}$
  - (Otherwise, there is a less costly way to reduce pollution)

- **What is $\boldsymbol{Q^*}$**, the optimal level of pollution reduction?
  - The point in which the marginal benefit of reducing one extra unit of pollution is equal to the marginal cost of reducing it (MC=MB)

  - Notice: Setting $\boldsymbol{MC_a + MC_b = MB}$ is **not efficient**: The cost of firm A reducing one extra unit plus the cost of firm B reducing one extra unit (in total, **reducing two extra units**) should not be equal to the benefit of reducing one extra unit!

3- Externalities and Public Goods

3.2) Public Goods (Chapter 7 Gruber)

Public Goods

A good which **cannot be excluded from anyone (non-excludable),** and where **the use of one agent does not decrease the availability to others (non-rival).**

- E.g.: Air, Public Defense

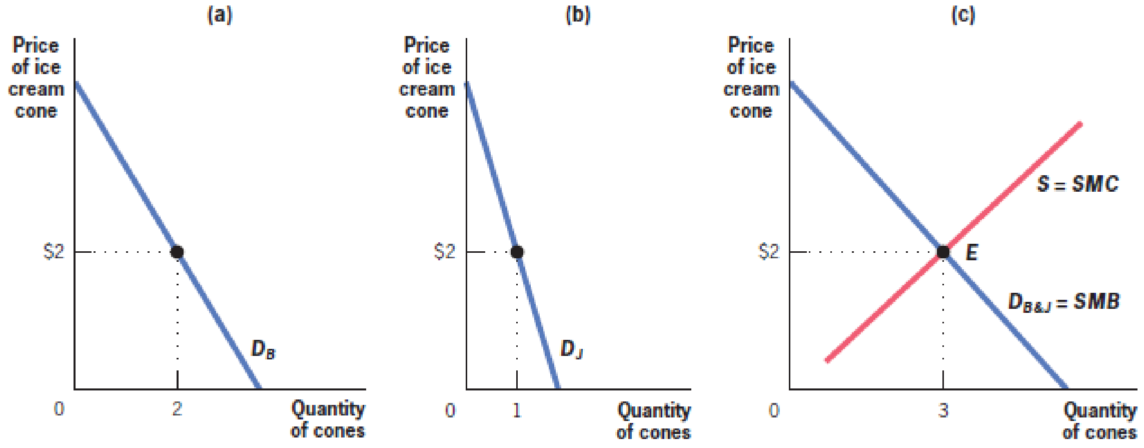
Two important implications:

- **Non-excludability: Free-riding** issues – if you can rely on others to pay for the good, will you pay it? This market failure (externality) leads to an under provision of the public good
- **Non-rivalry:** Since every unit of the public good can be consumed as many times, and by as many people, as possible aggregate demand is made through a vertical sum instead of the usual horizontal sum

Aggregating demands – Private vs Public goods

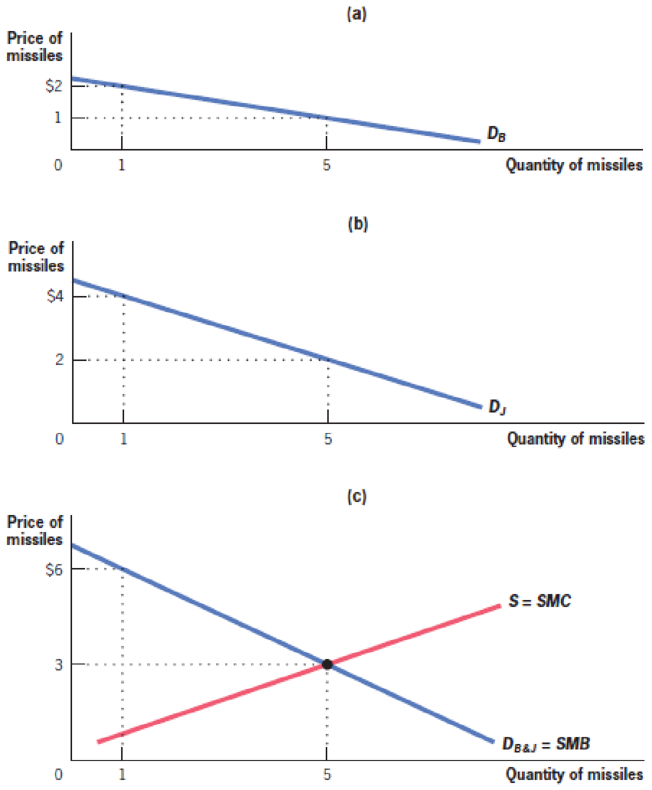
Private goods

Sum horizontally!



Public goods

Sum vertically!



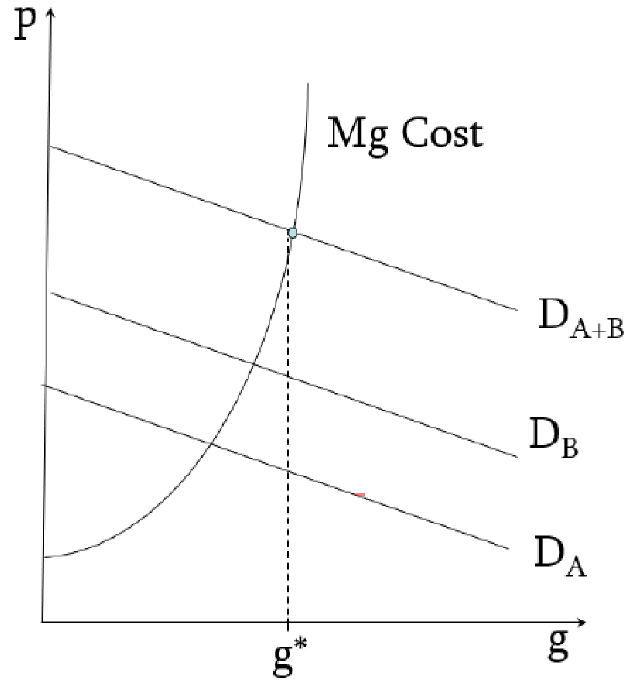
Public Goods – Optimal Provision

Samuelson Condition: The optimal provision is given by the allocation in which the **sum of every agent's Marginal Rate of Substitution** between the Public and the Private Good is **equal to the economy's Marginal Rate of Transformation** between both goods:

$$\sum_{i=1}^{n} MRS_i = MRT$$

If the **private good is money**, the condition can be rewritten as:

$$\sum_{i=1}^{n} MB_i = MC$$



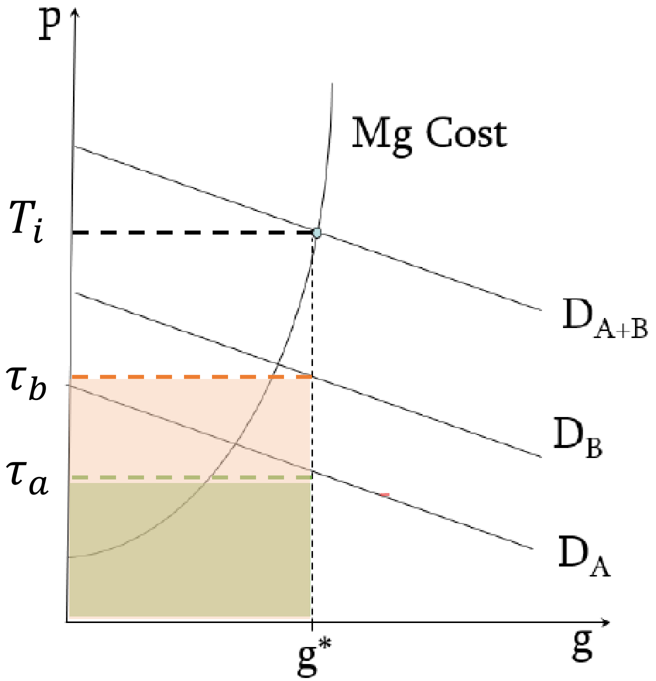
Public Goods – Lindahl Taxes

Lindahl Tax: If we charge every agent the marginal benefit they retrieve from the public good, evaluated at the optimum, the optimal quantity will be chosen unanimously, and the provision of the Public Good will be fully funded:

$$\boldsymbol{\tau_i = MB_i(G^*)}$$

Where the total amount of taxes collected is:

$$\boldsymbol{T_i = \tau_i * G^*}$$



PS 5: Public Goods

Ex.1) Think about the rival and excludable properties of public goods. To what degree is radio broadcasting a public good? To what degree is a highway a public good?

Radio
- Non-rival: many consumers can listen simultaneously without deterioration in sound quality
- Non-excludable (to a certain extent): although most are not, some radio signals can be made excludable

Highways
- Rival: many consumers simultaneously using the same highway will lead to congestions;
- Excludable: Can be made excludable through the use of tolls.

Ex.2) To determine the right amount of public good to provide, the government of West Essex decides to survey its residents about how much they value the good. **It will then finance the public good provision by taxes on residents**.

Describe a tax system that would lead residents to:

- underreport their valuations;

  Each resident must pay according to their reported value of the good – Incentive to underreport

- overreport their valuations.

  Public good is financed through a tax on non-residents – Incentive to overreport

PS 5: Public Goods

Ex.3) The town of Musicville has two residents: Bach and Mozart. The town currently funds its free outdoor concert series solely from the individual contributions of these residents.

Each of the two residents has a utility function over private goods (X) and total concerts (C) of the form **U = 3 ln(X) + 2 ln(C).** The total number of concerts given, C, is the sum of the number paid for by each of the two persons: $\mathbf{C = C_b + C_m}$.

Bach and Mozart both have income of 60, and the price of both the private good and a concert is 1. Thus, they are limited to providing between 0 and 60 concerts.

a) How many concerts are given if the **government does not intervene**?

$$\mathbf{C_b = C_m = 15 \ \Rightarrow C = 30}$$

b) Suppose the **government** is not happy with the private equilibrium and decides to **provide 8 concerts** in addition to what Bach and Mozart may choose to provide on their own. It **taxes** Bach and Mozart **equally to pay for the new concerts**. What is the new total number of concerts? How does your answer compare to your answer to a)? Have we achieved the social optimum? Why or why not?

$$\mathbf{C_b = C_m = 11 \ \Rightarrow C = 11 + 11 + 8 = 30}$$

PS 5: Public Goods

Ex.3) The town of Musicville has two residents: Bach and Mozart. The town currently funds its free outdoor concert series solely from the individual contributions of these residents.

Each of the two residents has a utility function over private goods (X) and total concerts (C) of the form **$U = 3 \ln(X) + 2 \ln(C)$.** The total number of concerts given, C, is the sum of the number paid for by each of the two persons: $C = CB + CM$.

Bach and Mozart both have income of 60, and the price of both the private good and a concert is 1. Thus, they are limited to providing between 0 and 60 concerts.

c) Suppose that instead an **anonymous benefactor pays for 8 concerts.** What is the new total number of concerts? Is this the same level of provision as in b? Why or why not?

PS 5: Public Goods

Ex.4) Consider an economy with three types of individuals, differing only with respect to their preferences for monuments.

Individuals of the first type get a **fixed benefit of 100** from the mere existence of monuments, whatever their number. Individuals of the second and third type get benefits according to:

$B_{ii} = 200 + 30M - 1.5M^2$, and $B_{iii} = 150 + 90M - 4.5M^2$, where M denotes the number of monuments in the city.

Assume that there are **50 people of each type**. Monuments **cost \$3,600 each** to build. How many monuments should be built?

PS 5: Public Goods

Ex.5) Andrew, Beth, and Cathy live in Lindhville. Andrew's demand for bike paths, a public good, is given by $Q = 12 - 2P$. Beth's demand is $Q = 18 - P$, and Cathy's is $Q = 8 - \frac{P}{3}$. The marginal cost of building a bike path is $MC = 21$.

The town government decides to use the following procedure for deciding how many paths to build. It **asks each resident** how many paths they want, and it **builds the largest number** asked for by any resident. To pay for these paths, it then taxes Andrew, Beth, and Cathy the prices a, b, and c per path, respectively, where a + b + c = MC. (The residents know these tax rates before stating how many paths they want.)

a) If the taxes are set so that each resident shares the cost evenly (a = b = c), how many paths will get built?

b) Show that the government can achieve the social optimum by setting the correct tax prices a, b, and c. What prices should it set?

Midterm Fall '22

Consider an economy with three agents and two goods, where X is a pure private good (money, with a unit price of 1) and G is a pure public good. Let the marginal cost of the public good be 9 monetary units.

Let x_i denote the amount of the private good consumed by agent i. Agent 1's preferences can be represented by utility function $U_1(x_1, G) = x_1G^2$. Agent 2's preferences can be represented by utility function $U_2(x_2, G) = x_2G$. Agent 3's preferences can be represented by utility function $U_3(x_3, G) = x_3$. The incomes of the agents before the provision of the public good are m_1=6, m_2=10 and m_3=5.

a. (1.25 points) Show that agent 1's demand for the public good is $p_1 = 4/G$ and that agent 2's demand for the public good is $p_2 = 5/G$.

For agent 1 we solve: $\max U_1 = x_1G^2 \quad st.\ 6 = x_1 + P_1G \Leftrightarrow x_1 = 6 - P_1G$

The solution is: $MRS_1 = P_1 \Leftrightarrow \frac{2x_1G}{G^2} = P_1 \Leftrightarrow 2(6 - P_1G) = P_1G \Leftrightarrow P_1 = \frac{4}{G}$

For agent 2 we solve: $\max U_2 = x_2G \quad st.\ 10 = x_1 + P_2G \Leftrightarrow x_1 = 10 - P_2G$

The solution is: $MRS_2 = P_2 \Leftrightarrow \frac{x_2}{G} = P_2 \Leftrightarrow 10 - P_2G = P_2G \Leftrightarrow P_1 = \frac{5}{G}$

Grading: 0.25 for setting up the utility maximization problems, 0.5 for solving the problem for each agent.

b. (1.25 points) Find the socially optimal quantity of the public good.

Samuelson Condition (when private good is money): $\sum MB_i = MC$

We have the MB for agent 1 and 2 from above. It is straightforward to conclude that the MB for agent 3 is zero.

Solving: $\frac{4}{G} + \frac{5}{G} + 0 = 9 \Leftrightarrow G^* = 1$

Grading: 0.5 for stating the Samuelson Condition, 0.75 for the solution.

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