Assignment 1 – Public Economics

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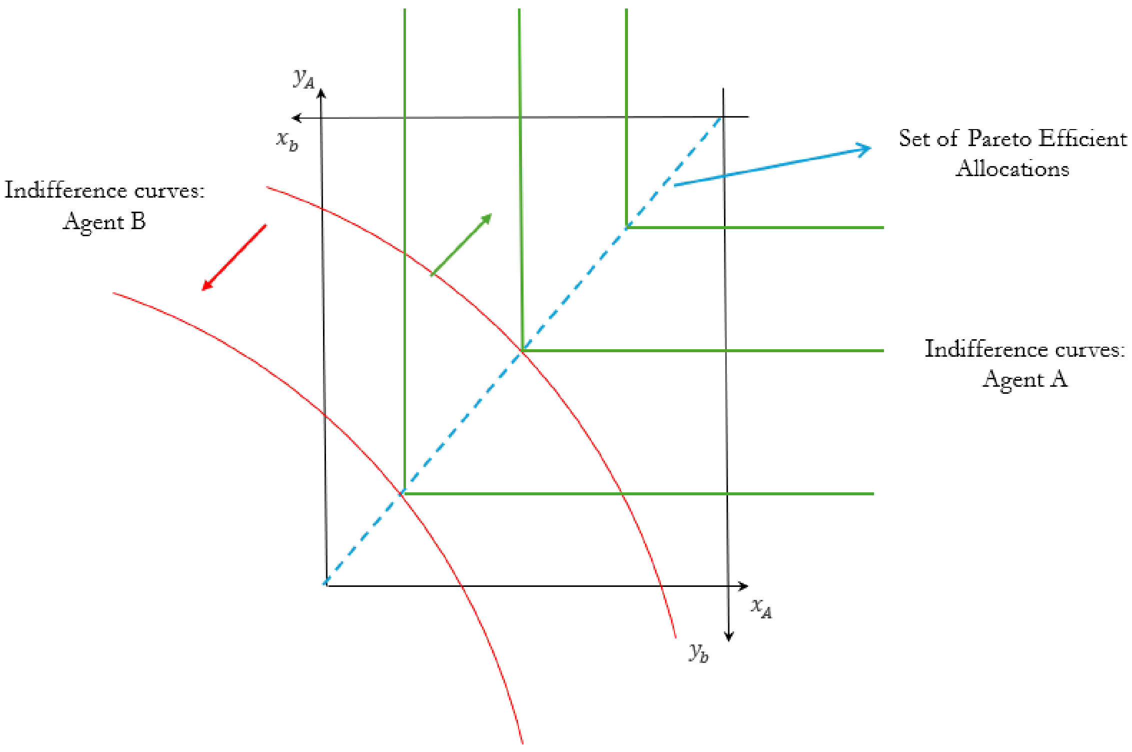
Exercise 1) An economy has two agents A and B and two private goods x and y.
Agent A's preferences can be represented by $u_A(x_A, y_A) = \min\{2x_A, y_A\}$ and agent B's preferences can be represented by $u_B(x_B, y_B) = x_B y_B$.
There are 6 units of x and 12 units of y in the economy.

a. Show in an Edgeworth box the set of efficient allocations. (2)

Since $u_A(x_A, y_A) = \min\{2x_A, y_A\}$, efficiency implies that: $2x_A = y_A$, represented in the Edgeworth box by the blue dashed line.



b. Find the utility possibility frontier for this economy. (2)

$Max_{x_A, y_A}\, U_a = \min\{2x_A, y_A\}$
$s.t.\!: \bar{U}_b = x_b\, y_b$
$6 = x_a + x_b \Leftrightarrow 6 - x_a = x_b$
$12 = y_a + y_b \Leftrightarrow 12 - y_a = y_b$

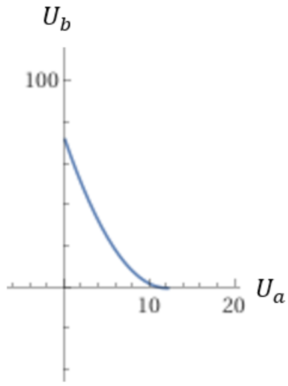
From the set of P.E.A, and using resource constraints, we can write:

$$\begin{cases} U_a = 2x_A = y_A \\ U_b = (6 - x_a) * (12 - y_a) \end{cases} \Leftrightarrow \begin{cases} \rule{4cm}{0.4pt} \\ U_b = \frac{(12-2x_a)*(12-y_a)}{2} \end{cases} \Leftrightarrow$$

$$\begin{cases} \rule{4cm}{0.4pt} \\ U_b = \frac{(12-U_a)^2}{2} \end{cases}$$

Since $U_a = 2x_A$, and $x_A \in [0,6]$:

UPF: $\boldsymbol{U_b = \frac{(12-U_a)^2}{2}}$**, for** $\boldsymbol{U_a \in [0, 12]}$



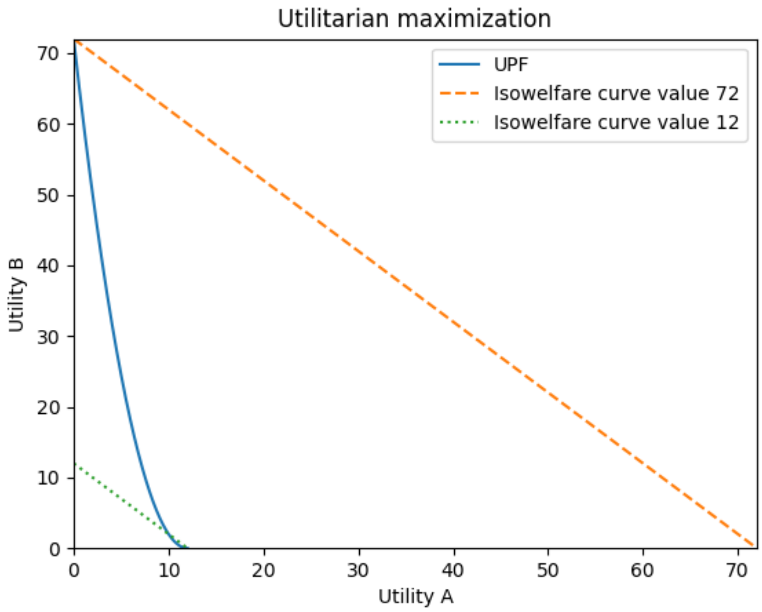
c. Determine the utilitarian and Rawlsian allocations and the associated utilities for the agents. (4)

Utilitarian: $Max\ U_a + U_b$
$s.t.\!: U_b = \frac{(12-U_a)^2}{2}$

As the UPF is convex we obtain a minimum from the FOC, instead of a maximum.
The result will be a corner solution.

$U_a = 0 : \ U_b = \frac{(12)^2}{2} = 72$
$U_b = 0 : U_a = 12$

Utilitarian allocation: $(x_a, y_a) = (0,0)$, $(x_b, y_b) = (6,12)$.
Leading to: $U_a = 0, U_b = 72$



Rawlsian: $Max \min \{U_a ; U_b\}$

$s.t.: U_b = \frac{(12-U_a)^2}{2}$

$U_a = U_b \Leftrightarrow U_a = \frac{(12-U_a)^2}{2} \Leftrightarrow U_a = \frac{26 \pm 10}{2} \Leftrightarrow U_a = 8$, since $U_a \in [0,12]$.

Since $U_a = 2x_A = y_A$, we know that $(x_A, y_A) = (4,8)$.

Rawlsian allocation: $(x_a, y_a) = (4,8)$, $(x_b, y_b) = (2,4)$.
Leading to: $U_a = 8, U_b = 8$

d. Using this example, discuss the following claim: «If an allocation gives all agents the same utility, it must be envy-free.» (max. 10 lines). (2)

An allocation is envy-free if no agent prefers the allocation of the other agent over her own, i.e., that $U_a(x_A, y_A) \geq U_a(x_B, y_B) \wedge U_B(x_B, y_B) \geq U_B(x_A, y_A)$. Thus, one can see that all agents having the same utility $(U_a(x_A, y_A) = U_B(x_B, y_B))$ will not imply that such allocation is envy-free. An illustration is precisely the allocation in the last exercise, where although $U_a(x_A, y_A) = U_B(x_B, y_B) = 8$, the allocation was not envy-free since agent B would prefer the bundle of agent A (4,8) over her own (2,4), since:
$U_B(4,8) = 32 > 8 = U_B(2,4)$.

2. Try to model a real-life allocation problem (such as the example of splitting a cake, but involving two goods) where you can apply the concepts of equal division lower bound and no-envy. Describe the problem (including resources and preferences) and discuss whether the concepts lead to the same recommendations. Are they compatible with efficiency? (10)

You should build your own example. Here are the criteria we valued:

- The student builds a real-life example, presenting a specific and realistic situation.
- There is creativity in the example (and the utility functions chosen make sense and are not just a copy of available examples).
- The contract curve is correctly found.
- The equal-division lower bound criteria is explained and the correct area is found.
- The no-envy criteria is explained and the correct area is found.
- The points that are efficient (contract curve) and respect equal-division lower bound are identified.
- The points that are efficient (contract curve) and respect no-envy are identified.
- The recommendations of equal-division lower bound and no-envy are compared.