In-Class Lecture

Portfolio Theory and the CAPM

Advanced Financial Management

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Key takeaways

01 **Understand mean-variance optimization with risky stocks and a risk-free asset.**

02 **Capital Asset Pricing Model and intuition of its derivation.**

03 **Understand the differences between SML and CML.**

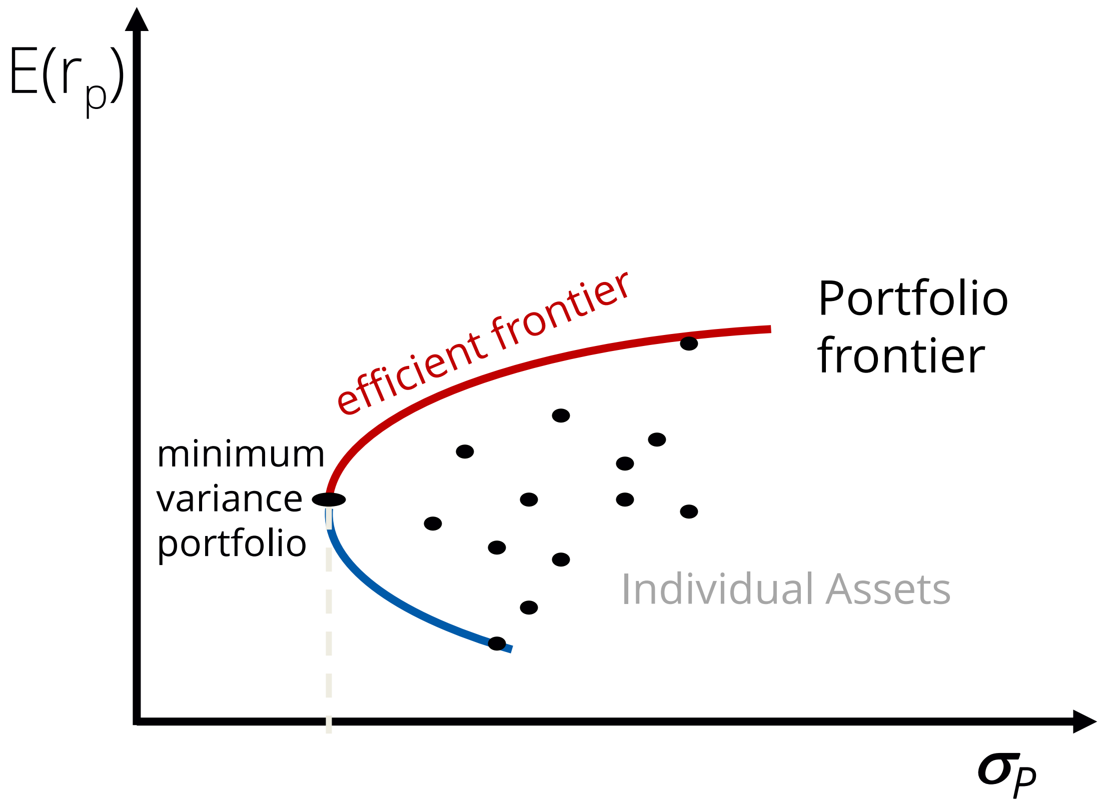
Mean-variance optimization

We take the following steps:

1. Derive the feasible set: the set of portfolio risk and return pairs that it is possible to generate from a given set of securities.
2. Identify the portfolio frontier: this is the set of portfolios with smallest risk for each level of expected return.
3. Identify the efficient frontier: this is the set of portfolios with the highest expected return for each level of risk.
4. Identify the optimal portfolio: it must lie on the portfolio frontier and we use the investor’s indifference curves to identify it.

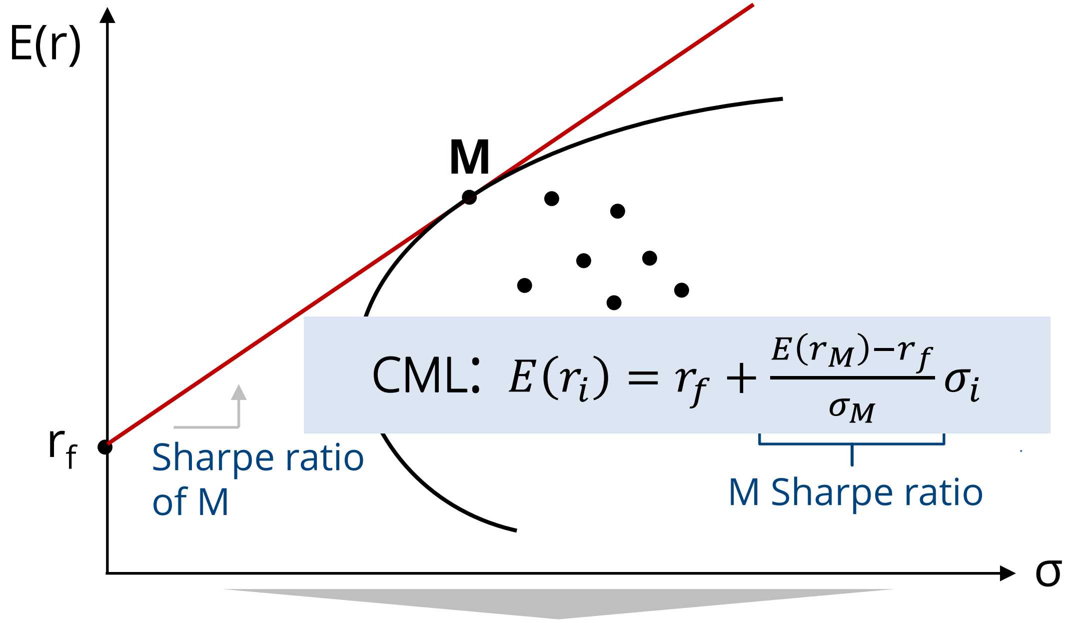
Efficient frontier for many securities

- The portfolio frontier is a hyperbola (once we allow unlimited short sales).
- Points to the right of the portfolio frontier are portfolios that can be formed, but which are not minimum variance. Such portfolios have high risk for their respective levels of expected return.
- Often, you will not find an individual asset on the portfolio frontier!



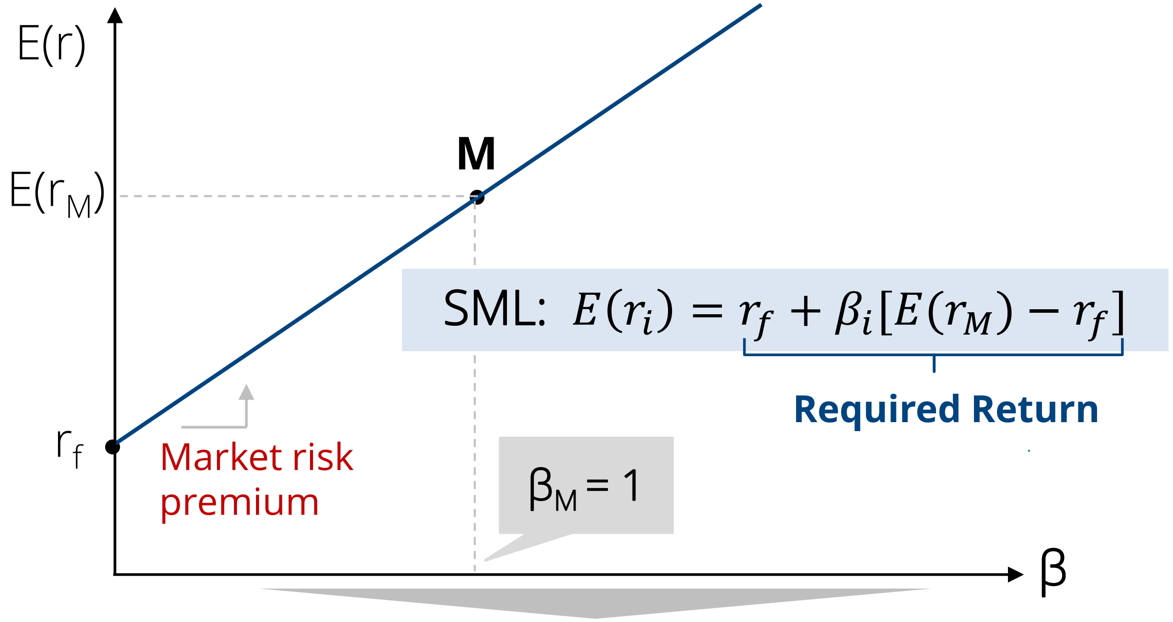
CAPM implications

Capital Market Line (CML)



- Efficient frontier is linear and consists of combinations of the risk-free asset and the market portfolio only.
- All investors locate somewhere on the CML, but all risky portfolios or securities other than the market portfolio lie to its right.

Security Market Line (SML)



- In equilibrium, all portfolios or securities lie on the SML.
- The only reason why one stock's expected return should differ from that of another, is if their βs are different.

Intuition behind the CAPM

High beta stocks are risky, and must therefore offer a higher return on average to compensate for the risk, i.e., for investors to be willing to buy these stocks.

Example

	Depression	Recession	Normal	Boom	E(r)
r_A	-20%	10%	30%	50%	17.5%
r_B	5%	20%	-12%	9%	5.5%

Intuition

Why are high beta stocks risky?

- Because they pay up just when you need the money least – when the overall market is doing well.
- And they lose money when you really need it – when the overall market is doing poorly.

Uses of the CAPM

Among many other things, the CAPM is used for:

1. Portfolio choice
   - Use CAPM equation to estimate expected returns of stocks
2. Capital budgeting
   - CAPM expected return is opportunity cost of capital for a firm's equity (and the firm as a whole if it has zero debt)
     - What rate of return we could obtain for comparable risk in the market?
     - Take a project only if NPV>0 using the CAPM expected return
3. Benchmarking of portfolio managers who claim to outperform the CAPM benchmark (and get positive "alpha")

Exercise 1

Your investment consists of €10,000 invested in only one stock, stock X with the following characteristics: E(r) = 12% and σ = 40%. The risk-free rate is 5% and the market portfolio has an expected return of 10% and volatility of 18%. Under the CAPM assumptions:

a. What alternative investment has the lowest possible volatility while having the same expected return as stock X? What is the volatility of this alternative investment? How does it compare to stock X?

b. What investment has the highest possible expected return while having the same volatility as stock X? What is the expected return of this investment? How does it compare to stock X?

Exercise 2

An economy contains **only** two risky assets X and Y, plus a risk-free asset. Information on these assets is given below and the correlation between the two assets is one third. Asset prices satisfy the CAPM.

Stock	Shares	Price	E(r)	Volatility
X	50	20	21%	15%
Y	80	25	15%	9%

a. Compute the market portfolio weights, the expected return on the market and the standard deviation of the market return.

b. If the risk free rate is 3.5%, what are the betas on both assets?

c. Are X and Y on the CML? What are the Sharpe ratios of X, Y and the market?

Exercise 3

	FIRM A	FIRM B
Volatility	0.25	0.43
Market Cap	100	50

The correlation between A and B is 0.6 and they are the only firms in the market.

a. Compute the market portfolio weights.
b. Compute the variance of the market return.
c. If the beta for Firm A is 0.75 (it is not), what is the ratio between the volatility of expected returns and the volatility of Firm A (this ratio is labelled percentage of systematic volatility)?
d. Compute the beta of Firm A.

Reminder: If a and b are constants, and x and y are random variables (e.g. returns):

$$V(a) = 0 \qquad Cov(a, x) = 0$$

$$V(ax + by) = a^2V(x) + b^2V(y) + 2abCov(x, y)$$

$$Cov(x, ax + by) = aCov(x, x) + bCov(x, y) = aV(x) + bCov(x, y)$$

Exercise 4

Consider two portfolios that lie on the capital market line. The first one has an expected return of 15% and a volatility of 30%. The second one has an expected return of 25% and a volatility of 60%.

a. Compute the Sharpe ratio of the market
b. Compute the Sharpe ratio of each of the portfolios
c. Compute the risk-free rate
d. If the efficient frontier is given by $\mathrm{E(r)} = 0.06 + a\sqrt{\sigma - 0.09} \quad for \ \sigma > 0.1$, compute a and the market portfolio volatility. Warning: the volatility is very high!