Exercises Week 3

1. VAR Models

1. Consider the following VAR:

$$y_t = (1+\beta)y_{t-1} - \beta\alpha x_{t-1} + \epsilon 1t$$
$$x_t = \gamma y_{t-1} + (1-\gamma\alpha)x_{t-1} + \epsilon_{2t}$$

   Show that this VAR is non-stationary

2. Consider the following VAR model:

$$\begin{bmatrix} 1 & c_{12} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} = \begin{bmatrix} 0.85 & 0.15 \\ -0.25 & 0.65 \end{bmatrix} \begin{bmatrix} x_{1,t-1} \\ x_{2,t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix} \tag{1}$$

   where

$$\begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix} \sim iid \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0.8 \\ 0.8 & 1 \end{bmatrix} \right).$$

   a) (a) Is this model in structural or reduced form? Carefully justify your answer.

   b) (b) Is the model under-, just-, or over-identified? What type of identification scheme is applied? What type of ordering between the two endogenous variables does it imply? Carefully justify your answers.

   c) (c) Write the reduced-form of the model in (1). Compute the means, variances, and the covariance of the reduced-form shocks, to be called $[v_{1t}, v_{2t}]'$. Carefully justify your answer and show the details. Note that:

$$\begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -b \\ 0 & 1 \end{bmatrix}.$$

   d) (d) Compute the impulse response function (IRF) for both variables to a shock of 0.5 to x_{2t} at impact and step 1.

2. VAR Estimation - Computational Problem

You have recently been recruited as a data analyst at RodNob Fund, a hedge fund specializing in algorithmic trading. Your team is exploring predictive strategies to improve portfolio allocation and is particularly interested in modeling stock market returns using a **Vector Autoregressive (VAR) Model**.

Your primary task is to design a **forecast-based trading strategy**. The investment universe consists of the following stocks:

- 'MSFT': Microsoft
- 'AAPL': Apple
- 'TSLA': Tesla
- 'NFLX': Netflix
- 'META': Meta
- 'AMZN': Amazon
- 'GOOGL': Google

The dataset spans from **January 2000 to September 2023** and consists of daily closing prices obtained from a financial data provider (e.g., Yahoo Finance).

Phase 1: Data Preprocessing and Model Estimation

1. Examine the **stationarity** of each return series by determining its order of integration using appropriate statistical tests (e.g., Augmented Dickey-Fuller test).
2. Estimate a **VAR model** using the most recent **1000 trading days** of data as input.
3. Select the optimal lag length by comparing information criteria and choose the model with the lowest AIC.

Phase 2: Portfolio Allocation Strategy

You are required to construct a **dynamic trading strategy**. For that, you will need to use the previous 1000 days' returns as input for the VAR model, fit up to 20-lag VARs and select the model with the lowest AIC, and using that VAR, forecast the next day's returns. Next, you will construct a Portfolio Allocation Strategy based on the following rules:

- If the model estimation fails for a given day, assume a **neutral position** by assigning zero forecasted returns to all stocks.

- For each stock, **go long**[1] if its forecasted return is **positive**; otherwise, hold no position in that stock for the day.
- Construct an **equally-weighted portfolio** consisting only of the stocks with a positive return forecast.

As the lead analyst on this project, the firm's investment committee has requested a full report detailing your findings. Present an evaluation of the strategy's **cumulative returns** over time, comparing its performance against a simple buy-and-hold strategy.

[1] Going long means buying an asset with the expectation that its price will rise.