

## Exercises Week 3

## **1.ARCH and GARCH Models**

1. Explain how an ARCH Effect can be detected in a time series. What statistical tests can be used to check for the presence of ARCH effects in residuals? **Solution:** 

To detect ARCH effects in a time series, we analyze the presence of autocorrelation in the squared residuals of a fitted model. The following statistical tests are commonly used:

- Lagrange Multiplier (LM) Test: This test checks for serial correlation in squared residuals.
- Correlogram of Squared Residuals: By examining the autocorrelation function (ACF) and partial autocorrelation function (PACF) of  $\epsilon_t^2$ , significant correlations at higher lags suggest ARCH effects.

If ARCH effects are detected, a conditional heterosked asticity model such as ARCH or GARCH should be considered.

2. Derive the conditional variance equation for an ARCH(1) model and show how it evolves over time.

Solution:

The ARCH(1) model is defined as:

$$y_t = \mu + \epsilon_t, \quad \epsilon_t \sim N(0, h_t),$$
  
$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2,$$

where  $h_t$  is the conditional variance. Substituting recursively, we get:

 $h_t = \alpha_0 + \alpha_1(\alpha_0 + \alpha_1\epsilon_{t-2}^2) = \alpha_0 + \alpha_1\alpha_0 + \alpha_1^2\epsilon_{t-2}^2.$ 

Repeating this process for multiple lags gives:

$$h_t = \alpha_0(1 + \alpha_1 + \alpha_1^2 + \dots) + \alpha_1^q \epsilon_{t-q}^2.$$

For stability,  $\alpha_1$  must satisfy  $0 \leq \alpha_1 < 1$ .

3. Explain how the GARCH(1,1) model generalizes ARCH models. Write down the variance equation for GARCH(1,1) and discuss why GARCH(1,1) is often preferred over ARCH(q) models. Solution:



The GARCH(1,1) model extends ARCH by incorporating lagged conditional variance:

$$h_t = \omega + \alpha \epsilon_{t-1}^2 + \beta h_{t-1},$$

where  $\omega > 0$ ,  $\alpha \ge 0$ , and  $\beta \ge 0$ . Why GARCH(1,1) is Preferred Over ARCH(q):

- Reduces the number of estimated parameters while still capturing long memory in volatility.
- More parsimonious than ARCH(q), which requires many lags.
- Allows for greater flexibility in modeling volatility clustering.



## 2. Modeling Volatility - Practice

TechNOVA Inc. is a new AI-driven tech company whose stock has been highly volatile. Analysts are interested in modeling its volatility. You can find their stock values in the file **techNova.dta**.

- 1. Using the estimated residuals, test for ARCH effects. What does it imply about the residuals?
- 2. Estimate an ARCH(1) and a GARCH(1,1) model for the TechNOVA stock returns. Compare the estimated coefficients.
- 3. Suppose you also estimate EGARCH and GJR-GARCH models. What features do these models capture that GARCH(1,1) does not? Show the estimation outputs and interpret the results.
- 4. Use the GARCH(1,1) model to forecast the next 5 periods of volatility.
- 5. What does high volatility persistence tell us about the stock market? Why would investors care about forecasting volatility?