1. Consumers A and B. Goods X and Y. $U_i = x_i y_i$, with $i = A, B$. Equilibrium: $x_A = 9, y_A = 9, x_B = 1, y_B = 1$ and $\frac{p_x}{p_y} = 1$.

a) Compute the welfare levels according to the Utilitarian, Rawlsian and Cobb-Douglas social welfare functions.

Before we compute the social welfare levels, we need information about the utility levels of each individual. Therefore, we first have to compute the utilities of consumers A and B under this equilibrium:

$$U_A = (9, 9) = 9 \times 9 = 81$$

$$U_B = (1, 1) = 1 \times 1 = 1$$

Then, we can compute the social welfare according to each type of social welfare function.

(1) Utilitarian Social Welfare Function:

The Utilitarian social welfare function is given by $W = U_A + U_B$, meaning that social welfare will be: [1]

$$W = 81 + 1 = 82$$

(2) Rawlsian Social Welfare Function:

The Rawlsian social welfare function is given by $W = min\{U_A, U_B\}$, meaning that social welfare will be:

$$W = min\{81, 1\} = 1$$

(3) Cobb-Douglas Social Welfare Function:

The Cobb-Douglas social welfare function is given by $W = U_A U_B$, meaning that social welfare will be: [2]

$$W = 81 \times 1 = 81$$

[1] We can generalize this function as $W = \alpha U_A + \beta U_B$. This is a weighted sum of utilities, where α and β are the weights we give to each agent defining social welfare.

[2] We can generalize this social welfare function by giving different weights to each agent. It can be generalized as $W = U_A^{\alpha} U_B^{\beta}$, where α and β are, namely, (positive) constants reflecting the weights given to each individual.

Assume a new equilibrium: $x_A = 5, y_A = 5, x_B = 5, y_B = 5$ and $\frac{p_x}{p_y} = 1$.

b) What can be done from the perspective of the Government to achieve this equilibrium?

Let us recall the first and the second welfare theorems:

First Welfare Theorem

If agents have **weakly monotonic preferences**, then **all Walrasian equilibria are Pareto efficient**.

Second Welfare Theorem

If agents have **weakly monotonic** and **weakly convex preferences**, then **for any Pareto efficient allocation there is a price vector and a redistribution of the initial endowment that allows the allocation to constitute a Walrasian equilibrium**.[3]

Moving to the answer for the exercise:

First, we can assess that both agents have Cobb-Douglas preferences, which are strictly monotonic and strictly convex (and, necessarily, also weakly monotonic and weakly convex).

Since both agents have weakly monotonic preferences, we can use the First Welfare Theorem to conclude that this new equilibrium must be Pareto efficient.[4]

Moreover, because this allocation is Pareto efficient and both agents have weakly monotonic and weakly convex preferences, we can use the Second Welfare Theorem to conclude that it can be reached as a market equilibrium through a redistribution of the initial endowment.

Therefore, the only thing the government needs to do is a lump-sum redistribution. For instance, if the government sets $(w_x^A, w_y^A, w_x^B, w_y^B)$=(5,5,5,5) then, with a price ratio $\frac{p_x}{p_y} = 1$, we can indeed move from the initial equilibrium to this new one.[5]

[3] A redistribution of the initial endowment just means that we are changing how the initial endowment is distributed between the two agents. That is, we are changing the $(w_x^A, w_y^A, w_x^B, w_y^B)$ while keeping the total quantity of both goods available fixed and without changing anything else.

[4] You can also see that this new equilibrium is Pareto efficient by deriving the contract curve as we have been doing (CC: $MRS^A_{x,y} = MRS^B_{x,y} \Leftrightarrow ... \Leftrightarrow y_A = x_A$)

[5] In fact, the government can redistribute the initial equilibrium to any allocation that is on the budget constraint that goes through the second equilibrium, because if the two welfare theorems are satisfied, we will always be able to achieve the new equilibrium from any allocation on that budget constraint.

c) Would this change be a Pareto Improvement?

Remember that we can have a Pareto improvement when we can guarantee that, when moving from one allocation to the other, no one would get worse off and that at least one person would be better off. We check this by simply comparing the utilities for both agents in the first and in the second equilibrium.

From **a)**, we have that:

$$U_A^{initial} = U_A(9, 9) = 9 \times 9 = 81$$

$$U_B^{initial} = U_B(1, 1) = 1 \times 1 = 1$$

Now, we have that:

$$U_A^{new} = U_A(5, 5) = 5 \times 5 = 25$$

$$U_B^{new} = U_B(5, 5) = 5 \times 5 = 25$$

Because $U_A^{new} < U_A^{initial}$, agent A got worse off, meaning that **this change cannot be a Pareto Improvement** (even if Agent B is better off in the new equilibrium).

d) Under which of the social welfare functions in a) would society be better off?

(1) Utilitarian Social Welfare Function:

Remember that, under a Utilitarian social welfare function, initial social welfare is $W^{initial} = 82$. On the other hand, currently social welfare is given by:

$$W^{now} = 25 + 25 = 50$$

Given that $W^{initial} > W^{now}$, we conclude that **from a Utilitarian point of view, society got worse off**.

(2) Rawlsian Social Welfare Function:

Remember that, under a Rawlsian social welfare function, initial social welfare is $W^{initial} = 1$. On the other hand, currently social welfare is given by:

$$W^{now} = min\{25, 25\} = 25$$

Given that $W^{now} > W^{initial}$, we conclude that **from a Rawlsian point of view, society got better off now**.

(3) Cobb-Douglas Social Welfare Function:

Remember that, under a Cobb-Douglas social welfare function, initial social welfare is $W^{initial} = 81$. On the other hand, currently social welfare is given by:

$$W^{now} = 25 \times 25 = 625$$

Given that $W^{now} > W^{initial}$, we conclude that **from a Cobb-Douglas point of view, society got better off now**.