Risk and return. Diversification

Advanced Financial Management

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Key takeaways

01 **Compute historical returns and standard deviations of firms.**

02 **Understand the tools to compute portfolio expected returns and volatilities.**

03 **Understand the benefits of diversification and the difference between market risk and idiosyncratic risk.**

Expected vs realized return

Expected return	**Realized return**
The percentage *expected* return from holding the stock in a given period is:	The **realized** return takes into account the *observed* level of dividends and prices:

$$E_0[r_1] = \frac{E_0\,[D_1 + P_1 - P_0]}{P_0} =$$

$$= \underbrace{\frac{E_0\,[D_1]}{P_0}}_{\text{Dividend yield}} + \underbrace{\frac{E_0\,[P_1 - P_0]}{P_0}}_{\text{Capital gain}}$$

$$r_1 = \frac{D_1}{P_0} + \frac{P_1 - P_0}{P_0}$$

Historical risk and return

Historical return	**Historical risk**

The average return for a given frequency of an investment during some period is the average of the *realized* frequency returns:

$$\overline{r_m} = \frac{1}{T}\sum_{t=1}^{T} r_{m,t}$$

where T is the number of frequency intervals in in the period you are considering

- Sample Variance:

$$\sigma^2 = \frac{1}{T-1}\sum_{t=1}^{T}(r_t - \bar{r})^2 \text{ , with } \bar{r} = \frac{1}{T}\sum_{t=1}^{T} r_t$$

  - The units of the sample variance are not in the same units as *r* because they have been squared

- Standard deviation: $\sigma = \sqrt{\sigma^2}$

Forward looking returns: (use probabilities, *p, of* states ω)

$$\bar{r} = \sum_{\omega} p_{\omega}.r_{\omega}$$

Forward looking variance: (use probabilities, *p, of* states ω)

$$\sigma^2 = \sum_{\omega} p_{\omega}(r_{\omega} - \bar{r})^2$$

Historical risk and return

Historical covariance	**Historical correlation**

The covariance between the return of two stocks (A,B) is:

$$Cov(r^A, r^B) = \frac{1}{T-1} \sum_{t=1}^{T} (r_t^A - \bar{r})(r_t^B - \bar{r}) \quad ,$$

$$\text{with } \bar{r} = \frac{1}{T} \sum_{t=1}^{T} r_t^j$$

where T is the number of frequency intervals in in the period you are considering

We define correlation as:

$$Corr(r^A, r^B) = \frac{Cov(r^A, r^B)}{\sigma^A \sigma^B}$$

Where σ^j is the standard deviation of asset j

Forward looking returns: (use probabilities, *p, of states ω*)

$$Cov(r^A, r^B) = \sum_{\omega} p_\omega \, (r_\omega^A - \bar{r}^A)(r_\omega^B - \bar{r}^B)$$

Portfolios

Formulas

Expected portfolio return:

$$E(R_p) = w_A E(R_A) + w_B E(R_B)$$

$$\text{where } w_A + w_B = 1$$

Variance of the portfolio:

$$\sigma_p^2 = Var(R_p) = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B \sigma_{A,B} =$$
$$= w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B \sigma_A \sigma_B \rho_{A,B}$$

Diversification

- As the number of securities in a portfolio increases you can diversify more risk

- The reduction in volatility can't happen indefinitely ⇒ **Market risk** is the risk that cannot be eliminated

- 2 components of total risk (σ):

σ = systematic risk + idiosyncratic risk

Exercise 1

You can form a portfolio of two assets which have the following characteristics:

Stock	Expected return %	Standard deviation %	Correlation
A	10	20	0.5
B	15	40	

a. If you demand an expected return of 12% what is the portfolio standard deviation?

b. If the correlation increases, what happens to expected return of the portfolio and the standard deviation?

Exercise 2

The historical standard deviation of Dell and Home Depot are respectively 29.32% and 29.27%. The correlation coefficient between the two stocks is 0.59.

a.   What is the standard deviation of a portfolio invested half in Dell and half in Home Depot?

b.   What is the standard deviation of a portfolio invested 1/3 in Dell, 1/3 in Home Depot and 1/3 in Treasury Bills (these have zero risk)? [HINT: think of this portfolio as a portfolio invested in Treasury Bills and the portfolio in part a.]

c.   What is the standard deviation of a portfolio that is split evenly between Dell and Home Depot and is financed at 50% margin, i.e., the investor puts up only 50% of the total amount and borrows the balance from his broker at risk free rate?

Exercise 3

Consider Firm A is developing a new vaccine. If the vaccine turns out successful, the firm will obtain an EPS of 100 per year from next year onwards. If the vaccine fails, the firm will liquidate, so it will not pay any dividends. Assume the result of the vaccine research is known before the first year ends and the vaccine is successful with 50% probability. Assume the discount rate is constant at 5% and equal for every firm.

a)   What is the realized return if the vaccine is successful from today until the day before the ex-dividend date for the first dividend?

b)   What is the expected return from today until the day before the ex-dividend date for the first dividend?

c)   Consider Firm B is also developing a vaccine for the same diseases. In this case, if one of the firms is successful but the other is not; the successful firm distributes 100 as dividend per year. However, if both firms are successful, each distributes 50. If the vaccine is unsuccessful, the firm always go bankrupt. Assume the probability of success of Firm B is also 50% and the success of one firm is independent of the other. What is the expected value and standard deviation of the return of a portfolio that invests 50% in Firm A and 50% in Firm B?

Exercise 4

Consider the following data for returns:

	FIRM A	FIRM B	Portfolio
Mean	4%	6%	4.5%
Standard deviation	0.2	0.3	0.198

Consider Portfolio is a portfolio of Firm A and Firm B.

a) What is the weight of firm A in Portfolio

b) What is the correlation between Firm A and Firm B returns