Microeconomics II

Spring 2024

Midterm Exam

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You have a total of 120 minutes (2 hours) to solve the exam.
Identify each sheet with your Student Number and Name.
Good luck!

I (4.5 points)

In the context of an economy with a constant returns to scale technology, show that a walrasian equilibrium has the core property.

II (6.5 points)

Consider a pure exchange economy with two goods, x and y, and two consumers, 1 and 2. The respective endowments are: $\omega_1 = (0,3)$ $\omega_2 = (2,0)$. The preferences of consumer 1 are represented by the utility function $U_1(x_1, y_1) = \sqrt{x_1} + y_1$ for $x_1 \geq 0$ and $y_1 \geq 0$. The preferences of consumer 2 are represented by $U_2(x_2, y_2) = \sqrt{1 + x_2}$ for $x_2 \geq 0$ and $y_2 \geq 0$.

Answer the following questions in a clearly drawn Edgeworth box:

(a) If this economy has Pareto optimal allocations, show them clearly on the diagram. If there are none, state that.
(b) If this economy has competitive allocations, show them clearly on the diagram. If there are none, state that.
(c) Explain whether we can ensure that a price quasiequilibrium with transfers can also be a price equilibrium with transfers in this economy.

III (4 points)

Consider the following variation of the CAPM model. There is one good and S states of nature at date t=1 (s=1,...,S). Let x_0 denote the consumption at t=0 and x_s (s=1,...,S) denote the consumption at t=1 in state s. The utility function for each agent is now a CRRA utility:

$$u^h(x_0, x_1, \ldots, x_S) = \frac{x_0^{1-\rho}}{1-\rho} + \beta \sum \gamma_s \frac{x_s^{1-\rho}}{1-\rho}$$

Normalize p_0=1 and let the aggregate endowment be denoted by:

$$\omega = (\omega_0, \tilde{\omega}) = \sum_{h=1}^{H} \omega^h$$

(a) Show that each equilibrium state price can be written as a function of $\frac{\omega_0}{\omega_s}$.
(b) Using the result in (a), determine the risk-free return.
(c) [EXTRA CREDIT] Show that, in equilibrium, the price of an asset $\tilde{x} = (x_1, \ldots, x_S)$ is of the form

$$\pi(\tilde{x}) = \frac{E(\tilde{x})}{R} - \operatorname{cov}\left[f\left(\frac{\tilde{\omega}}{\omega_0}\right), \tilde{x} \right]$$

where f is increasing.

IV (5 points)

The quality of a used car q can be measured in the interval $[0,1]$ and follows a uniform distribution. Sellers know q, buyers do not. A buyer's valuation for a used car is $1.2q$. A seller's valuation is q.

a) Will the market outcome be efficient?
b) Assume that sellers can choose to submit to a costless test that reliably identifies the quality of the car. After that, two buyers will make competing offers for the car. What would be the subgame perfect Nash equilibrium of this game?