Macroeconomics II

– Preliminary –

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Optimal Taxation

- Main reference:
  - Ljungqvist and Sargent 'Recursive macroeconomic theory' (2018), Chapter 16.
  - Chari, Nicolini and Teles 'Optimal Capital Taxation Revisited' (2020).
- Up to now: how taxes affect consumption, capital, output.
  - Taking government policies as given.
- Next: given government expenditures, how to set taxes in an optimal way.
  - Taking into account optimal decisions of households and firms.
  - And how these respond to tax policies.

Optimal Taxation

Economy: Preferences

An infinitely lived representative household has preferences over streams of consumption c_t and leisure l_t, according to:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \tag{1}$$

- $\beta \in (0, 1)$: discount factor.
- u is strictly increasing, strictly concave.
- The household is endowed with one unit of time each period that can be used for leisure l_t and labor n_t:

$$l_t + n_t = 1. \tag{2}$$

Optimal Taxation

Economy: Technology

The single good is produced with capital k_t and labor n_t. Output can be consumed by the household, used by the government, or invested in future capital:

$$c_t + g_t + k_{t+1} = f(k_t, n_t) + (1 - \delta)k_t \tag{3}$$

- $\delta \in (0, 1)$ is the depreciation rate of capital,
- $f(k, n)$ is a constant returns to scale production function.
  - Homogeneous of order 1.
  - $f(\lambda k, \lambda n) = \lambda f(k, n)$
  - $\implies f(k, n) = f_k k + f_n n \quad$ (f_x: derivative w.r.t. x)

Optimal Taxation

Economy: Government

The government finances a stream of expenditure $\{g_t\}_{t=0}^{\infty}$ with taxes on:

- capital earnings, at rate τ_{kt},
- labor earnings, at rate τ_{nt},
- and by borrowing with one-period bonds b_t:

$$g_t = \tau_{kt} r_t k_t + \tau_{nt} w_t n_t + \frac{b_{t+1}}{R_t} - b_t \tag{4}$$

- r_t rental rate of capital,
- w_t wage rate for labor,
- R_t gross rate of return on debt from t to $t+1$.

Optimal Taxation

Household

The representative household chooses $\{c_t, n_t, k_{t+1}, b_{t+1}\}_{t=0}^{\infty}$ to maximize:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \tag{5}$$

subject to the sequence of budget constraints:

$$c_t + k_{t+1} + \frac{b_{t+1}}{R_t} = (1-\tau_{nt})w_t n_t + (1-\tau_{kt})r_t k_t + (1-\delta)k_t + b_t, \; t \geq 0. \tag{6}$$

and $l_t = 1 - n_t$.

(Note that we kept the assumption that the household owns the capital stock and rents it to the firm.)

Set $\beta^t \lambda_t$ as the Lagrange multiplier on time t budget constraint. The first order conditions are:

$$
\begin{aligned}
c_t: &\qquad u_c(t) = \lambda_t, && (7)\\
n_t: &\qquad u_l(t) = \lambda_t(1-\tau_{nt})w_t, && (8)\\
k_{t+1}: &\qquad \lambda_t = \beta\lambda_{t+1}[1-\delta+(1-\tau_{kt+1})r_{t+1}], && (9)\\
b_{t+1}: &\qquad \lambda_t \frac{1}{R_t} = \beta\lambda_{t+1}, && (10)
\end{aligned}
$$

together with (6), for $t = 0, 1, ...$

$u_x(t)$ denotes the first derivative of u with respect to x, evaluated at the time t allocation.

Optimal Taxation

Household's Problem

Substituting out λ_t, we obtain:

$$u_l(t) = u_c(t)(1 - \tau_{nt})w_t, \quad (11)$$

$$u_c(t) = \beta u_c(t+1)[1 - \delta + (1 - \tau_{kt+1})r_{t+1}], \quad (12)$$

$$R_t = [1 - \delta + (1 - \tau_{kt+1})r_{t+1}]. \quad (13)$$

As before, we can iterate on successive budget constraints to eliminate b_{t+j}, to obtain the household's present value budget constraint:

$$\sum_{t=0}^{\infty} q_t c_t = \sum_{t=0}^{\infty} q_t (1 - \tau_{nt}) w_t n_t + [1 - \delta + (1 - \tau_{k0}) r_0] k_0 + b_0 \quad (14)$$

with $q_t \equiv (R_1 R_2 ... R_t)^{-1}$, $q_0 = 1$, where we have imposed:

$$\lim_{T \to \infty} q_T k_{T+1} = 0, \quad \lim_{T \to \infty} \frac{q_T}{1 + r_{T+1}} b_{T+1} = 0. \quad (15)$$

Optimal Taxation

Firm's Problem

The representative firm rents capital and hires labor from households, taking prices (r_t, w_t) as given. Period profits are given by:

$$\Pi_t = f(k_t, n_t) - r_t k_t - w_t n_t \tag{16}$$

The first order conditions are:

$$r_t = f_k(t), \tag{17}$$
$$w_t = f_n(t). \tag{18}$$

With f constant returns to scale, profits are zero ($\Pi_t = 0$).

Optimal Taxation

Competitive Equilibria with Taxes

For a given sequence of government purchases $\{g_t\}_{t=0}^{\infty}$, and budget-feasible tax policy, i.e. taxes that satisfy:

$$\sum_{t=0}^{\infty} q_t g_t + b_0 = \sum_{t=0}^{\infty} q_t \{\tau_{kt} r_t k_t + \tau_{nt} w_t n_t\} \tag{19}$$

Such as:

- $\tau_{kt} = \tau_k, \tau_{nt} = 0$;
- $\tau_{kt} = 0, \tau_{nt} = \tau_n$;
- ...
- $\{\tau_{kt}, \tau_{nt}\}_{t=0}^{\infty}$;

there is a **Competitive Equilibrium (CE)** with a potentially different allocation (c_t, n_t, k_{t+1}, g_t) and prices (u_t, w_t, r_t).

Each CE, with a different allocation, corresponds to a different value of

$$\sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_t) \tag{20}$$

Ramsey Problem:

To find the CE that yields the highest value of (20).

Formally, given $\{g_t\}_{t=0}^{\infty}$, b_0 and k_0, the problem is to find:

- A *feasible allocation*: $\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}$ that satisfies:

$$c_t + g_t + k_{t+1} = f(k_t, n_t) + (1-\delta)k_t,$$

- A *price system*: $\{R_t, w_t, r_t\}_{t=0}^{\infty}$,
- A budget-feasible *government policy*: $\{\tau_{kt}, \tau_{ct}, b_t\}_{t=0}^{\infty}$,

such that, given the price system and the government policy, the allocation solves the household's problem and the firm's problem, and maximizes (20).

Optimal Taxation

Ramsey Problem

Choose $\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}$ and $\{\tau_{kt}, \tau_{ct}, b_{t+1}\}_{t=0}^{\infty}$ to maximize:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, 1-n_t) \quad \text{subject to}$$

$$\sum_{t=0}^{\infty} q_t g_t + b_0 = \sum_{t=0}^{\infty} q_t \{\tau_{kt} r_t k_t + \tau_{nt} w_t n_t\}$$

$$u_l(t) = u_c(t)(1-\tau_{nt})w_t,$$

$$u_c(t) = \beta u_c(t+1)[1-\delta+(1-\tau_{kt+1})r_{t+1}]$$

$$\sum_{t=0}^{\infty} q_t c_t = \sum_{t=0}^{\infty} q_t (1-\tau_{nt}) w_t n_t + [1-\delta+(1-\tau_{k0})r_0]k_0 + b_0$$

$$r_t = f_k(k_t, n_t)$$

$$w_t = f_n(k_t, n_t)$$

$$c_t + g_t + k_{t+1} = f(k_t, n_t) + (1-\delta)k_t$$

The former is a complex problem involving the allocation, prices and taxes.

In order to simplify it, we can use the FOC of the household's problem and the firm's problem to express prices and taxes as a function of the allocation:

$$(R_t, r_t, w_t, \tau_{kt}, \tau_{nt}) \longrightarrow (c_t, n_t, k_{t+1})$$

Optimal Taxation

Ramsey Problem: Primal Approach

Let λ be a Lagrange multiplier on the household's budget constraint.

The FOC for $t = 0, 1, ...$ in the household's problem are:

$$c_t: \qquad \beta^t u_c(t) - \lambda q_t = 0, \tag{21}$$

$$n_t: \qquad -\beta^t u_l(t) + \lambda q_t (1 - \tau_{nt}) w_t = 0 \tag{22}$$

Recall that $q_0 = 1$. These conditions imply:

$$q_t = \beta^t \frac{u_c(t)}{u_c(0)}, \tag{23}$$

$$(1 - \tau_{nt}) w_t = \frac{u_l(t)}{u_c(t)} \tag{24}$$

Optimal Taxation

Ramsey Problem: Primal Approach

Use these two conditions to rewrite the household's budget constraint:

$$\sum_{t=0}^{\infty} q_t c_t = \sum_{t=0}^{\infty} q_t (1-\tau_{nt}) w_t n_t + [1-\delta+(1-\tau_{k0}) r_0] k_0 + b_0$$

as

$$\sum_{t=0}^{\infty} \beta^t \frac{u_c(t)}{u_c(0)} c_t = \sum_{t=0}^{\infty} \beta^t \frac{u_l(t)}{u_c(0)} n_t + [1-\delta+(1-\tau_{k0}) r_0] k_0 + b_0 \quad (25)$$

or:

$$\sum_{t=0}^{\infty} \beta^t [u_c(t) c_t - u_l(t) n_t] = u_c(0) \{[1-\delta+(1-\tau_{k0}) r_0] k_0 + b_0\} \quad (26)$$

Optimal Taxation

Ramsey Problem: Primal Approach

Simpler formulation of the Ramsey Problem (the Primal Approach):

$$\max_{\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_t) \quad \text{subject to}$$

$$\sum_{t=0}^{\infty} \beta^t [u_c(t) c_t - u_l(t) n_t] = u_c(0)\{[1 - \delta + (1 - \tau_{k0}) r_0] k_0 + b_0\} \tag{27}$$

$$c_t + g_t + k_{t+1} = f(k_t, n_t) + (1 - \delta) k_t \tag{28}$$

Optimal Taxation

Ramsey Problem

- Equation (27) is an "implementability condition".
- It is an intertemporal constraint, consolidating the household's budget constraint, the equilibrium prices and taxes, in one condition.
- After solving this problem, use the equilibrium conditions left behind to back out the remaining endogenous variables (prices and taxes) as a function of the allocation.
- The household's budget constraint and the resource constraint imply the government's budget constraint (need only 2 out of the 3).

- Period zero wealth:

$$W_0 \equiv [1 - \delta + (1 - \tau_{k0}) r_0] k_0 + b_0 \tag{29}$$

- We assume Planner sets taxes such that household must keep at least some *value* of initial wealth $\mathcal{W}_0 = u_c(0) W_0 \geq \bar{\mathcal{W}}$:

$$\sum_{t=0}^{\infty} \beta^t [u_c(t) c_t - u_l(t) n_t] \geq \bar{\mathcal{W}} \tag{30}$$

- τ_{k0} (possibly > 1) is set so that (30) is satisfied.

Ramsey Problem:

$$\max_{\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_t) \quad \text{subject to}$$

$$\sum_{t=0}^{\infty} \beta^t [u_c(t) c_t - u_l(t) n_t] - \bar{\mathcal{W}} \geq 0 \tag{31}$$

$$f(k_t, n_t) + (1 - \delta) k_t - c_t - g_t - k_{t+1} \geq 0 \tag{32}$$

Let φ be the Lagrange multiplier on constraint (31). Define:

$$V(c_t, n_t, \varphi) = u(c_t, 1 - n_t) + \varphi [u_c(t) c_t - u_l(t) n_t] \tag{33}$$

Optimal Taxation

Solving the Ramsey Problem

Let θ_t is the Lagrange multiplier on resource constraints (32). Then form the Lagrangian:

$$
\begin{aligned}
J = \sum_{t=0}^{\infty} \beta^t \{ V(c_t, n_t, \varphi) + \theta_t [f(k_t, n_t) + (1-\delta)k_t \\
- c_t - g_t - k_{t+1}]\} - \varphi \bar{\mathcal{W}} \quad (34)
\end{aligned}
$$

First order conditions:

$$
\begin{aligned}
c_t : &\quad V_c(t) - \theta_t &= 0, t \geq 0 \quad (35) \\
n_t : &\quad V_n(t) + \theta_t f_n(k_t, n_t) &= 0, t \geq 0 \quad (36) \\
k_{t+1} : &\quad -\theta_t + \beta\theta_{t+1}[f_k(k_{t+1}, n_{t+1}) + 1 - \delta] &= 0, t \geq 0 \quad (37) \\
\theta_t : &\quad f(k_t, n_t) + (1-\delta)k_t - c_t - g_t - k_{t+1} &= 0, t \geq 0. \quad (38)
\end{aligned}
$$

Rearranging the FOC:

$$\frac{V_c(t)}{\beta V_c(t+1)} = f_k(k_{t+1}, n_{t+1}) + 1 - \delta, \tag{39}$$

$$\frac{V_n(t)}{V_c(t)} = f_n(k_t, n_t), \tag{40}$$

$$f(k_t, n_t) + (1-\delta)k_t - c_t - g_t - k_{t+1} = 0. \tag{41}$$

Optimal Taxation

Steady State Results

Assume that $g_t \to g$ as $t \to \infty$. Suppose that, as $t \to \infty$, the solution to the Ramsey Problem converges to a constant steady-state solution (c_s, n_s, k_s).
Then:

$$V_c(t) = V_c(t+1), \tag{42}$$

and (39) implies that:

$$\frac{1}{\beta} = f_k(k_s, n_s) + 1 - \delta \tag{43}$$

Comparing to the Competitive Equilibrium condition:

$$\frac{1}{\beta} = (1 - \tau_{kt}) f_k(k_s, n_s) + 1 - \delta \tag{44}$$

Then, it must be that:

$$\Rightarrow \tau_{kt} = 0$$

Optimal Taxation

Steady State Results

Result 1: Zero capital income tax at the steady state

In the steady state solution of the Ramsey problem, the optimal capital income tax is zero:

$$\tau_k = 0$$

It is optimal not to distort intertemporal decisions, when the economy converges to the steady state.

Additionally, at the steady state, condition (40) becomes:

$$\frac{V_n}{V_c} = f_n(k_s, n_s), \tag{45}$$

with V_n, V_c constant (only depend on c_s, n_s, φ). Comparing to the Competitive Equilibrium counterpart:

$$\frac{u_l}{u_c} = (1 - \tau_{nt}) f_n(k_s, n_s). \tag{46}$$

Result 2: Constant labor income tax at the steady state
In the steady state solution of the Ramsey problem, the optimal labor income tax is constant:

$$\tau_{nt} = \tau_n$$

It is optimal to distort intratemporal decisions with a constant (possibly zero) tax, when the economy converges to the steady state.

Optimal Taxation

More Results

Below we assume that preferences are given by:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, n_t) = \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma} - 1}{1-\sigma} - \eta n_t^{\psi} \right] \qquad (47)$$

Standard in macro models (constant elasticities). We can now evaluate V_c, V_n under this assumption.

$$\begin{aligned} V_c(t) &= u_c(t) + \varphi[u_{cc}(t)c_t + u_c(t)] \\ &= u_c(t) \left[1 + \varphi \left(\frac{u_{cc}(t)c_t}{u_c(t)} + 1 \right) \right] \end{aligned}$$

Given (47), this simplifies to

$$V_c(t) = u_c(t) \left[1 + \varphi(1-\sigma) \right]$$

Back to condition (39) of the the Ramsey problem solution:

$$\frac{V_c(t)}{\beta V_c(t+1)} = f_k(k_{t+1}, n_{t+1}) + 1 - \delta, t \geq 0$$

becomes

$$\frac{u_c(t)}{\beta u_c(t+1)} = f_k(k_{t+1}, n_{t+1}) + 1 - \delta, t \geq 0.$$

Compare to the Competitive Equilibrium condition:

$$\frac{u_c(t)}{\beta u_c(t+1)} = (1 - \tau_{kt+1}) f_k(k_{t+1}, n_{t+1}) + 1 - \delta, t \geq 0.$$

The Ramsey solution requires:

$$\Rightarrow \tau_{kt+1} = 0, t \geq 0. \tag{48}$$

Result 3: Zero capital income tax from period $t = 1$.

In the solution of the Ramsey problem, the optimal capital income tax is zero during the transition:

$$\tau_{kt+1} = 0, t \geq 0.$$

- With standard macro preferences, since elasticities (σ, ψ) are constant over time, it is optimal to have no intertemporal distortions from period one onward.
- τ_{k0}, capital tax in period $t = 0$, is set so that household gets at least value $\mathcal{W}_0$ from initial wealth.
- Intuition: A positive (constant) capital income tax has the same distortion on consumption and savings as a growing consumption tax $(\tau_{ct+1}/\tau_{ct}) > 1$.

Optimal Taxation

With preferences as in (47), the leisure-consumption condition becomes:

$$\frac{V_n(t)}{V_c(t)} = f_n(k_t, n_t) \iff \frac{u_n(t)}{u_c(t)} \frac{(1-\varphi\psi)}{1+\varphi(1-\sigma)} = f_n(k_t, n_t), t \geq 0$$

Compare to the Competitive Equilibrium condition:

$$\frac{u_n(t)}{u_c(t)} = (1-\tau_{nt}) f_n(k_t, n_t)$$

The two are consistent if and only if:

$$\Rightarrow \tau_{nt} = \tau_n, t \geq 0. \tag{49}$$

Result 4: Constant labor income tax from period $t = 0$.
In the solution of the Ramsey problem, the optimal labor tax is constant.

Optimal Taxation

Optimal Tax Results

- In general, optimal taxes in the long-run steady state have:
  - **R1.** No tax on capital,
  - **R2.** Constant tax on labor (or consumption).
- For standard preferences over consumption and labor:
  - **R3.** Optimal capital income tax is zero always (except eventually at $t = 0$),
  - **R4.** Labor (or consumption) is taxed at a constant rate.
- The result that intertemporal decisions should not be distorted extends to other taxes:
  - time-varying consumption taxes, wealth taxes, dividend taxes are sub-optimal.