Uncertainty

Advanced Microeconomics - Pratical Lecture 9

João Bonito Gomes

Nova School of Business and Economics

April 15, 2025

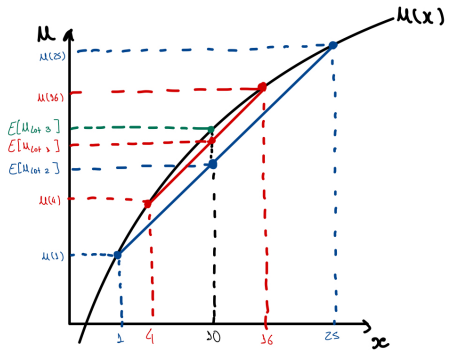
a) Which lottery is preferred by a risk-loving, risk-neutral or risk-averse decision-maker, respectively, and why?

The agent chooses the lottery that gives the **highest expected utility** for each type of attitude towards risk.

Risk-Averse Agent

Prefers certainty.
Dislikes volatility of values across states.

- $u(.)$ is strictly concave.
- $u''(.) < 0$
- $E(u_{Lot}) < u(E[Lot])$



Exercise 1
Problem Set 9

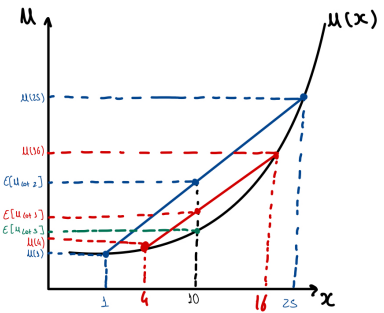
a) Which lottery is preferred by a risk-loving, risk-neutral or risk-averse decision-maker, respectively, and why?

The agent chooses the lottery that gives the **highest expected utility** for each type of attitude towards risk.

Risk-Loving Agent

Prefers uncertainty.
Prefers lotteries that can provide the greatest payoffs, regardless of the probability of achieving them.

- $u(.)$ is strictly convex.
- $u''(.) > 0$
- $E(u_{Lot}) > u(E[Lot])$



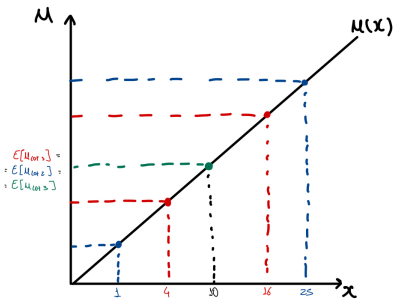
a) Which lottery is preferred by a risk-loving, risk-neutral or risk-averse decision-maker, respectively, and why?

The agent chooses the lottery that gives the **highest expected utility** for each type of attitude towards risk.

Risk-Neutral Agent

Indifferent between all alternatives as long as they deliver the same expected value.

- $u(.)$ is linear.
- $u''(.) = 0$
- $E(u_{Lot}) = u(E[Lot])$



b) For $u(x) = x^{\frac{1}{2}}$ and $u(x) = x^2$ calculate and compare the certainty equivalent and risk premium of each lottery.

Certainty Equivalent

The Certainty Equivalent (CE) of a given lottery is the value that solves $u(CE) = E(u_{Lot})$, i.e., it is the value for which the agent is indifferent between receiving that value for sure or facing the lottery.

Risk Premium

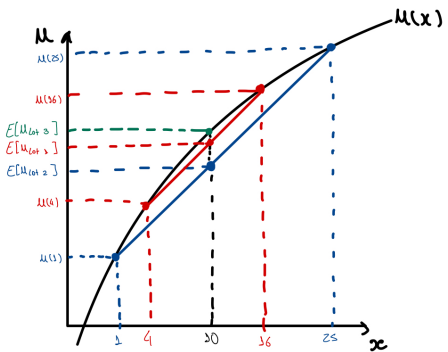
The Risk Premium (RP) of a given lottery is computed as $RP = E(Lot) - CE$, i.e., it is the maximum value the agent is willing to give up from the expected payment in order to avoid risk.

b) For $u(x) = x^{\frac{1}{2}}$ and $u(x) = x^2$ calculate and compare the certainty equivalent and risk premium of each lottery.

Risk-Averse Agent

Prefers certainty.
Dislikes volatility of values across states.

- $u(.)$ is strictly concave.
- $u''(.) < 0$
- $E(u_{Lot}) < u(E[Lot])$
- $RP > 0 \leftrightarrow E(Lot) > CE$

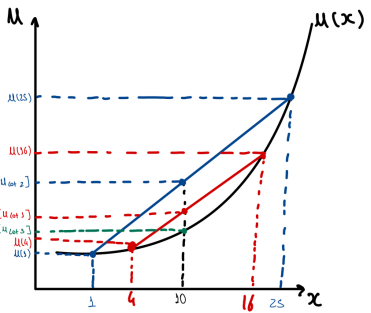


b) For $u(x) = x^{\frac{1}{2}}$ and $u(x) = x^2$ calculate and compare the certainty equivalent and risk premium of each lottery.

Risk-Loving Agent

Prefers uncertainty.
Prefers lotteries that can provide the greatest payoffs, regardless of the probability of achieving them.

- $u(.)$ is strictly convex.
- $u''(.) > 0$
- $E(u_{Lot}) > u(E[Lot])$
- $RP < 0 \leftrightarrow E(Lot) < CE$

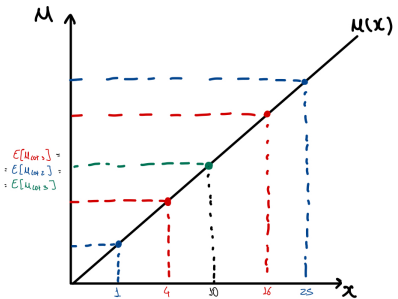


b) For $u(x) = x^{\frac{1}{2}}$ and $u(x) = x^2$ calculate and compare the certainty equivalent and risk premium of each lottery.

Risk-Neutral Agent

Indifferent between all alternatives as long as they deliver the same expected value.

- $u(.)$ is linear.
- $u''(.) = 0$
- $E(u_{Lot}) = u(E[Lot])$
- $RP = 0 \leftrightarrow E(Lot) = CE$



$U = \ln(4C)$ | Income of 30 000 per year | 5% chance of facing a health problem and a loss of 20 000

a) Suppose you can buy insurance. How much does a unit of insurance cost if the insurance is actuarially fair? How much insurance would you buy in this case?

Actuarially fair insurance

An insurance is actuarially fair if the insurance firm makes zero profits in expectation:

$$\pi_1(P - I) + (1 - \pi_1)P = 0$$

Is the insurance is actuarially fair: Unit Cost = Probability of Loss.
A **risk-averse agent** faced with **actuarially fair insurance** will choose to buy **full insurance.**

b) What is the maximum amount that you would be willing to pay for full insurance?

The maximum willingness to pay for full insurance is equal to the insurance premium that makes you indifferent between having full insurance or no insurance.

Exercise 3
Problem Set 9

Probability p that an illegally parked car will be ticketed.
$\$X$ is the price of parking legally | $\$F$ is the price of the ticket by parking illegally

a) Suppose that the aim of the traffic department is to discourage illegal parking. If all drivers have constant marginal utility of income, what is the minimum fine F that the traffic department should impose, in order to achieve this aim?

b) Suppose now that all drivers have diminishing marginal utility of income and experience neither utility nor disutility from obeying the law. How does this affect your answer to (a)? What if all drivers have increasing marginal utility of income?