Game Theory II

Advanced Microeconomics - Pratical Lecture 6

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Midterm: March 26th (Wednesday), 2:30 pm
Duration: 1 hour and 20 minutes
Material: Until this class (including what we are going to see today)

Office Hour: 20th March (Thrusday) at 5PM, Room B003 (please come with your doubts and questions prepared).

Last Class: Static Games with Complete Information

Today's Class: Dynamics Games with Complete Information

- Perfect Information
- Imperfect Information

Plan for Today

Dynamic Games with Complete Information

Why **Dynamic**?

The agents take their **actions sequentially** - a strategy may be chosen at some point in the game with knowledge about previous moves carried out by other players.

Why with **Complete Information**?

Agents are **fully aware of the relevant characteristics of the remaining players** (such as they types, available options and the respective payoffs).

Why with **Perfect Information** or **Imperfect Information**?

Games with **Perfect Information** have the feature that players know what their rivals chose before them.

In games with **Imperfect Information**, it can be the case that players are not aware of the preceding choices of their rivals.

Plan for Today

Dynamic Games with Complete Information - Game Representation (Game Tree)

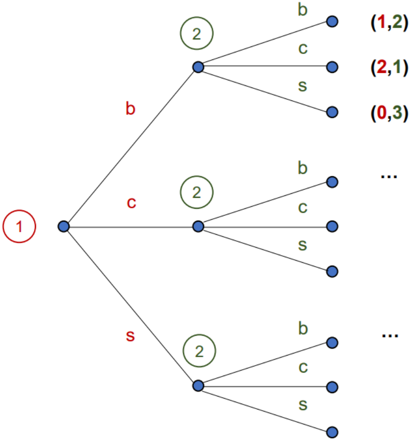
Node

Point in a game where one player has to make a decision.

Information Set

It represents the level of detail in the information that a player has. Does the player know the choice the preceding player made? Or does he only know that the player made a choice?

> When there is **one node** in each information set, the player knows what his rival chose and we have a game with **Perfect Information**.
>
> When there is **more than one node** in each information set, the player only knows that his rival made a choice. But he does not know which one. Hence, we have a game with **Imperfect Information**.

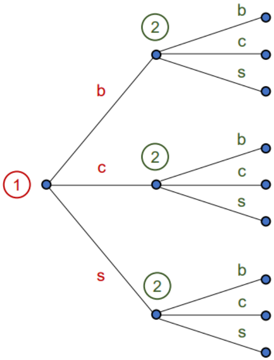


Strategy

Conditional and complete plan of actions.

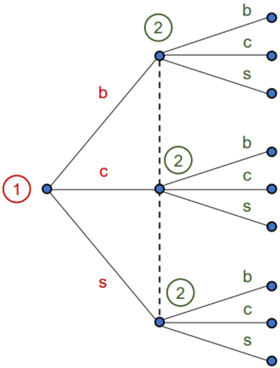
> **Conditional** because it tells each player which branch to follow at each information set.
>
> **Complete** because it tells what to choose at every information set.

Perfect Information



Player 1: 1 information set and 3 strategies.
Player 2: 3 information sets and 3 x 3 x 3 strategies

Imperfect Information



Player 1: 1 information set and 3 strategies.
Player 2: 1 information sets (with 3 nodes) and 3 strategies

Subgame

Subset of the game which does not break any information set.

There are Nash Equilibria that players would never choose - they are not **credible**.

- A choice by other player is **credible** if, after the other player makes his choice, he has incentives (in terms of payoffs) to make that choice.
- A Nash Equilibrium is a **Subgame Perfect Nash Equilibrium (SPNE)** if it is a Nash Equilibrium for **every** subgame.
- We can rule out unreasonable Nash Equilibria by using **Backward Induction**.

Subgame Perfect Nash Equilibrium (SPNE)

Vector of strategies that, when confined to any subgame of the original game, have the players playing a Nash Equilibrium within that subgame.

If players do not know the preceding action, it is as they are playing a **simultaneous game**, which we can represent (and solve) in the usual manner through the payoff matrix (strategic form).

1/2	L	R
A	1 , 2	1 , 2
B	0 , 0	2 , 1

a) Draw the extensive form of this game.
(assuming **player 1** plays first)

b) Find the Nash Equilibria. Which one of them makes more sense?

c) Use Backward Induction to solve the problem.

a) How many information sets are there for each player? Specify the pure strategies for each player.

Information sets:

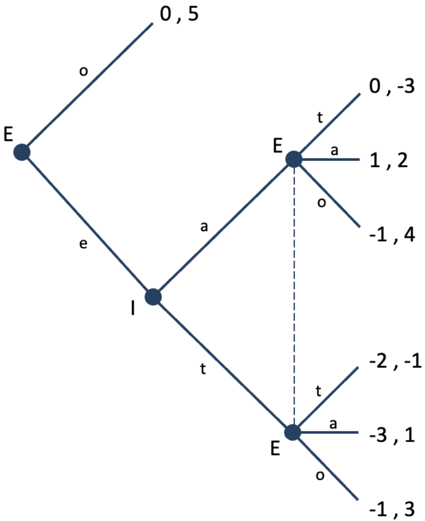
Player E: 2 information sets

Player I: 1 information set

Pure strategies:

Player E: et, ea, eo, ot, oa, oo

Player I: t, a



Exercise 2
Problem Set 6

NOT THE ACTUAL EXERCISE, JUST AN EXAMPLE!
What if it was a game with perfect information?

a) How many information sets are there for each player? Specify the pure strategies for each player.

Information sets:

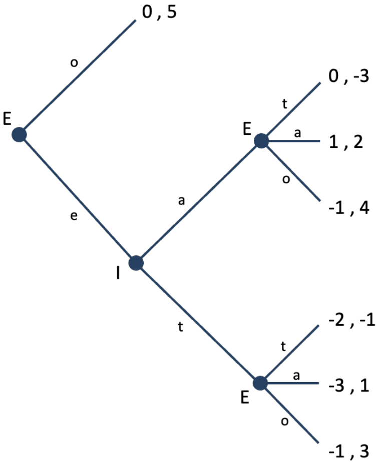
Player E: 3 information sets

Player I: 1 information set

Pure strategies:

Player E: ett, eta, eto, eat, eaa, eao, eot, eaa, eoo, ott, ota, oto, oat, oaa, oao, oot, ooa, ooo

Player I: t, a



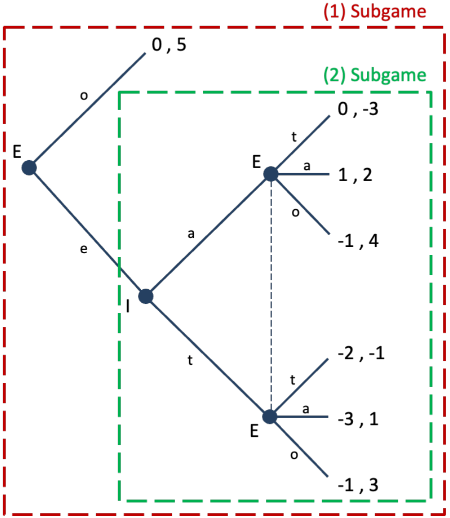
b) How many subgames are there, Is the game solvable by backward induction?

The game has **2 subgames**:

(1) The whole game.

(2) The subgame that starts on the node where player I is choosing between t, a and o after observing player E choosing e.

The game **cannot be solved using backward induction** because it is not a game with perfect information (because some information set contains more than one node).



NOT THE ACTUAL EXERCISE, JUST AN EXAMPLE!

What if it was a game with perfect information?

b) How many subgames are there, Is the game solvable by backward induction?

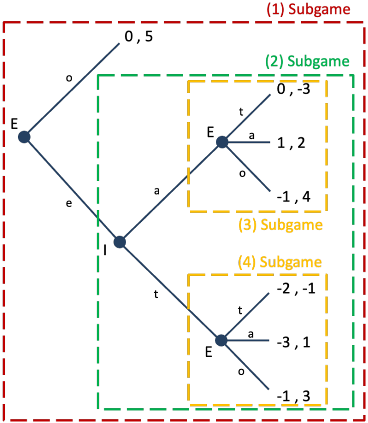
The game has **4 subgames**:

(1) The whole game.

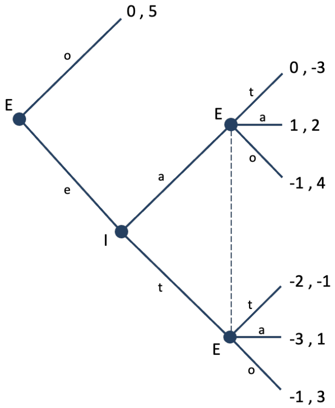
(2) The subgame that starts on the node where player I is choosing between t, a and o after observing player E choosing e.

(3) The subgame that starts on the node where player E is choosing between t, a and o after firstly choosing e, and player I previously choosing t.

(4) The subgame that starts on the node where player E is choosing between t, a and o after firstly choosing e and player I previously choosing a.



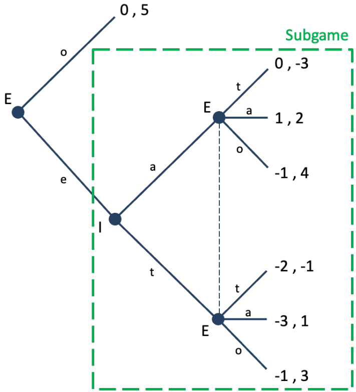
c) Find the Nash Equilibria and subgame perfect Nash Equilibria.
Remember: **Player E's strategies:** et, ea, eo, ot, oa, oo | **Player I's strategies:** t, a
Finding the Nash Equilibria:



We can represent the game in matrix form:

E / I	t	a
et	-2, -1	0 , -3
ea	-3 , 1	1 , 2
eo	-1 , 3	-1 , 4
ot	0 , 5	0 , 5
ao	0 , 5	0 , 5
oo	0 , 5	0 , 5

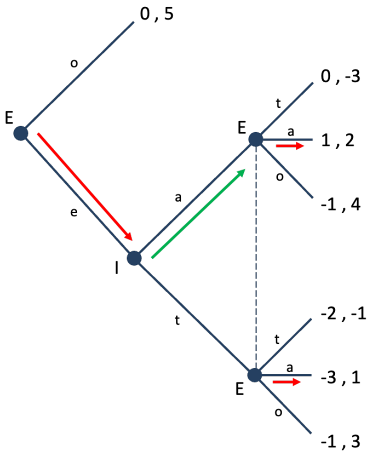
c) Find the Nash Equilibria and subgame perfect Nash Equilibria.
Finding the SPNE (in pure strategies): Find the NE of the last/smallest subgame, and then start moving up.



We can represent the game in matrix form (because it is a game with incomplete information, so it is equivalent as if they were playing at the same time):

E / I	t	a
t	-2, -1	0 , -3
a	-3 , 1	1 , 2
o	-1 , 3	-1 , 4

c) Find the Nash Equilibria and subgame perfect Nash Equilibria.
Now that we know the NE of the last subgame, we can move upwards and consider the whole game:



Two players: Firm 1 and Firm 2.
Firm 1 **chooses first**.
Firm 2 observes Firm 1's choice and **chooses q_2 afterwards**.
Inverse demand: $P = a - bQ$, with $Q = q_1 + q_2$
Marginal cost: c for both firms.

a) What is firm 2's best response?

b) Find the subgame perfect Nash equilibrium (SPNE) by backward induction.

c) Consider a market with inverse demand given by $P = 1000 - Q$ and a constant marginal cost equal to 100. Determine the SPNE.