Game Theory I

Advanced Microeconomics - Pratical Lecture 5

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What is **Game Theory**?

It is concerned with the analysis of **strategic interactions** between players. It provides a framework to analyze the outcomes and decisions of agents in cases where their interactions are **independent**, that is, when the outcome (payoff) of each agent's actions depends on other agents' actions/decisions.

Example: International climate negotiations, firms' price and quantity decisions, political choices.

Key components of a game

(1) Players: Who is playing the game.

(2) Information and strategies: Possible decisions/course of actions.

(3) Payoffs: Outcome of decisions.

Why **static**?

The agents choose their action without knowing the action carried out by the remaining players. In fewer words, a game in which every player plays at the **same time**.

Why with **complete information**?

Agents are **fully aware of the relevant characteristics of the remaining players** (such as they types, available options and the respective payoffs).

a) Identify all pairs of strategies where one weakly dominates the other.

1/2	D	E	F
A	1,0	3,1	1,1
B	1,1	3,2	0,1
C	2,2	3,3	0,2

Weakly dominant strategy

s_i^1 is a **weakly dominant strategy** for player i if for all $s_i' \neq s_i^1$ and for all $s_j \in S_j$, $\pi_i(s_i^1, s_j) \geq \pi_i(s_i', s_j)$

Strategy s_i^1 weakly dominates strategy s_i' if, for any given action carried out by the other player (player j), playing s_i^1 results in at least the same payoff than playing s_i').

a) Identify all pairs of strategies where one weakly dominates the other.

For **Player 1**:

A weakly dominates B

1/2	D	E	F
A	**1**,0	**3**,1	**1**,1
B	**1**,1	**3**,2	**0**,1
C	2,2	3,3	0,2

C weakly dominates B

1/2	D	E	F
A	1,0	3,1	1,1
B	**1**,1	**3**,2	**0**,1
C	**2**,2	**3**,3	**0**,2

No dominance relation between A and C

1/2	D	E	F
A	**1**,0	**3**,1	**1**,1
B	1,1	3,2	0,1
C	**2**,2	**3**,3	**0**,2

No weakly dominant strategy since there is no strategy that weakly dominates all the other strategies of player 1.

a) Identify all pairs of strategies where one weakly dominates the other.

For **Player 2**:

E weakly dominates D

1/2	D	E	F
A	1,**0**	3,**1**	1,1
B	1,**1**	3,**2**	0,1
C	2,**2**	3,**3**	0,2

F weakly dominates D

1/2	D	E	F
A	1,**0**	3,1	1,**1**
B	1,**1**	3,2	0,**1**
C	2,**2**	3,3	0,**2**

E weakly dominates F

1/2	D	E	F
A	1,0	3,**1**	1,**1**
B	1,1	3,**2**	0,**1**
C	2,2	3,**3**	0,**2**

E is a weakly dominant strategy since it weakly dominates all the other strategies of player 2.

b) Do the same for strategies where one strongly dominates the other.

Strictly dominant strategy

s_i^1 is a **strictly dominant strategy** for player i if for all $s_i' \neq s_i^1$ and for all $s_j \in S_j$, $\pi_i(s_i^1, s_j) > \pi_i(s_i', s_j)$

Strategy s_i^1 weakly dominates strategy s_i' if, for any given action carried out by the other player (player j), playing s_i^1 results in a higher payoff than playing s_i').

For **Player 1**: No relationships of strong/strict dominance.
For **Player 2**: E strongly dominates D.

c) Does the game have an equilibrium in dominant strategies?.

Equilibrium in dominant strategies

There is an equilibrium in dominant strategies when both players are playing a dominant strategy.

Player 1 does not have a dominant strategy (not even a weakly dominant strategy) and therefore the game **cannot have an equilibrium in dominant strategies**.

a) Are there strategies that cannot be part of a Nash Equilibrium of this game? If so, which and why? Represent the reduced game obtained by eliminating these strategies.

1/2	A	B	C	D
a	1,0	3,1	2,2	8,-1
b	2,4	7,1	-1,3	6,0
c	3,6	8,0	0,5	4,1
d	0,0	6,1	1,6	3,2

Nash Equilibrium

Each player is playing a **best response to the others' strategy choice**, i.e., each playing is playing her optimal strategy, given what the other players are playing. In a NE, no player has an incentive to unilaterally deviate.

This implies that a **strictly dominated strategy will never** be part of a NE.

a) Are there strategies that cannot be part of a Nash Equilibrium of this game? If so, which and why? Represent the reduced game obtained by eliminating these strategies.

1/2	A	B	C	D
a	1,0	3,1	2,**2**	8,-1
b	2,**4**	7,1	-1,3	6,0
c	3,**6**	8,0	0,5	4,1
d	0,0	6,1	1,**6**	3,2

1/2	A	B	C	D
a	1,0	3,1	2,**2**	8,-1
b	2,**4**	7,1	-1,3	6,0
c	3,**6**	8,0	0,5	4,1
d	0,0	6,1	1,**6**	3,2

For **Player 2**: B and D are never best responses, so we can eliminate them.

a) Are there strategies that cannot be part of a Nash Equilibrium of this game? If so, which and why? Represent the reduced game obtained by eliminating these strategies.

1/2	A	C
a	1,0	**2**,2
b	2,4	-1,3
c	**3**,6	0,5
d	0,0	1,6

1/2	A	C
a	1,0	**2**,2
b	2,4	-1,3
c	**3**,6	0,5
d	0,0	1,6

For **Player 1**: b and d are never best responses, so we can eliminate them.

a) Are there strategies that cannot be part of a Nash Equilibrium of this game? If so, which and why? Represent the reduced game obtained by eliminating these strategies.

1/2	A	C
a	1,0	2,2
c	3,6	0,5

There are no more strategies that are never best responses, so we cannot eliminate further strategies.

Therefore, this is the **reduced game** we obtain after iteratively eliminating the never best responses.

b) Find all pure and mixed strategy Nash Equilibria of the reduced game.

Finding all Nash Equilibria

(1) Nash Equilibria in **pure strategies**.

Agents make that choice 100% of the time.

(2) Nash Equilibria in **mixed strategies**.

Assign probability to each choice and play according to assigned probabilities.
Why mixed strategies? **Theorem:** A game with a finite set of strategies and players always has a Nash Equilibrium, at least in mixed strategies.

b) Find all pure and mixed strategy Nash Equilibria of the reduced game.

In **Pure Strategies**:

1/2	A	C
a	1,0	**2,2**
c	**3,6**	0,5

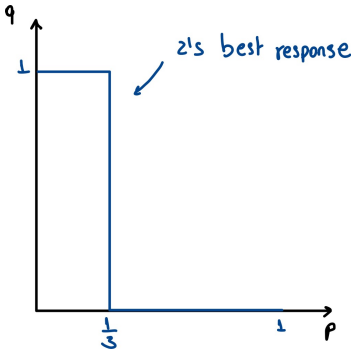
b) Find all pure and mixed strategy Nash Equilibria of the reduced game.

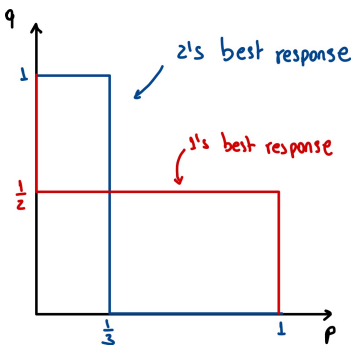
In **Mixed Strategies**:

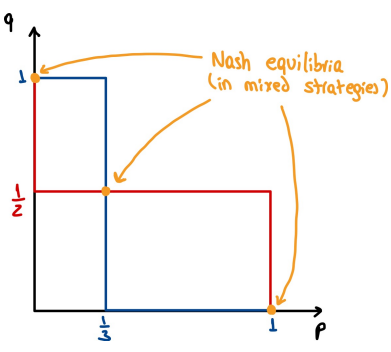
		q	**1 - q**
	1/2	**A**	**C**
p	**a**	1,0	2,2
1 - p	**c**	3,6	0,5

b) Find all pure and mixed strategy Nash Equilibria of the reduced game.

In **Mixed Strategies**:







Exercise 3
Problem Set 5

a) What kind of game is this?

This is a simultaneous game (more precisely, a static game with complete information).

b) Can you represent it in a strategic form?

In order to represent this game in a strategic form, we must be able to represent the **3 key components of a game: players, strategies and payoffs.**

(1) Players:

Firm 1
Firm 2

(2) Strategies:

$q_1 \in [0, +\infty[$
$q_2 \in [0, +\infty[$

(3) Payoffs

$\pi_1 = (a - c)q_1 - bq_1^2 - bq_1q_2$
$\pi_2 = (a - c)q_2 - bq_2^2 - bq_1q_2$

c) What are the firms best responses?

d) What is the Nash Equilibrium?

e) Consider a market inverse demand given by $P = 120 - 4Q$ and a constant marginal cost equal to 20. Determine the Nash Equilibrium.

$a = 120 \mid b = 4 \mid c = 20$