General Equilibrium II

Advanced Microeconomics - Pratical Lecture 2

João Bonito Gomes

Nova School of Business and Economics

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Exercise 1.2

Problem Set 1 (continuation from last class)

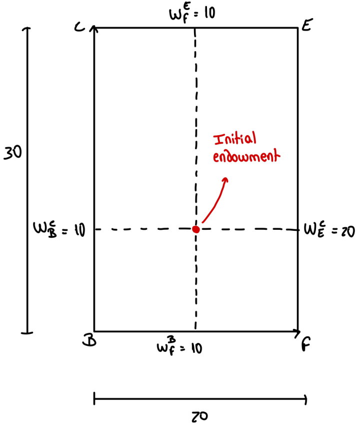
Bert: B **Initial Endowments ($f \rightarrow$ food, $c \rightarrow$ clothing):** $w_f^B = 10, w_c^B = 10 \mid w_f^E = 10, w_c^E = 20$

Ernie: E

a) Represent these initial endowments in an Edgeworth box.

Total amount of f: $w_f^B + w_f^E = 10 + 10 = 20$

Total amount of c: $w_c^B + w_c^E = 10 + 20 = 30$



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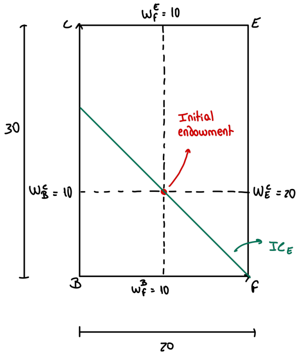
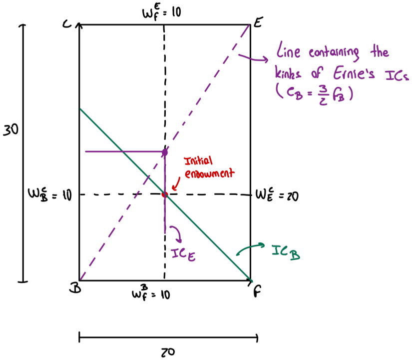
Ernie: E

b) Bert regards food and clothing as perfect 1-for-1 substitutes. Ernie regards them as perfect complements, always wanting 3 units of clothing for every 2 units of food. Describe the set of allocations that are Pareto preferred to initial endowment.





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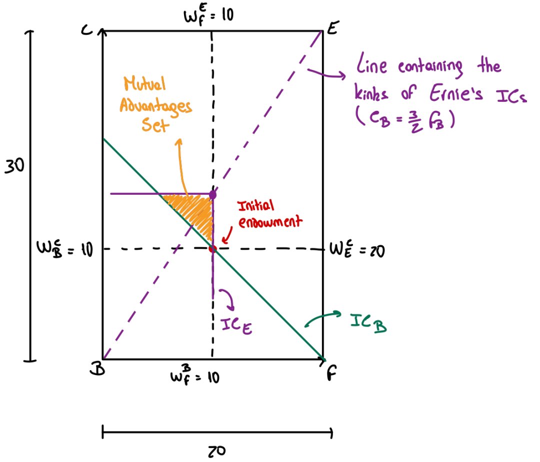

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Ernie: E

c) Describe the contract curve for that allocation.

How to graphically derive a contract curve

Do both agents have well-behaved (e.g., Cobb-Douglas preferences)? No! We have to think.
We can try the following process to find the contract curve graphically:

(1) Fix one agent's utility (e.g., Ernie).

(2) Think about the highest utility level the other agent (Bert) can achieve given the fixed level of utility of the initial agent (Ernie).

(3) With this new highest level of utility for the other agent fixed, see if the utility level of the initial agent cannot increase even further.

Generalize the process by repeating these steps for different initials levels of Ernie's utility.

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Ernie: E

d) What price ratio will be required to sustain an allocation on the contract curve?

Exercise 1.2

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Ernie: E

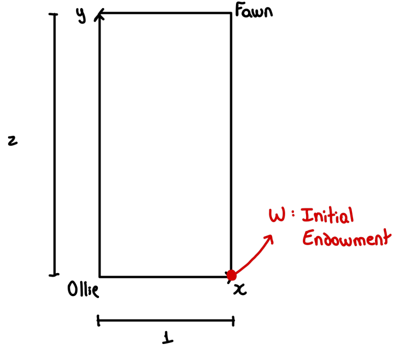
e) How will your answers differ if 5 units of Ernie's clothing endowment are given to Bert?

Ollie: $U_O(x_O, y_O) = 3x_O + y_O$ **Initial Endowments:** $w_x^O = 1, w_y^O = 0 \mid w_x^F = 0, w_y^F = 2$
Fawn: $U_F(x_F, y_F) = x_F y_F$

a) Mark the initial allocation with the letter W.

Total amount of x: $w_x^O + w_x^F = 1 + 0 = 1$

Total amount of y: $w_y^O + w_y^F = 0 + 2 = 2$



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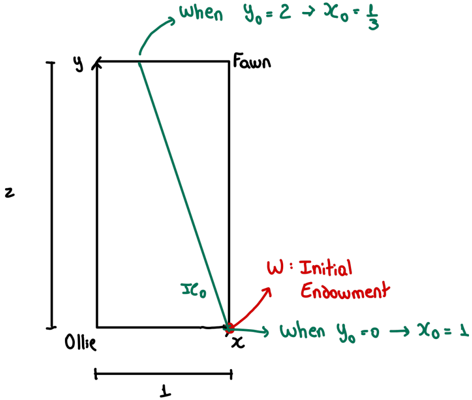
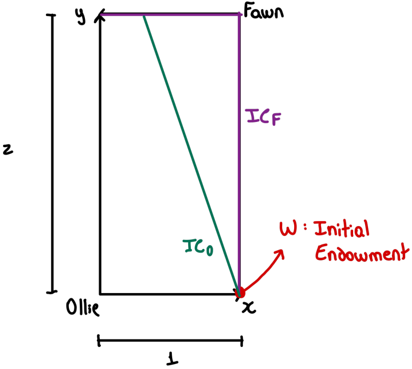
b) Draw the indifference curves that go through the initial endowments.





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Problem Set 2

Ollie: $U_O(x_O, y_O) = 3x_O + y_O$ **Initial Endowments:** $w_x^O = 1, w_y^O = 0 \mid w_x^F = 0, w_y^F = 2$

Fawn: $U_F(x_F, y_F) = x_F y_F$

c) Find the contract curve.

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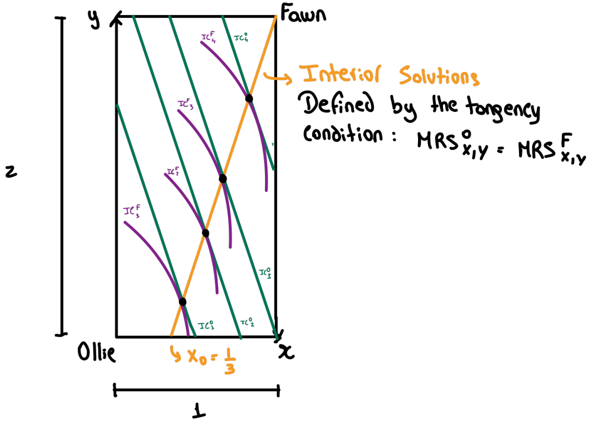

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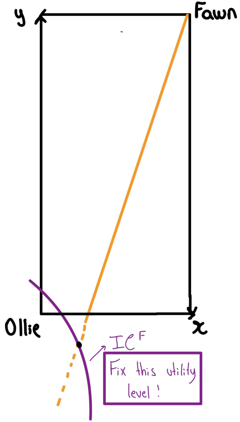

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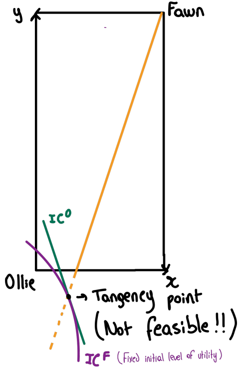

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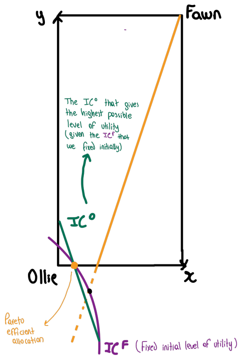

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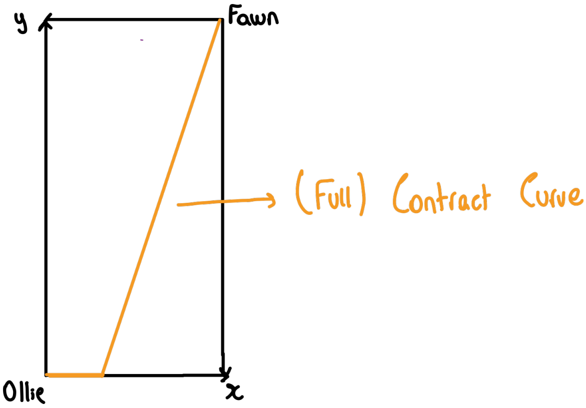

Exercise 1.2
Problem Set 2

Agent a: $U_a(x_a, y_a) = 4x_a^{1/2}y_a^{1/2}$ **Initial Endowments:** $w_x^a = 1, w_y^b = 1 \mid w_x^b = 1, w_y^b = 1$
Agent b: $U_b(x_b, y_b) = x_b^{1/3}y_b^{2/3}$

a) State the budget constraint and solve the problem for each agent.

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Problem Set 2

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Agent b: $U_b(x_b, y_b) = x_b^{1/3} y_b^{2/3}$

b) Find the competitive equilibrium and draw the edgeworth box.

Competitive (or Walrasian) Equilibrium

(1) All agents maximize their utility subject to their budget constraint.

(2) All markets clear, i.e., the price ratio $\frac{p_x}{p_y}$ must be such that:

$$x_a^* + x_b^* = w_x^a + w_x^b$$
$$y_a^* + y_b^* = w_y^a + w_y^b$$

Walras' Law

The value of the aggregate excess demand is zero for all prices, i.e.:

$$p_x(x_a + x_b - w_x^a - w_x^b) + p_y(y_a + y_b - w_y^a - w_y^b) = 0$$

The implication is that, if there are markets for n goods, then we only need to compute the prices for $n-1$ markets.

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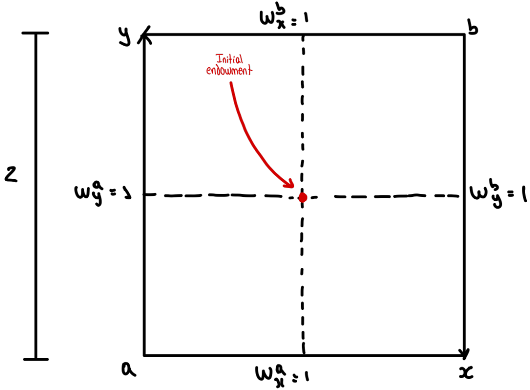

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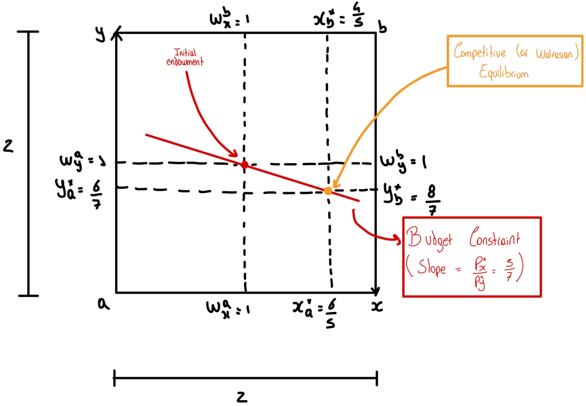

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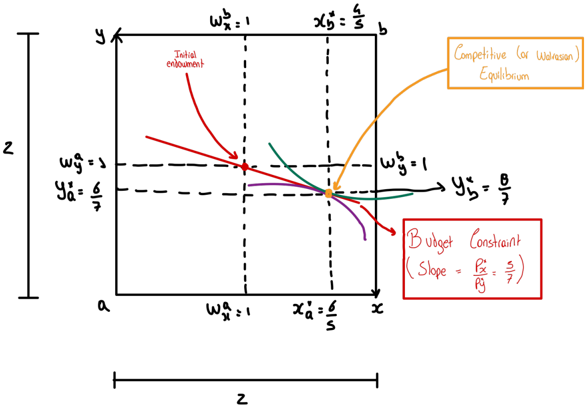

Summing up

How to **usually** derive the Contract Curve (some specific cases may require different solutions)

3 Types of Preferences: Cobb-Douglas, Perfect Substitutes, and Perfect Complements

- **Cobb-Douglas & Cobb-Douglas:** Solve $MRS_A = MRS_B$, and you are good to go **(Problem Set 1 - Exercise 1.1)**!
- **Perfect Substitutes & Perfect Complements:** Solve the kinks of the perfect complements' utility function **(Problem Set 1 - Exercise 1.2)**.
- **Cobb-Douglas & Perfect Substitutes:** Solve $MRS_A = MRS_B$, and add the points up till the corner of the agent with the Perfect Substitutes **(Problem Set 2 - Exercise 1.1)**.

Summing up

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3 Types of Preferences: Cobb-Douglas, Perfect Substitutes, and Perfect Complements

- **Cobb-Douglas & Perfect Complements:** Solve the kinks of the perfect complements' utility function and add the points up till the corner of the agent with Cobb-Douglas.
- **Perfect Substitutes & Perfect Substitutes:** The entire box if $MRS_A = MRS_B$, and the axis' on which agents have the full amount of the good they prefer relatively more if $MRS_A \neq MRS_B$
- **Perfect Complements & Perfect Complements:** Area between kinks functions for both agents (and the functions themselves).