Auctions

Advanced Microeconomics - Practical Lecture 12

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English/Dutch Style Auction

English Style Auction: The price for an item starts low and increases as bidders compete, until only one bidder remains. The bidder pays the second-highest bid plus some increment.

Dutch Style Auction: The price for an item starts high and decreases until someone places a bid. The bidder pays the highest bid.

First-Price Sealed Bid Auction

Highest bidder gets the object and pays his own bid.

- If bids are identical, equal chance of getting the object.
- Winner gets payoff equal to difference between valuation and his own bid. Loser gets 0.

Second-Price Sealed Bid Auction

Highest bidder gets the object, but only pays the second highest submitted bid.

- If bids are identical, equal chance of getting the object.
- Winner gets payoff equal to difference between valuation and second highest bid. Loser gets 0.

Strategic Equivalence:

- English and second-price auctions.
- Dutch and first-price auctions.

Revenue Equivalence

Principal (the seller) will get the same expected revenue either from a first-price or from a second-price auction as long as agents (buyers):

- Have **independent** valuations.
- Are **risk neutral**.

If first-price and second-price lead to the same revenue, is there a better auction mechanism?

Yes! One that satisfies both the Individual Rationality (IR_i) and the Incentive Compatibility (IC_i) constraints.

- There is a **single unit** of a good to be sold.
- There are **two buyers** (agents).
- Each buyer values the good at either θ or μ, with $\theta > \mu > 0$.
- The seller (principal) cannot observe valuation but knows the probability of type θ.
- Type θ and type μ's payoffs are respectively:
  ($\theta - \textit{price}$**)** if she gets the object and 0 otherwise and
  ($\mu - \textit{price}$**)** if she gets the object and 0 otherwise.
- Principal wants to **maximize expected sale price**.

Higher valuation bidders, type θ

$P(\theta)$ is the total probability type θ wins the object:

$$P(\theta) = Pr(other\ agent\ is\ \theta) \cdot P(\theta, \theta) + Pr(other\ agent\ is\ \mu) \cdot P(\theta, \mu)$$

Where:

- $P(\theta, \theta)$ is the probability of θ getting object if other agent is also θ
- $P(\theta, \mu)$ is the probability of θ getting object if other agent is also μ

Thus:

$$P(\theta) = Pr(type\ \theta) \cdot P(\theta, \theta) + Pr(type\ \mu) \cdot P(\theta, \mu)$$

$M(\theta)$ is the expected payment type θ makes to the seller.

Lower valuation bidders, type μ

$P(\mu)$ is the total probability type μ wins the object:

$$P(\mu) = Pr(other\ agent\ is\ \theta) \cdot P(\mu, \theta) + Pr(other\ agent\ is\ \mu) \cdot P(\mu, \mu)$$

Where:

- $P(\mu, \theta)$ is the probability of μ getting object if other agent is also θ
- $P(\mu, \mu)$ is the probability of μ getting object if other agent is also μ

Thus:

$$P(\mu) = Pr(type\ \theta) \cdot P(\mu, \theta) + Pr(type\ \mu) \cdot P(\mu, \mu)$$

$M(\mu)$ is the expected payment type μ makes to the seller.

Participation Constraints (Individual Rationality)

Expected Payoff ≥ 0

Type θ, IR_θ: $\theta P(\theta) - M(\theta) \geq 0$

Type μ, IR_μ: $\mu P(\mu) - M(\mu) \geq 0$

Incentive Compatibility Constraints

Expected payoff bidding own valuation $\geq$ Expected payoff bidding other type's valuation

Type θ, IC_θ: $\theta P(\theta) - M(\theta) \geq \theta P(\mu) - M(\mu)$

Type μ, IC_μ: $\mu P(\mu) - M(\mu) \geq \mu P(\theta) - M(\theta)$

Principal (seller)'s Problem

- **Maximize expected revenue:** $E[\pi_{seller}] = Pr(type\ \theta)M(\theta) + Pr(type\ \mu)M(\mu)$
- **Choosing:** $P(\theta,\theta), P(\theta,\mu), P(\mu,\theta), P(\mu,\mu), M(\theta), M(\mu)$
- **Subject to:** $IC_\theta, IC_\mu, IR_\theta, IR_\mu$

Therefore, we have to solve the problem:

$$\max_{P(\theta,\theta),P(\theta,\mu),P(\mu,\theta),P(\mu,\mu),M(\theta),M(\mu)} \quad E[\pi_{seller}] = Pr(type\ \theta)M(\theta) + Pr(type\ \mu)M(\mu)$$

$$\begin{aligned} \text{s.t.} \quad \theta P(\theta) - M(\theta) \geq 0 \quad & (\text{IR}_\theta) \\ \mu P(\mu) - M(\mu) \geq 0 \quad & (\text{IR}_\mu) \\ \theta P(\theta) - M(\theta) \geq \theta P(\mu) - M(\mu) \quad & (\text{IC}_\theta) \\ \mu P(\mu) - M(\mu) \geq \mu P(\theta) - M(\theta) \quad & (\text{IC}_\mu) \end{aligned}$$

Principal (seller)'s Problem

To solve this problem we follow the steps:

(1) Show that IR_θ constraint is implied by the IC_θ constraint and the IR_μ constraint.

(2) Conclude that IR_θ can be ignored, and that IC_θ and IR_μ bind at the optimum.

(3) Also ignore IC_μ since we can check at the end that it holds.

(4) Solve the simplified problem of maximizing $E[\pi_{seller}]$ subject only to IC_θ and IR_μ holding with strict equality.

Exercise 1
Problem Set 12

Type θ: Value of the album at 20 $\rightarrow$ Probability of $\frac{1}{4}$
Type μ: Value of the album at 4 $\rightarrow$ Probability of $\frac{3}{4}$
We also know that agents are **risk-neutral** because the surplus for each type of buyer will be equal to the difference between their valuation and the price paid for the object in case they get it and zero otherwise (i.e., they have linear utility functions).

a) The owner is planning a second-price auction but a friend of the owner suggests a first-price auction instead, claiming the revenue would be higher. Do you think that the friend is right?

Revenue Equivalence

Principal (the seller) will get the same expected revenue either from a first-price or from a second-price auction as long as agents (buyers):

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Both types of auctions will yield the same expected revenue - it is indifferent for the owner to implement a first-price or a second-price auction.

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b) You are also a friend of the owner and want to recommend the optimal auction mechanism. Formalize and solve the problem that will allow you to make a recommendation.

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c) Can you think of a real-life mechanism that would lead to the optimal solution for the auctioneer?

A real life mechanism that would lead to the optimal solution for the auctioneer must be in accordance with the probabilities and payments we just found on the last question. This means that such mechanism must ensure bidders with a low valuation for the vinyl album will never win the auction.

Type t (tourists): Willingness to pay of 600 $\rightarrow$ Probability of $\frac{1}{2}$
Type e (executives): Willingness to pay of 1000 $\rightarrow$ Probability of $\frac{1}{2}$
We also know that agents are **risk-neutral** because the surplus for each type of buyer will be equal to the difference between their valuation and the price paid for the flight in case they get it and zero otherwise (i.e., they have linear utility functions).

a) Which prices will TAP charge if it knows whether the traveler is an executive or a tourist? What is the profit associated with those prices?

Type t (tourists): Willingness to pay of 600 $\rightarrow$ Probability of $\frac{1}{2}$
Type e (executives): Willingness to pay of 1000 $\rightarrow$ Probability of $\frac{1}{2}$
We also know that agents are **risk-neutral** because the surplus for each type of buyer will be equal to the difference between their valuation and the price paid for the flight in case they get it and zero otherwise (i.e., they have linear utility functions).

b) TAP is hesitating between a first-price sealed bid auction and a second-price sealed bid auction for that remaining seat. Without making any calculations, what would your recommendation be? Why?

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c) Assume that TAP can offer a menu of contracts (for instance, a given price may correspond to a certain probability of actually getting the seat).

- **(i)** Formalize TAP's maximization problem. What type of problem is this?
- **(ii)** Which constraints will be binding? Solve the optimization problem.