Mechanism Design

Advanced Microeconomics - Practical Lecture 11

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Moral Hazard and Mechanism Design

Moral Hazard: Actions of the agent are unobservable.

Mechanism Design: Types/characteristics of the agent are unobservable.

Here, we have an **agent** who can be of different types (which affects his/her payoffs) and a **principal** trying to maximize his/her payoff (which depends on the choices of the players and their outcome).

As with Moral Hazard, there is a **conflict** between the goals of the principal and the agent $\rightarrow$ A given type can claim to be a different type of agent in order to increase benefits, which negatively affects the principal.

What can the principal do? Design a **mechanism** which allows him/her to "separate" each type of agent and to maximize the payoff.

Mechanism

A **mechanism** is defined by a set of **rules**, specifying available **choices** for the agent, as well as the **outcome** under each choice.

The principal designs a mechanism in order to achieve some goal (payoff maximization). The mechanism is:

- **Incentive Compatible (IC):** Each type prefers his own assignment/contract to that of the other type.
- **Individually Rational (IR):** Mechanism yields a higher payoff than the outside option.

Direct Revelation Mechanism

Agent's set of strategies is to **report his/her type**. In this case:

- IC implies that each type **tells the truth** (about their type).
- IR guarantees **voluntary participation**.

Observable Types (First-best contract)

The principal maximizes her utility/profits/revenue (depending on the context), taking into account that agent(s) must be willing to participate, i.e., agent's utility must not fall below their reservation utility $\bar{u}_i$.

Therefore, for each type $i \in \{A, B\}$, we have to solve the problem:

$$\max_{x_i, y_i} \quad U = U(x_i, y_i)$$
$$\text{s.t.} \quad u_i(x_i, y_i) \geq \bar{u}_i \quad (\text{IR}_i)$$

To solve this problem, we plug the constraint into the objective function and then solve the (unconstrained) maximization problem.

Unobservable Types (Second-best contract)

The principal maximizes her utility taking into account that:

- Agents must be **willing to participate** i.e., agents' utility must not fall below their reservation utility $\bar{u}_i$.
- An agent's utility under the contract for her type is greater or equal than under the contract for the other type.

Therefore, we have to solve the problem:

$$\max_{x_A, y_A, x_B, y_B} \quad E[U] = p \cdot U(x_A, y_A) + (1-p) \cdot U(x_B, y_B)$$

$$\text{s.t.} \quad u_A(x_A, y_A) \geq \bar{u}_A \quad (\text{IR}_A)$$

$$u_B(x_B, y_B) \geq \bar{u}_B \quad (\text{IR}_B)$$

$$u_A(x_A, y_A) \geq u_A(x_B, y_B) \quad (\text{IC}_A)$$

$$u_B(x_B, y_B) \geq u_B(x_A, y_A) \quad (\text{IC}_B)$$

Unobservable Types (Second-best contract)

To solve this problem we follow the steps:

(1) Show that one of IR_i constraint is implied by the IC_i constraint of the corresponding type and the IR_j constraint of the opposing type.

(2) Conclude that IR_i can be ignored, and that IC_i and IR_j bind at the optimum.

(3) Also ignore IC_j since we can check at the end that it holds.

(4) Solve the simplified problem of maximizing U subject only to IC_i and IR_j holding with strict equality.

$u(t, e) = t - e^2$
Where:

$t \rightarrow$ Payment received

$e \rightarrow$ Effort

Firms can be of **two types**:

- Advice is **extremely useful** $\rightarrow$ Type H
- Advice is **very useful** $\rightarrow$ Type L

For each firm, $\pi_i(e, t) = \alpha_i e - t$
With:

$\alpha_H = 4$ and $\alpha_L = 2$

Both firms have a reservation payoff of zero.

a) Assume that you can observe firms' type. Derive and explain the consulting contracts that maximize your utility and are accepted by the firms.

b) Now assume that you cannot observe firm's type, but that you have a prior belief that both types are equally likely. Find the profit maximizing menu of contracts (t_H, e_H) and (t_L, e_L) that makes firms accept the contract and reveal their type. Explain.

Utility of the **government**:
$u_g = 2\sqrt{q} - t$
Where:

$q \rightarrow$ Quantity produced

$t \rightarrow$ Payment to the firm

Utility of the **firms**:
$u_f = t - \alpha q - \frac{41}{90}$
Where:

$q \rightarrow$ Quantity produced

$t \rightarrow$ Payment from the government

Firms can be of **two types**:

- **Type L** (when $\alpha_L = 1$) $\rightarrow$ Probability of $\frac{1}{2}$
- **Type H** (when $\alpha_H = 2$) $\rightarrow$ Probability of $\frac{1}{2}$

Both firms have a reservation utility of zero.

a) What are the incentive compatibility and the participation constraints in this case?

b) Compute the first best contract.

c) Compute the second best contract.