

# Moral Hazard

Advanced Microeconomics - Practical Lecture 10

João Bonito Gomes

Nova School of Business and Economics

April 22, 2025

## Exercise 1

### Problem Set 10

#### Moral Hazard

Moral Hazard refers to the fact that when economic agents know others will bear the costs (consequences) of their actions, they take on more risks than they would otherwise.

**Examples:** Insurance firm/insured agent, owner/manager, service provider/cliente.

Here, we will be referring to **principal-agent problems**, which arise when an economic agent (the **agent**) takes an action which affects other agent(s) (the **principal**) who are unable to force the agent to change their action and act according to their interests.

**Why are the principals unable to change the actions of the agents?** Because they may not observe their actions (e.g., their level of effort) or they may not be able to prove that the agent acted knowing all the risks involved. → **Result:** Conflict of interests/goals.

**Then, what can the principal do?** Set up an **incentive scheme** in order to induce the agent to act in accordance to the actions preferred by the principal.

## Exercise 1

### Problem Set 10

1 unit of **wealth** |  $u(x, e) = \sqrt{x} - e$ , with  $e \in \{0, 0.1\}$

a) Will a policy that insures the consumers completely generate a moral hazard problem?

	Careful ( $e = 0.1$ )	Not careful ( $e = 0$ )
$p$	0.25	0.75
$1 - p$	0.75	0.25

To solve this, we have to consider two cases:

(1) Check what consumers will choose to do if they **don't have insurance**.

(2) Check what consumers will choose to do if they **have complete (full) insurance**.

If the introduction of insurance changes their choice, we might have a problem of moral hazard.

## Exercise 1

### Problem Set 10

1 unit of **wealth** |  $u(x, e) = \sqrt{x} - e$ , with  $e \in \{0, 0.1\}$

**b)** If this a competitive market, and given your answer in (a), what could be the price of a full insurance policy? Would the consumers buy it?

Insurance companies are:

- **Risk-neutral** (given by the exercise).
- In a **competitive market** for insurance (also given by the exercise).

This means that expected profit for insurance firms must be 0, which in turn imply that the insurance premium is actuarially fair (recall PS9). This means that:

$$p(P - I) + (1 - p)P = 0 \leftrightarrow P = pl$$

Where  $p$  is the probability of losing all wealth,  $P$  is the (actuarially fair) premium, and  $I$  is the amount of insurance (in this case,  $I = 1$  because we are considering a full insurance policy that pays back all wealth in case of loss).

## Exercise 2

### Problem Set 10

**Jesualdo's utility** depends on two factors:

**(1) Ice-cream:**

- Eating ice cream (*eat*)  $\rightarrow$  No disutility
- Not eating ice cream (*not eat*)  $\rightarrow$  Disutility of 10

**(2) Pocket money:**

- $u(w) = 2\sqrt{w} - d_e$

So we can write Jesualdo utility function as:

$$u_J(w, e) = 2\sqrt{w} - d_e$$

Where if:

$$e = \text{eat} \rightarrow d_{\text{eat}} = 0$$

$$e = \text{not eat} \rightarrow d_{\text{not eat}} = 10$$

Eating ice cream (*eat*) or not eating ice cream (*not eat*) influences the probability of having a major illness (and having no illness):

	<i>eat</i>	<i>not eat</i>
Major illness: $p_M$	0.5	0.25
Minor illness: $p_m$	0.25	0.25
No illness: $p_n$	0.25	0.5

Eating ice cream increases the probability of a major illness and decreases the probability of having no illness, while the probability of having no illness remains unchanged.

## Exercise 2

### Problem Set 10

**Ana's utility** depends on two factors:

**(1) Jesualdo's health:**

- If major illness: Utility of 0
- If minor illness: Utility of 100
- If no illness: Utility of 1000

**(2) Money given to Jesualdo:**

- For each payment of  $w$ , Ana has a disutility of  $w$ .

So we can write Ana's utility function as:

If **major illness**:  $u_A = -w$

If **minor illness**:  $u_A = 100 - w$

If **no illness**:  $u_A = 1000 - w$

## Exercise 2

### Problem Set 10

**Jesualdo's** utility function:  $u_J(w, e) = 2\sqrt{w} - d_e$

Where if:

$$e = \text{eat} \rightarrow d_{\text{eat}} = 0$$

$$e = \text{not eat} \rightarrow d_{\text{not eat}} = 10$$

**Ana's** utility function:

- If **major illness**:  $u_A = -w$
- If **minor illness**:  $u_A = 100 - w$
- If **no illness**:  $u_A = 1000 - w$

	<i>eat</i>	<i>not eat</i>
Major illness: $p_M$	0.5	0.25
Minor illness: $p_m$	0.25	0.25
No illness: $p_n$	0.25	0.5

- a) What is Ana's expected utility if Jesualdo eats a lot of ice cream?
- b) What is Jesualdo's attitude towards risk?
- c) If Ana gives Jesualdo some pocket money that does not depend on his health, will he eat a lot of ice cream or none at all? Does your answer depend on the amount of pocket money - why or why not?

## Exercise 2

### Problem Set 10

**Jesualdo's** utility function:  $u_J(w, e) = 2\sqrt{w} - d_e$

Where if:

$$e = \text{eat} \rightarrow d_{\text{eat}} = 0$$

$$e = \text{not eat} \rightarrow d_{\text{not eat}} = 10$$

**Ana's** utility function:

- If **major illness**:  $u_A = -w$
- If **minor illness**:  $u_A = 100 - w$
- If **no illness**:  $u_A = 1000 - w$

	<i>eat</i>	<i>not eat</i>
Major illness: $p_M$	0.5	0.25
Minor illness: $p_m$	0.25	0.25
No illness: $p_n$	0.25	0.5

Now, the amount of pocket money also depends on the state of illness:

- If **major illness**:  $w = 0$
- If **minor illness**:  $w = B$
- If **no illness**:  $w = B$

**d)** Ana is considering a payment scheme in which she gives a bonus ( $B$ ) to Jesualdo if no major illness is observed. Normalize the basic pocket money to zero. How much should  $B$  be in order to make Jesualdo eat no ice cream? Will she want to offer  $B$ ?



## Exercise 3

### Problem Set 10

**Type** of days:

- Bad ( $b$ )  $\rightarrow$  Revenue of 3
- Medium ( $m$ )  $\rightarrow$  Revenue of 6
- Very good ( $g$ )  $\rightarrow$  Revenue of 12

**Employee** effort can take two values:

- High effort  $\rightarrow e_H = 1$
- Low effort  $\rightarrow e_L = 0$

$$u_E(w, e) = \sqrt{w - 4} - e$$

where  $w \rightarrow$  Wage

Utility of the **owner** is his net profit:

$$\pi = R - w$$

where  $R \rightarrow$  Revenue

The fact that the employee exerts high effort ( $e_H = 1$ ) or low effort ( $e_L = 0$ ) influences the probability of each type of day:

	$e_H = 1$	$e_L = 0$
Bad day: $p_b$	1/6	1/3
Medium day: $p_m$	1/6	1/3
Very good day: $p_g$	2/3	1/3

Exerting high effort increases the probability of having a very good day, at the same that it decreases the probability of having a bad or medium day.

## Exercise 3

### Problem Set 10

**Type** of days:

- Bad ( $b$ )  $\rightarrow$  Revenue of 3
- Medium ( $m$ )  $\rightarrow$  Revenue of 6
- Very good ( $g$ )  $\rightarrow$  Revenue of 12

**Employee** utility:

$$u_E(w, e) = \sqrt{w - 4} - e$$

where  $w \rightarrow$  Wage

Utility of the **owner** is his net profit:

$$\pi = R - w$$

where  $R \rightarrow$  Revenue

	$e_H = 1$	$e_L = 0$
Bad day: $p_b$	1/6	1/3
Medium day: $p_m$	1/6	1/3
Very good day: $p_g$	2/3	1/3

**a)** Determine the optimal incentive scheme under no moral hazard.

Under no moral hazard, effort is **observable**.

Principal can design contracts where the wage directly depends on the effort level exerted by the employee:

- $w_L \rightarrow$  Wage for  $e_L = 0$
- $w_H \rightarrow$  Wage for  $e_H = 1$

## Exercise 3

### Problem Set 10

**Type** of days:

- Bad ( $b$ )  $\rightarrow$  Revenue of 3
- Medium ( $m$ )  $\rightarrow$  Revenue of 6
- Very good ( $g$ )  $\rightarrow$  Revenue of 12

**Employee** utility:

$$u_E(w, e) = \sqrt{w - 4} - e$$

where  $w \rightarrow$  Wage

Utility of the **owner** is his net profit:

$$\pi = R - w$$

where  $R \rightarrow$  Revenue

	$e_H = 1$	$e_L = 0$
Bad day: $p_b$	1/6	1/3
Medium day: $p_m$	1/6	1/3
Very good day: $p_g$	2/3	1/3

**b)** Determine the optimal incentive scheme under moral hazard.

Under no moral hazard, effort is **not observable**.

Principal must design a contract curve where wages depend on observable outcomes (in this case, revenues):

- $w_g \rightarrow$  Wage for a very good day
- $w_m \rightarrow$  Wage for a medium day
- $w_b \rightarrow$  Wage for a bad day

## Exercise 3 - No Moral Hazard (Process detailed)

### Problem Set 10

Because effort is **observable**, both wages and effort can be included in the contract and **wages depend directly on effort**.

If the employee's reservation utility is  $\bar{u}$ , for an effort level  $e_i$ , the owner must offer a wage  $w_i$  in order to guarantee that the employee accepts the contract:

$$u_E(w_i, e_i) \geq \bar{u}$$

Thus, to solve the problem under no moral hazard we:

- (1) Find the value of  $w_L$  that minimizes the owner's costs and for which the employee accepts the contract with  $e_L$ .
- (2) Find the value of  $w_H$  that minimizes the owner's costs and for which the employee accepts the contract with  $e_H$ .
- (3) Select the contract for which the owner's expected profits/utility are higher.

## Exercise 3 - Moral Hazard (Process detailed)

### Problem Set 10

Because effort is **not observable**, only wages can be included in the contract and **wages depend on observable outcomes** (e.g., revenue levels).

The employee observes the wage offer then individually selects the desired effort level. Thus, the employee only selects high effort if:

$$E[u_E | e = e_H] \geq E[u_E | e = e_L]$$

Thus, to solve the problem under moral hazard we:

- (1) Find the conditions when the employee chooses  $e_H$  over  $e_L$ .
- (2) Find the cost minimizing wage offer which induces the employee to choose  $e_H$ .
- (3) Find the cost minimizing wage offer which induces the employee to choose  $e_L$ .

(In general, these wages will all be equal between themselves and equal to the wage offered for low effort when effort is observable)

- (4) Select the contract for which the owner's expected profits/utility are higher.