Exercise Solutions for Industrial Organization

May 5, 2025

1 The Extremes: Perfect Competition and Monopoly

1. Consider the industry of Portuguese footwear, which is a perfectly competitive one with a demand function given by $P = a - bQ$. In this market every firm sells the same product and they all have the same cost structure, $TC = cQ$.

   (a) Define the long-run equilibrium price, quantity (aggregate and firm level) and profit.

   (b) Draw and calculate the consumer surplus, the producer surplus and the total welfare.

2. Assume that the demand of a given perfectly competitive industry is given by $P = 40 - Q$. Furthermore, consider that in this industry there are 10 companies, all presenting the same cost function $TC = 5 + 4q + q^2$.

   (a) Compute the firm-level supply function.
   Solution:
   $MC = \frac{\partial TC}{\partial q_i} = 2q_i + 4$
   $S_i = \frac{p}{2} - 2$

   (b) Derive the industry-level supply function.
   Solution:
   $Q = n \cdot q_i = 10q_i = 5p - 20$

   (c) Calculate the price and aggregated quantity of equilibrium, as well as firm-level quantity.
   Solution:
   $S = D \leftrightarrow 5p - 20 = 40 - p \leftrightarrow p = 10 \wedge Q = 30 \wedge q_i = 3$

   (d) What is the profit of each firm?
   Solution:
   $\pi_i = pq_i - TC_i = 10 \cdot 3 - 5 - 4 \cdot 3 - 3^2 = 30 - 26 = 4$

   (e) Is this a short-run equilibrium or a long-run one? Explain. What do you expect to happen in this market?
   Solution: This is a short-run equilibrium since $\pi_i > 0$. Therefore, it is expected that new firms enter the market. This process of entrance of new firms will only stop once $\pi_i = 0$

3. Consider a market supplied by a single firm, The Monopolist. This company faces a market demand of $P = a - bQ$ and it has a total cost $TC = cQ$.

   (a) Formalize the problem of the Monopolist, and find the equilibrium quantity and price.

   (b) Draw and calculate the consumer surplus, the producer surplus and the total welfare.

   (c) Compare the results of the Perfect Competition exercise (first one in this section) and point out the effect over:

      i. Price
      ii. Total quantity
      iii. Firm profits
      iv. Consumer Surplus
      v. Producer Surplus
      vi. Total welfare

   (d) Draw and calculate the Deadweight loss (excess burden) created by this monopoly.

4. Grace Inc. is the only firm in that operates in the market of product Z, a valuable good that is produced in Burkina Faso. This monopolist has a total cost function of $TC = q^2 + 12$. Additionally it faces a demand curve that is given by $P = 24 - q$.

   (a) Calculate the marginal revenue for Grace Inc.
   Solution:
   $TR = P(q) \cdot q = (24 - q) \cdot q = 24q - q^2$
   $MR = \frac{\partial TR}{\partial q} = 24 - 2q$

   (b) Compute the equilibrium price and quantity of the monopolist.
   Solution:
   $MC = \frac{\partial TC}{\partial q} = 2q$
   To obtain the optimum: $MR = MC \leftrightarrow 24 - 2q = 2q \leftrightarrow q = 6 \wedge p^M = 18$

   (c) Determine the profit of Grace Inc. in equilibrium.
   Solution:
   $\pi = pq - TC = 6 \cdot 18 - 6^2 - 12 = 60$

   (d) Calculate the elasticity of demand at the equilibrium. What would happen to total revenue if Grace Inc. decreased its price? Is it advisable to do so?
   Solution:
   $|\varepsilon_D(P^M, Q^M)| = |\frac{\partial Q}{\partial P} \cdot \frac{P}{Q}| = | - 1 \cdot \frac{18}{6}| = 3$

5. Show that the optimal price in a Monopoly satisfies the following condition:

$$\frac{P - MC}{P} = -\frac{1}{\epsilon}$$

where MC is the marginal cost and ϵ is the elasticity of demand. When does this solution converge to the perfect competitive outcome? Explain.

Solution:

$$\max_Q \pi^M = P(Q) \cdot Q - TC(Q)$$

$$\frac{\partial \pi^M}{\partial Q} = 0 \leftrightarrow \frac{\partial P}{\partial Q} \cdot Q + P - MC = 0 \leftrightarrow P - MC = -\frac{\partial P}{\partial Q} \cdot Q \leftrightarrow \frac{P - MC}{P} = \frac{\partial P}{\partial Q} \cdot \frac{Q}{P} \leftrightarrow \frac{P - MC}{P} = -\frac{1}{\epsilon}$$

2 Market Structure

1. With table 1, find the interval of values for the Herfindahl Index for the PC market in the US. Find the interval of values for the instability index.

Firm	**Market Share**	
	2020	**2024**
HP	30%	24,2%
Dell	25%	22,3%
Lenovo	15%	17,2%
Apple	8%	15,8%
Others	22%	20,5%

Table 1: Exercise 1.

2. (*) Alexis Jacquemin presented the following characteristics for a good measure of concentration:

   - **Non-ambiguous character** Given two markets, it should be possible to say without any doubt which one is more concentrated.
   - **Scale invariance** The measure should depend only upon the relative dimension of each firm.
   - **Transference** The measure should increase when we reduce the market share of a small firm in favor of a bigger firm.
   - **Monotonicity in the number of firms** If the N firms have identical market shares, then the measure should be decreasing in N.
   - **Cardinality** Dividing each firm in k equal firms, the measure should decrease in the same proportion.

   Verify if the Concentration Ratio (C_k) and the Herfindahl-Hirschman Index (HHI) satisfy these conditions.

   Solution: Check the PDF available in the 'Material' section on Moodle.

3. (*) Show that $H = \frac{1}{N} + N\sigma_i$, where H is the Herfindahl index, N the number of firms and σ_i the variance of the market shares. With this equation explain the meaning of the measure "equivalent number" of Adelman defined as $NE \equiv \frac{1}{H}$.

   Solution:

$$\begin{aligned}
\text{HHI} &= \sum S_i^2 \\
&= \sum \left[(S_i - \bar{S}) + \bar{S}\right]^2 \\
&= \sum (S_i - \bar{S})^2 + 2(S_i - \bar{S})\bar{S} + \sum \bar{S}^2 \\
&= \sum (S_i - \bar{S})^2 + N\bar{S}^2 \\
&= N\left(\frac{\sum (S_i - \bar{S})^2}{N} + \bar{S}^2\right) \\
&= N\sigma + N\bar{S}^2 \\
&= N\sigma + \frac{1}{N}. \\
&= \frac{1}{N} + N\sigma
\end{aligned}$$

4. Table 2 presents the market shares of the 20 biggest insurance firms operating in the life branch. Calculate the possible variation interval for the Herfindahl index in both years. Calculate the variation interval for the Instability Index. What is the equivalent number of Adelman?

Firms	**1995**	**1996**
Tranquilidade	16,50%	16,70%
Fidelidade	11,60%	11,60%
Ocidental	13,10%	11,60%
BPI	12,10%	11,20%
Mundial Confiança	6,10%	7,40%
Barclays	2,50%	6,25%
Império	5,40%	5,85%
BPA	4,90%	4,70%
Aliança UAP	5,00%	4,20%
BFE	2,50%	2,60%
Victoria	2,40%	2,00%
Bonança	2,30%	1,90%
Europeia	1,80%	1,40%
Alico	1,70%	1,40%
Abeille	1,40%	1,30%
Gan	1,40%	1,30%
Eagle Star	1,40%	1,30%
Portugal Previdente	1,50%	1,30%
Kusitânia	0,40%	1,00%
Génesis	1,20%	0,60%
Others	4,80%	4,40%
Total	100,00%	100,00%

Table 2: Exercise 4.

Solution:

HHI-Index: $max_{95} = 8,82\%$, $min_{95} = 8,80\%$, $max_{96} = 8,70\%$, $min_{96} = 8,68\%$

Adelman's: $max_{95} = 11,37$, $min_{95} = 11,34$, $max_{96} = 11,53$, $min_{96} = 11,49$

Instability: $min = 6,4\%$, $max = 10,8\%$

5. In 2010, the diaper industry in Portugal consisted of 5 firms producing identical diapers. However, in 2020, other firm(s) entered the market, obtaining a market share of 11%. Showing your computations, fill-in the missing items in table 3. Then, according to each concentration measure that you have studied, find out in which year the industry was more concentrated. Explain and calculate Adelman's equivalent number. Compute the volatility index.

Year	**Firms**						**Concentration Index**		
	1	2	3	4	5	Other(s)	C4	Inf H	Max H
2010	40%	15%	15%	15%	15%	0%			
2020	45%	11%	11%	11%	11%	11%			

Table 3: Exercise 5.

Solution:

2010:

$C^4 = 0.4 + 3 \cdot 0.15 = 0.85 \wedge HHI_{min} = HHI_{max} = 0.4^2 + 4 \cdot 0.15^2 = 0.25 \wedge ENA = \frac{1}{HHI} = 4$

2020:

$C^4 = 0.45 + 3 \cdot 0.11 = 0.78 \wedge HHI_{min} = 0.4^2 + 4 \cdot 0.11^2 = 0.25 \wedge HHI_{max} = 0.4^2 + 5 \cdot 0.11^2 = 0.26 \wedge ENA_{min} = \frac{1}{HHI_{max}} = 3.8 \wedge ENA_{max} = \frac{1}{HHI_{min}} = 3.99$

Instability-Index:

$I = \frac{1}{2} \cdot (|0.45 - 0.4| + 4 \cdot |0.11 - 0.15| + |0.11 - 0|) = 0.16$

6. In 2008, the Portuguese diapers' industry was characterized by the existence of 8 firms producing identical diapers. Let s_i denote the market share of firm i, $i = 1, 2, ..., 8$. It has recently been observed that the market shares are given by table 4:

Firm	1	2	3	4	5	6	7	8
s_i	60%	10%	5%	5%	5%	5%	5%	5%

Table 4: Exercise 6

(a) Compute the measures C_4 and H for this industry.

Solution:

$C^4 = 0.6 + 0.1 + 0.05 + 0.05 = 0.8$

$H = 0.6^2 + ... + 0,05^2 = 0.385$

(b) Suppose now that firms 2 and 3 merge and become a single firm. Compute the post-merger concentration measures (C_4 and H). Compute the change[1] in concentration resulting from this merger.

Solution:

$\hat{C}_4 = 0.6 + 0.15 + 0.05 + 0.05 = 0.85$

[1] 'Change' implies computing $\Delta I = I_1 - I_0$, where I_1 is the post-merger index, and I_0 is the original value of the index.

$\hat{H} = 0.6^2 + ... + 0.05^2 = 0.395$

$\Delta C_4 = 0.05 \wedge \Delta H = 0.01$

(c) Now suppose that the merger between 2 and 3 does not work out, so treat them separately. Instead, suppose firms 6, 7 and 8 merge (consider that for both this and the next item of this exercise). Compute the post-merger values (C_4 and H).

Solution:

$\overline{C}_4 = 0.6 + 0.15 + 0.1 + 0.05 = 0.9$

$\overline{H} = 0.6^2 + ... + 0.05^2 = 0.4$

(d) Compute the change in concentration resulting from this merger.

Solution:

$\Delta C_4 = 0.1 \wedge \Delta H = 0.015$

(e) Now suppose that the Portuguese Competition Authority suggests that a merger should not be challenged if the post-merger Herfindahl index and its change due to the merger are such that:

- $H < 0.1$, or
- $0.1 \leq H < 0.18$ and $\Delta H < 0.01$, or
- $H \geq 0.18$ and $\Delta H \leq 0.005$

Determine if any of both mergers is likely to be challenged by the Regulator.

Solution: Both mergers are likely to be challenged.

7. Consider that the market for coffee in New Zealand is composed by 5 firms that have the market shares shown in table 5:

Kiwi Coffee	NZ Coffee Inc.	CNZ	Kafé Auckland	Wellington Coffees
40%	10%	20%	12%	18%

Table 5: Exercise 7

(a) Compute the concentration ratios C_4 and H for this market.

Solution:

$C^4 = 0.4 + 0.2 + 0.18 + 0.12 = 0.9 \wedge H = 0.4^2 + 0.2^2 + 0.18^2 + 0.12^2 + 0.1^2 = 0.2568$

Now consider that CNZ takes a risky move and decides to acquire NZ Coffee Inc. and Kafé Auckland in order to get closer to its main rival, KiwiCoffee.

(b) Compute the post-acquisition concentration ratios (C_4 and H).

Solution:

$\hat{C}^3 = 0.42 + 0.4 + 0.18 = 1 \wedge H = 0.42^2 + 0.4^2 + 0.18^2 = 0.369$

(c) What reaction do you think the Antitrust Regulatory Body of New Zealand could have to this operation? Explain.

Solution: It is likely that the New Zealand Antitrust Agency will challenge the transaction due to the significant increase in concentration in an already highly concentrated market.

3 Game Theory: Basics

1. Determine the equilibrium using iterative elimination of dominated strategies in the following game and by checking the best response to each player's strategy. Determine the Nash Equilibrium.

	U	D
U	8,8	0,15
D	15,0	2,2

2. Consider a game in which two firms, firm A and firm B, decide simultaneously about the price they want to charge for a product that they both sell. The following payoff matrix displays the outcomes of the different moves they can take:

		B	
		high	low
A	high	2,2	15,-3
	low	-3,15	10,10

Compute the Nash Equilibrium of this game.

Solution: (High,High)

3. Determine the equilibrium using iterative elimination of dominated strategies in the following game.

	L	C	R
T	-1,-2	-2,0	0,0
M	-2,0	0,-2	0,0
B	0,0	0,0	1,1

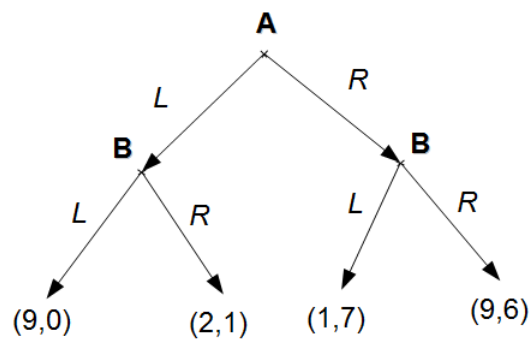
4. Determine the Nash Equilibrium of the following game:

	L	C	R
T	0,4	4,0	5,3
M	4,0	0,4	5,3
B	3,5	3,5	6,6

Solution: (B,R)

5. Firms 1 and 2 are both considering expanding into a small new geography. Firm 1 has been studying this possibility for a while and is ready to go ahead with the expansion now. Firm 2, on the other hand, will only be able to do so in 6 months. The investment needed to enter the new geography is €5M. If they both enter, they will have operational profits of €2M. If one of the firms enters the new geography alone, it will have an operational profit of €10M. How do you expect Firm 1 to act? Draw the game and find its solution.

6. Consider the following sequential game in extensive form.[2] Find the equilibrium using Backward induction. Represent the game in the normal form and find the Nash Equilibrium.



4 Oligopoly Models

4.1 Dominant Firm and Monopolistic Competition

1. Consider a market with demand given by $q = 200 - 2p$. In this market operates a dominant firm and a competitive fringe made of small firms. The small firms are price takers offering an aggregate quantity given by $S = p - 70$ ($p > 70$), where p is the price established by the dominant firm. The dominant firm satisfies the residual demand. Determine the optimal solution of the dominant firm when its marginal cost is constant and given by $c = 70$, $c = 45$ and $c = 20$. **Hint:** Drawing a plot may be useful to understand the second case.

   Solution:

   First Step: Get the Supply of the fringe - $S = p - 70$

   Second Step: Compute the Residual Demand

$$D'(p) = \begin{cases} 0 & \text{if } p > 90, \\ 270 - 3p & \text{if } 70 \leq p \leq 90, \\ 200 - 2p & \text{if } p < 70. \end{cases}$$

   Third Step: Solve the quasi-monopolist problem

$$\max_{P} \pi^{DF} = (P - MC) \cdot D'(p)$$

   One must solve this maximization problem for each value of the marginal cost. Moreover, before solving the problem, it is unknown which branch of the residual demand will contain the optimum. Therefore, all possibilities must be considered.

[2]Simultaneous games can also be drawn in extensive form, but this one is not simultaneous.

If $c = 20$: $P = 60$, $Q^D = 80$, $S^{DF} = 80$ (third branch)

If $c = 45$: $P = 70$, $Q^D = 60$, $S^{DF} = 60$ (corner solution)

If $c = 70$: $P = 80$, $Q^D = 40$, $S^{DF} = 30$, $S^F = 10$ (first branch)

2. Consider a market, with demand given by $p = 20 - 2q$, which is constituted by 3 firms (A, B and C). Firm A is more efficient than the other two, operating with a constant marginal cost equal to 9. The total costs of firms B and C are given by the function $TC_i = q_i * (q_i + 11)$, with $i = B, C$. Find the equilibrium of the market, supposing firm A acts like a dominant firm and firms B and C like a competitive fringe.

3. Comment the following statement: "A dominant firm does not care if the number of firms that compose the competitive fringe increases. Only their efficiency matters."

   Solution: False! An increase in the number of firms in the competitive fringe reduces the residual demand, thereby diminishing the dominant firm's profit.

4. Suppose that all firms in a market under a Monopolistic Competition environment face an individual inverse demand of $P = 90 + \frac{20}{n} - 4q$, then $n \geq 1$. The total cost function of each firm operating in this market is $TC(q) = q^2 + 414,05$.

   (a) Assume that in the short-run there are only 4 firms in the market, a firm named "NSBE" and three other similar competitors. Find the optimal quantity and price as well as the profit earned in the short-run by NSBE. What does the sign of the profit level tell you about potential market entry (or exit)?

   (b) Monopolistic Competition implies zero profits in long-run equilibrium. Use this fact to find the number of firms and NSBE's quantity and price in the long-run. What do you expect will happen in the long-run equilibrium if the fixed cost $F = 414,05$ increases? Justify.

   (c) Compare the short-run and long-run consumer surplus (CS).

   (d) Taking into account the computed equilibrium quantities in (a) and (b) and given the total cost function, what can you conclude about the productive efficiency of the firms operating in this market?

   (e) Compute the Lerner index $\left(L = \frac{P-MC}{P}\right)$ when $n = n_0$ and show that it is independent of the number of firms n_0. Explain intuitively the reason why NSBE's market power does not decrease with the number of competitors when $n \to \infty$.

5. Take the Portuguese market for breakfast cereals: 160 producers, each with a total cost function of $TC(q_i) = 5q_i + 5$. Each firm faces individual inverse demand of $P = \frac{100}{n\sqrt{q_i}}$ for which it tries to maximize its profits.

   (a) Compute the short run equilibrium (individual quantities, prices and profits).

   Solution:

   Short-run equilibrium:

   $n = 160 \to P = \frac{100}{160\sqrt{q_i}}$

   $TR = P(q)q = \frac{100\sqrt{q}}{160} \to MR = \frac{\partial TR}{\partial q} = \frac{50}{160\sqrt{q}}$

   $MC = \frac{\partial TC}{\partial q} = 5$

   $MR = MC \leftrightarrow \frac{50}{160\sqrt{q_i}} = 5 \leftrightarrow q_i = \frac{1}{156} \wedge P = 10 \wedge \pi_i = -\frac{1275}{256}$

(b) If there is free entry and exit in the long run, what will be the equilibrium (individual quantities, prices and profits)?

Solution:

Long-run equilibrium:

$$\begin{cases} MR = MC, \\ \pi_i = 0 \end{cases}$$

$1) MR = MC \leftrightarrow (...) \leftrightarrow \frac{10}{n} = \sqrt{q_i}$

$2) \pi_i = 0 \leftrightarrow P(q_i) \cdot q_i - TC(q) = 0 \leftrightarrow \frac{100}{n}\sqrt{q_i} - 5q_i - 5 = 0 \leftrightarrow 10q_i - 5q_i - 5 = 0$

$\leftrightarrow q_i = 1 \rightarrow n = 10 \wedge P = 10 \wedge \pi_i = 0$

(c) Among the firms still in the market, there is the well-known firm Killogg's. How much would it be willing to "pay" to each competitor in order to make them leave the market?

Solution:

If all firms except Killogg's exit the market, Killogg's will become a monopolist and will therefore earn a profit of:

$$\max_Q \pi^M = P(Q) \cdot Q - TC(Q)$$

$$\frac{\partial \pi^M}{\partial Q} = 0 \leftrightarrow (...) \leftrightarrow q = 100 \wedge P = 10 \wedge \pi = 495$$

Killogg's will be willing to pay each firm up to $\frac{\pi^M}{9} = 55$.

6. Consider a market under a monopolistic competition environment with $n = 101$ firms with identical demand and cost functions:

$$P = 150 - q_i - \frac{1}{50}\sum_{k=1}^{n-1} q_k$$

$$TC(q_i) = \frac{1}{2}q_i^3 - 20q_i^2 + 270q_i$$

with $i \neq k$ and $i = 1, ..., n$.

(a) Assume that the number of firms in the market does not change. Find the optimal quantity and price as well as the profit earned in the short-run by each firm.

Solution:

Short-run equilibrium:

$TR = P(q_i)q_i = 150q_i - q_i^2 - \frac{q_i}{50}\sum_{k=1}^{n-1} q_k$

$MR = \frac{\partial TR}{\partial q} = 150 - 2q_i - \frac{1}{50}\sum_{k=1}^{n-1} q_k \rightarrow$ by symmetry $q_i = q_k$

$MR = 150 - 2q_i - \frac{100}{50}q_i = 150 - 4q_i$

$MC = \frac{\partial TC}{\partial q} = \frac{3}{2}q_i^2 - 40q_i + 270$

$MR = MC \leftrightarrow (...) \leftrightarrow q_i = 20 \wedge P = 90 \wedge \pi_i = 400$

(b) Assume now that there is free entry of new firms. What is the long-run equilibrium in this market?

Solution:

Long-run equilibrium:

$$\begin{cases} MR = MC, \\ \pi_i = 0 \end{cases}$$

$1)MR = \leftrightarrow 150 - 2q_i - \frac{1}{50}\sum_{k=1}^{n-1} q_k = 150 - 2q_i - \frac{n-1}{50}q_i$
$MR = MC \leftrightarrow 150 - 2q_i - \frac{n-1}{50}q_i = \frac{3}{2}q_i^2 - 40q_i + 270$
$2)\pi_i = 0 \rightarrow P = AC$
$150 - q_i - \frac{1}{50}\sum_{k=1}^{n-1} q_k = \frac{q_i^2}{2} - 20q_i + 270 \leftrightarrow 150 - q_i - \frac{n-1}{50}q_i = \frac{q_i^2}{2} - 20q_i + 270 \leftrightarrow (...)$
Given that from 1) $150 - 2q_i - \frac{n-1}{50}q_i = \frac{3}{2}q_i^2 - 40q_i + 270$**:**
$\leftrightarrow q_i = 19 \rightarrow n \approx 160 \wedge P \approx 70.5$

4.2 Cournot

1. Consider the market of an homogeneous product with demand given by $p = 100 - 2q$. There are two firms, both with constant marginal cost equal to 10.

   (a) Compute the Cournot-Nash equilibrium.

   (b) Calculate the efficiency loss as a percentage of the efficiency loss in the monopoly situation.

   (c) Repeat the exercise assuming now that there are 8 firms instead of 2 firms.

2. Consider a duopoly with demand given by $Q = 10 - \frac{p}{2}$. The total cost function of each firm is given by $C = 10 + q(q + 1)$. Determine the Cournot equilibrium values.

   Solution:

$$\max_{q_1} \pi_1 = p(q_1 + q_2) \cdot q_1 - TC(q_1) = (20 - 2(q_1 + q_2)) \cdot q_1 - 10 - q_1 - q_1^2$$

$$\frac{\partial \pi_1}{\partial q1} = 0 \leftrightarrow 20 - 4q_1 - 2q_2 - 1 - 2q_1 = 0 \leftrightarrow q_1 = \frac{19}{6} - \frac{q_2}{3}$$

   By symmetry:

$$q_1 = q_2 \rightarrow q_1 = \frac{19}{6} - \frac{q_1}{3} \leftrightarrow q_1^* = 2.375 = q_2^* \wedge Q = 4.75 \wedge P = 10.5 \wedge \pi_1^* = \pi_2^* = 10.5 \cdot 2.375 - 10 - 2.375^2 - 2.375 = 9.3$$

3. Repeat the previous exercise assuming that the cost functions are given by: $C_1 = 10 + 2q_1$ and $C_2 = 10 + 1,5q_2$.

   Solution:

   Get BR_1

$$\max_{q_1} \pi_1 = p(q_1 + q_2) \cdot q_1 - TC(q_1) = (20 - 2(q_1 + q_2)) \cdot q_1 - 10 - 2q_1$$

$$\frac{\partial \pi_1}{\partial q_1} = 0 \leftrightarrow 20 - 4q_1 - 2q_2 - 2 = 0 \leftrightarrow q_1 = 4.5 - \frac{q_2}{2}$$

Get BR_2

$$\max_{q_2} \pi_2 = p(q_1+q_2)\cdot q_2 - TC(q_2) = (20-2(q_1+q_2))\cdot q_2 - 10 - 1.5q_2$$

$$\frac{\partial \pi_1}{\partial q_1} = 0 \leftrightarrow 20 - 4q_2 - 2q_1 - 1.5 = 0 \leftrightarrow q_2 = 4.625 - \frac{q_1}{2}$$

Get $q_1^* \wedge q_2^*$

$$\begin{cases} q_1 = 4.5 - \frac{q_2}{2} \\ q_2 = 4.625 - \frac{q_1}{2} \end{cases} \leftrightarrow \begin{cases} q_1 = 2.91\overline{6} \\ q_2 = 3.1\overline{6} \end{cases}$$

4. Consider a duopoly with demand given by $Q = 500 - 50P$. The first firm has a constant marginal cost equal to 8. The second firm has a marginal cost equal to 6 and a limited production capacity of 25 units. Calculate the equilibrium values assuming Cournot competition.

5. The shoe industry in a country has 8 firms. 5 of them use an old technology producing 0,25 units per work hour. The others use a modern technology with productivity of 0,45 units per work hour. The market demand is given by $Q = 500.000 - 10p$ and the wage for each work hour is w=500.

   (a) Find the market's Cournot equilibrium.
   Solution:
   Get BR_o
   $MC_o = \frac{1}{0{,}25}\cdot 500q_o = 2000q_o$

$$\max_{q_o^1} \pi_o^1 = p(q_o^1 + ... + q_o^5 + q_N^1 + ... + q_N^3)\cdot q_o^1 - TC(q_o^1)$$

$$\frac{\partial \pi_o^1}{\partial q_o^1} = 0 \leftrightarrow 50000 - \frac{2q_o^1}{10} - \frac{q_o^2 + ... + q_o^5}{10} - \frac{q_N^1 + ... + q_N^3}{10} - 2000 = 0$$

   By symmetry $q_o^1 = ... = q_o^5 = q_o \wedge q_N^1 = ... = q_N^3 = q_N$
   $BR_o : q_o = 80000 - \frac{q_N}{2}$
   Get BR_N
   $MC_N = \frac{1}{0{,}45}\cdot 500q_N = 1111.1q_N$

$$\max_{q_N^1} \pi_N^1 = p(q_o^1 + ... + q_o^5 + q_N^1 + ... + q_N^3)\cdot q_N^1 - TC(q_N^1)$$

$$\frac{\partial \pi_N^1}{\partial q_N^1} = 0 \leftrightarrow 50000 - \frac{q_o^1 + ... + q_o^5}{10} - \frac{2q_N^1}{10} - \frac{q_N^2 + q_N^3}{10} - 1111.1 = 0$$

   By symmetry $q_o^1 = ... = q_o^5 = q_o \wedge q_N^1 = ... = q_N^3 = q_N$
   $BR_N : q_o = 122222.2 - \frac{5q_o}{4}$
   Get $q_o^* \wedge q_N^*$

$$\begin{cases} q_o = 80000 - \frac{q_N}{2} \\ q_N = 122222.2 - \frac{5q_o}{4} \end{cases} \leftrightarrow \begin{cases} q_o = 50370 \\ q_2 = 59259 \end{cases} \rightarrow \begin{cases} P = 7037 \\ Q = 429629 \end{cases}$$

(b) Find the maximum value at which a firm would be willing to buy the new technology, assuming that the rest of the firms would continue to use the same technology.

Solution: If a firm using the old technology adopts the new technology, there will be four firms operating with each technology. The firm's maximum willingness to pay for the new technology corresponds to the change in its profits resulting from adopting the new technology

Get BR_o

$$\max_{q_o^1} \pi_o^1 = p(q_o^1 + ... + q_o^4 + q_N^1 + ... + q_N^4) \cdot q_o^1 - TC(q_o^1)$$

$$\frac{\partial \pi_o^1}{\partial q_o^1} = 0 \leftrightarrow 50000 - \frac{2q_o^1}{10} - \frac{q_o^2 + ... + q_o^4}{10} - \frac{q_N^1 + ... + q_N^4}{10} - 2000 = 0$$

By symmetry $q_o^1 = ... = q_o^4 = q_o \wedge q_N^1 = ... = q_N^4 = q_N$

$BR_o : q_o = 96000 - \frac{4q_N}{5}$

Get BR_N

$$\max_{q_N^1} \pi_N^1 = p(q_o^1 + ... + q_o^4 + q_N^1 + ... + q_N^4) \cdot q_N^1 - TC(q_N^1)$$

$$\frac{\partial \pi_N^1}{\partial q_N^1} = 0 \leftrightarrow 50000 - \frac{q_o^1 + ... + q_o^4}{10} - \frac{2q_N^1}{10} - \frac{q_N^2 + ... + q_N^4}{10} - 1111.1 = 0$$

By symmetry $q_o^1 = ... = q_o 45 = q_o \wedge q_N^1 = ... = q_N^4 = q_N$

$BR_N : q_o = 97777.7 - \frac{4q_o}{5}$

Get $q_o^* \wedge q_N^*$

$$\begin{cases} q_o = 96000 - \frac{4q_N}{5} \\ q_N = 97777.7 - \frac{4q_o}{5} \end{cases} \leftrightarrow \begin{cases} q_o = 49382.7 \\ q_2 = 58271.6 \end{cases} \rightarrow \begin{cases} P = 6938.3 \\ Q = 430617.2 \end{cases}$$

$WTP = \Delta\pi = \pi_N - \pi_o$

$\pi_N = (6938.3 - 1111.1) \cdot 58271.6 = 339559620$

From a) $\pi_o = (7037 - 2000) \cdot 50370 = 253713690$

$WTP = \Delta\pi = \pi_N - \pi_o = 85845930$

6. The US has been pressuring Japan to open the automobile market to models made in the US. Consider that the automobile market in Japan presents a demand curve given by $Q = 10 - p$ and is served by three Japanese firms with total costs given by $TC = q$. The US producers are willing to export automobiles to the Japanese market in function of the price existing there, that is, acting like price-takers, following the function $S = p$, where S represents the number of US cars sold in Japan if the price in that market was p. The Japanese firms act like Cournot competitors.

   (a) Suppose Japan is forced to open its automobile market to US exports. What is the price, the quantity produced domestically and the quantity imported in equilibrium?

   (b) Suppose that the US, instead of asking permission for its firms to sell in Japan, establish an objective quantity for the sale of US-produced automobiles in the Japanese market. Suppose that such quantity is equal to 2. What are the new price, the quantity produced domestically and the quantity imported in equilibrium?

   (c) Compare, explaining the differences, your answers to the previous questions.

(d) Which of the two former systems do the US firms prefer? And the Japanese firms? And the Japanese consumers? And the Japanese society?

7. One firm imports computers in order to sell them in the domestic market. It can import each computer by paying 10 to the manufacturer. The domestic demand for this brand equals to $q = 100 - p$. Up to now, this firm has been the only importer of the brand into domestic market. However, the manufacturer of the computer is about to issue a second import license. This license, also allowing the winner to import computers at 10 per unit, will be awarded to the firm willing to pay the most for it. All potential importers interested in importing this brand of which there are more than one as well as the firm that owns the first license. If a firm other than the owner of the first license wins the second, they will compete *à* la Cournot.

(a) Who is going to win the second license? Explain. **Solution: A given firm will be willing to pay an amount equivalent to the Cournot profits for the second license. The incumbent, on the other hand, will be willing to pay an amount corresponding to the change in its profits, given by ($\Delta\pi^{Incumbent} = \pi^M - \pi^{Cournot}$).**
π^M:

$$\max_Q \pi^M = P(Q)Q - TC(Q) = (100 - Q) \cdot Q - 10Q$$

$$\frac{\partial \pi^M}{\partial Q} = 0 \leftrightarrow 100 - 2Q - 10 = 0 \leftrightarrow Q^M = 45 \rightarrow P = 55 \wedge \pi^M = 2025$$

$\pi^{Cournot}$:

$$\max_{q_1} \pi_1 = P(q_1 + q_2) \cdot q_1 - 10q_1$$

$$= (100 - q_1 - q_2) \cdot q_1 - 10q_1$$

$$\frac{\partial \pi_1}{\partial q_1} = 0 \leftrightarrow 100 - 2q_1 - q_2 - 10 = 0 \leftrightarrow q_1 = 45 - \frac{q_2}{2}$$

By symmetry: $q_1 = q_2 \rightarrow q_1 = 45 - \frac{q_1}{2} \leftrightarrow q_1^* = q_2^* = 30 \wedge Q = 60 \wedge P = 40 \wedge \pi_1^* = \pi_2^* = 900$
The incumbent is willing to pay $\Delta\pi^{Incumbent} = \pi^M - \pi^{Cournot} = 1125$ **for the second license, while a new entrant is only willing to pay** $\pi^{Cournot} = 900$**. Therefore, the incumbent will win the second license.**

(b) Will the manufacturer, who produces each computer at a cost of 5, be pleased with the outcome of the auction? What would be your advice to him?
Solution: If the second license is sold to the incumbent, then $Q^D = 45 \rightarrow \pi^{Manufacturer} = (10 - 5) \cdot 45 = 225$**. However, if the second license is sold to the new entrant, then** $Q^D = 60 \rightarrow \pi^{Manufacturer} = (10 - 5) \cdot 60 = 300$**. As a result, the manufacturer will be dissatisfied with the auction's outcome and should prevent the incumbent from participating in the bidding process.**

8. Consider a market with two firms simultaneously deciding on the technology that they wish to adopt. After this decision they will compete à la Cournot. Note that the competitors observe the first decision before they decide on the quantities. The two available technologies are characterized by the following total cost curves: $TC_a = 10q_a + 120$ and $TC_b = 25q_b + 5$. The demand function is given by $P = 40 - Q$. Find the chosen technologies and the equilibrium in the product market.

9. Consider two firms, 1 and 2, producing a homogeneous product, that simultaneously decide how much they want to produce. The market demand is given by $D(p) = 100 - p$. The marginal and average cost of production of both firms is constant and equal to 10. Firm 1 maximizes its profit but the manager of the other firm maximizes a weighted sum of the profit and the quantity produced, i.e., firm 2 maximizes $\pi(q_1, q_2) + aq_2$.

   (a) Represent graphically the best response functions of both firms.
   Solution:
   Firm 1:

$$\max_{q_1} \pi_1 = P(q_1 + q_2) \cdot q_1 - TC(q_1)$$

$$= (100 - q_1 - q_2) \cdot q_1 - 10q_1$$

$$\frac{\partial \pi_1}{\partial q_1} = 0 \leftrightarrow 100 - 2q_1 - q_2 - 10 = 0 \leftrightarrow q_1 = 45 - \frac{q_2}{2}$$

   Firm 2:

$$\max_{q_2} \pi_2 = P(q_1 + q_2) \cdot q_2 + aq_2 - TC(q_2)$$

$$= (100 - q_1 - q_2) \cdot q_2 + aq_2 - 10q_2$$

$$\frac{\partial \pi_2}{\partial q_2} = 0 \leftrightarrow 100 - 2q_2 - q_1 + a - 10 = 0 \leftrightarrow q_2 = \frac{90 + a}{2} - \frac{q_1}{2}$$

   (b) Find the equilibrium quantities.
   Solution:

$$\begin{cases} q_1 = 45 - \frac{q_2}{2} \\ q_2 = \frac{90+a}{2} - \frac{q_1}{2} \end{cases} \leftrightarrow (...) \leftrightarrow \begin{cases} q_1 = \frac{90-a}{3} \\ q_2 = \frac{90+2a}{3} \end{cases}$$

   (c) Which value of a maximizes the profit of firm 2?
   Solution:

$$Q = q_1 + q_2 = \frac{90 + 2a}{3} + \frac{90 - a}{3} = \frac{180 + a}{3}$$

$$P = 100 - \frac{180 + a}{3} \quad \Leftrightarrow \quad P = \frac{120 - a}{3}$$

$$\pi_1 = \left(\frac{120 - a}{3} - 10\right) \cdot \left(\frac{90 + 2a}{3}\right) = \left(\frac{90 - a}{3}\right) \cdot \left(\frac{90 + 2a}{3}\right)$$

$$\max_{a} \pi_2 = \left(\frac{90 - a}{3}\right) \cdot \left(\frac{90 + 2a}{3}\right)$$

$$\frac{\partial \pi_2}{\partial a} = 0 \leftrightarrow (...) \leftrightarrow a = 22.5$$

4.3 Stackelberg

1. Consider a duopoly in which two firms offer an homogeneous product and compete in q with no cost to produce. There is a leader firm, say L, and a follower firm, say F. Market demand is $p = 10 - q$.

   (a) Derive firm F's best response and the equilibrium quantities, price and profits.

   (b) Assume now that both firms choose q simultaneously. Compare your results and explain the differences.

2. Consider a market with 4 firms. Two national (A and B) and the other two foreign (C and D). The foreign firms are price takers and their total costs are: $TC_C(q_C) = \frac{3}{2}q_C^2$ and $TC_D(q_D) = \frac{3}{4}q_D^2$. On the other hand, the national firms compete *à* la Cournot with average and marginal costs given by $c_a = c_b = 1$. In this market the demand is given by $Q = 10 - p$.

   (a) Find the foreign supply in this market.

   Solution:

   Price-takers: $S = MC \to C : MC_C = 3q_C \to q_C = \frac{P}{3} \wedge D : MC_D = \frac{3q_D}{2} \to q_D = \frac{2P}{3} \wedge F(P) = q_C + q_D \leftrightarrow Q(P)^{Foreign} = P$

   (b) Determine the equilibrium quantities, price and profits.

   Solution: This exercise follows the logic of the dominant-firm model, as the national firms do not compete for the entire market demand but rather for the residual demand.

   First step: Get the supply of the fringe $\to F(P) = P$

   Second step: Get the residual demand

$$D'(P) = \begin{cases} 0 & \text{if } P > 5, \\ 10 - 2P & \text{if } P \leq 5. \end{cases}$$

   Third step: Solve the maximization problem of the national firms:

$$\max_{q_A} \pi_A = P(q_A + q_B) \cdot q_A - q_A$$

$$= \left(5 - \frac{q_A + q_B}{2}\right) \cdot q_A - q_A$$

$$\frac{\partial \pi_A}{\partial q_A} = 0 \leftrightarrow 5 - q_A - \frac{q_B}{2} - 1 = 0 \leftrightarrow q_A = 4 - \frac{q_B}{2}$$

   By symmetry $\to q_A = q_B \to q_A = 4 - \frac{q_A}{2} \leftrightarrow q_A^* = \frac{8}{3} = q_B^* \wedge P = \frac{7}{3} \wedge q_C = 0.\overline{7} \wedge q_D = 1.\overline{5}$

   (c) (*) Assume that each national firm can choose one and only one of the following strategies. Each of these strategies is associated to different costs.

Strategy	**Cost**
Eliminate national competitors	2
Eliminate foreign competitors	1
Do nothing	0

   Each firm will simultaneously choose its strategy. If both choose the strategy "Eliminate the foreign competitors," they will divide equally the costs.

i. Calculate the eventual Nash Equilibrium in this game.
Solution:
The decision of each firm will influence the decision of the other $\rightarrow$ **Game Theory**
$\rightarrow$ **If both eliminate a national competitor** $\rightarrow \pi_A = \pi_B = -2$
$\rightarrow$ **If one eliminates national competitor and the other eliminates foreign** $\rightarrow$ **The firm that eliminates the other national competitor becomes a monopolist:**

$$\max_Q \pi^M = P(Q) \cdot Q - TC(Q)$$

$$= (10 - Q) \cdot Q - Q$$

$$\frac{\partial \pi^M}{\partial Q} = 0 \leftrightarrow 10 - 2Q - 1 = 0 \leftrightarrow Q = 4.5 \wedge P = 5.5 \wedge \pi^M = 20.25 - 2 = 18.25$$

$\rightarrow$ **If both do nothing** $\rightarrow$ **b's results:**
$\rightarrow$ **If one does nothing and the other eliminates national** $\rightarrow$ **The remaining national firm becomes the dominant firm:**

$$\max_P \pi^{DF} = (P - MC) \cdot D'(P) = (P - 1)(10 - 2P)$$

$$\frac{\partial \pi^{DF}}{\partial Q} = 0 \leftrightarrow 10 - 4P + 2 = 0 \leftrightarrow P = 3 \wedge Q = 4 \wedge \pi^{DF} = 8 - 2 = 6$$

$\rightarrow$ **If both eliminate foreign competitors** $\rightarrow$ **National firms will compete** $\grave{a}$ **la Cournot:**

$$\max_{q_A} \pi_A = P(q_A + q_B) \cdot q_A - TC(q_A)$$

$$= (10 - q_A - q_B) \cdot q_A - q_A$$

$$\frac{\partial \pi_A}{\partial q_A} = 0 \quad \Leftrightarrow \quad 10 - 2q_A - q_B - 1 = 0$$

$$\Leftrightarrow \quad q_A = q_B = 3 \quad \wedge \quad P = 4 \quad \wedge \quad \pi_A = \pi_B = (4-1) \cdot 3 - \frac{1}{2} = 8.5$$

$\rightarrow$ **If one does nothing and the other eliminates foreign competitors** $\rightarrow$ **National firms will compete** $\grave{a}$ **la Cournot:**
$\pi^{DN} = 9 \wedge \pi^{EF} = 8$

	EN	EF	DN
EN	$(-2, -2)$	$(\underline{18.25}, -1)$	$(6, \underline{0})$
EF	$(-1, \underline{18.25})$	$(8.5, 8.5)$	$(\underline{8}, 9)$
DN	$(\underline{0}, 6)$	$(9, \underline{8})$	$(3.5, 3.5)$

$\rightarrow$ **No Nash Equilibrium**

ii. How would your answer change if one of the firms should choose first? Comment the results taking in consideration the eventual benefits and costs for the first firm if it decides to eliminate the national competitor.
Solution: SPNE=(DN,(DN,EN,EF))

4.4 Bertrand

1. Two firms, 1 and 2, with constant marginal and average costs equal to 10, operate in a market with demand $P = 100 - Q$, while competing *à* la Bertrand.

   (a) What is the market equilibrium?
   Solution: Bertrand Paradox $\rightarrow P = MC = 10 \rightarrow \pi_i = 0$

   Firm 1 announces a new marketing strategy: "Never knowingly undersold!" To make this strategy credible, firm 1 adopts as its own the price catalog of firm 2, while dropping its own catalog. Firm 2 knows all these decisions.

   (b) What is the market equilibrium after the implementation of this marketing strategy? Explain your answer while describing the economic intuition behind it.
   Solution: Firm 1 will always charge the same price as firm 2. Therefore, each firm will face half of the market demand $\frac{Q^D}{2} = Q = 50 - P$

   $$\max_P \pi = (P - 10) \cdot Q = (P - 10) \cdot (50 - \frac{P}{2})$$

   $$\frac{\partial \pi}{\partial P} = 0 \quad \Leftrightarrow \quad 50 - P + 5 = 0 \Leftrightarrow P = 55 \wedge q_i = 22.5 \wedge \pi_i = 1012.5$$

   (c) Suppose that firm 1, though having its competitor's catalog available for consultation, keeps its own, using the former just to prove that its own price is never higher. What is the market equilibrium resulting from this other marketing strategy?
   Solution: Same as a) as it would have incentives to undercut firm 2 to grab the whole market.

   (d) Should firm 1 use the "aggressive" marketing strategy described in b)? Answer qualitatively.
   Solution: Yes, as it results in a profit increase.

   (e) Is firm 2 worse off as a result of it? Quantify. **Solution: No, as its profits increase from 0 to 1012.5.**

   (f) Do consumers benefit? Quantify again. **Solution: No. Consumer Surplus is higher in a or c (4050) than in b (1012.5).**

2. Consider a duopoly where firms choose prices simultaneously and independently. The demand functions for firms 1 and 2 are given by: $p_1 = 14 - q_1 + p_2$ and $p_2 = 14 - q_2 + p_1$. Firms face a constant marginal cost equal to 1.

   (a) Determine the equilibrium prices, quantities and profits.
   Solution:

   $$\max_{P_1} \pi_1 = (P_1 - MC) \cdot q_1 = (P_1 - 1) \cdot (14 - P_1 + P_2)$$

   $$\frac{\partial \pi_1}{\partial P_1} = 0 \quad \Leftrightarrow \quad 14 - 2P_1 + P_2 + 1 = 0 \Leftrightarrow P_1 = 7.5 + \frac{P_2}{2}$$

   By symmetry: $P_1 = P_2 \rightarrow P_1 = 7.5 + \frac{P_1}{2} \Leftrightarrow P_1^* = P_2^* = 15 \wedge q_1^* = q_2^* = 14 \wedge \pi_1^* = \pi_2^* = 196$

(b) Compare these values with those obtained in the Bertrand model, explaining the differences (in terms of equilibrium prices and profits).

Solution: If these firms were competing *à* la Bertrand then each firm would have an incentive to undercut each other until both firms were charging its marginal cost and making zero profits. In this exercise, this situation is avoided because each firm produces a differentiated product - product differentiation is a way out of the Bertrand Paradox.

3. Consider a duopoly with a differentiated good where demand and cost functions are, for firm 1 and 2 respectively: $q_1 = 88 - 4p_1 + 2p_2$ and $c_1 = 10q_1$ for 1, and $q_2 = 56 + 2p_1 - 4p_2$ and $c_2 = 8q_2$ for 2. Find the reaction function in prices, assuming that each firm maximizes its profits. Determine the equilibrium price, indicating quantities and profits of each firm.

Solution:

First step: Get the BR_1

$$\max_{P_1} \pi_1 = (P_1 - MC) \cdot q_1 = (P_1 - 10) \cdot (88 - 4P_1 + 2P_2)$$

$$\frac{\partial \pi_1}{\partial P_1} = 0 \quad \Leftrightarrow \quad 88 - 8P_1 + 2P_2 + 40 = 0 \Leftrightarrow P_1 = 16 + \frac{P_2}{4}$$

Second step: Get the BR_2

$$\max_{P_2} \pi_2 = (P_2 - MC) \cdot q_2 = (P_2 - 8) \cdot (56 - 4P_2 + 2P_1)$$

$$\frac{\partial \pi_2}{\partial P_2} = 0 \quad \Leftrightarrow \quad 56 + 2P_1 - 8P_2 + 32 = 0 \Leftrightarrow P_2 = 11 + \frac{P_1}{4}$$

Third step: Find the intersection between the BR of each firm.

$$\begin{cases} P_1 = 16 + \frac{P_2}{4} \\ P_2 = 11 + \frac{P_1}{4} \end{cases} \leftrightarrow (...) \leftrightarrow \begin{cases} P_1^* = 20 \\ P_2^* = 16 \end{cases} \rightarrow \begin{cases} \pi_1^* = 400 \\ \pi_2^* = 256 \end{cases}$$

4. In the former exercise, suppose firms choose simultaneously one of the two available technologies before competing in prices. The available technologies have the following total cost functions: $ct_a = 10q_a + 120$ and $ct_b = 25q_b + 5$. Find the chosen technologies and the equilibrium in the goods market.

Solution: This exercise is similar to exercise 8 of Cournot.

→ If both choose A

$$\max_{P_1} \pi_1 = (P_1 - 10) \cdot q_1 - 120 \rightarrow P_1 = 16 + \frac{P_2}{4}$$

$$\max_{P_2} \pi_2 = (P_2 - 10) \cdot q_2 - 120 \rightarrow P_2 = 12 + \frac{P_1}{4}$$

$$\begin{cases} P_1 = 16 + \frac{P_2}{4}, \\ P_2 = 12 + \frac{P_1}{4}. \end{cases} \leftrightarrow (...) \leftrightarrow \begin{cases} P_1^* = 20.3 \\ P_2^* = 17.1 \end{cases} \rightarrow \begin{cases} \pi_1^* = 301.6 \\ \pi_2^* = 79.75 \end{cases}$$

→ **If firm 1 chooses A and 2 chooses B**

$$\max_{P_1} \pi_1 = (P_1 - 10) \cdot q_1 - 120 \rightarrow P_1 = 16 + \frac{P_2}{4}$$

$$\max_{P_2} \pi_2 = (P_2 - 25) \cdot q_2 - 5 \rightarrow P_2 = 19.5 + \frac{P_1}{4}$$

$$\begin{cases} P_1 = 16 + \frac{P_2}{4}, \\ P_2 = 19.5 + \frac{P_1}{4}. \end{cases} \leftrightarrow (...) \leftrightarrow \begin{cases} P_1^* = 22.3 \\ P_2^* = 25.1 \end{cases} \rightarrow \begin{cases} \pi_1^* = 472 \\ \pi_2^* = -5 \end{cases}$$

→ **If firm 1 chooses B and 2 chooses A**

$$\max_{P_1} \pi_1 = (P_1 - 25) \cdot q_1 - 5 \rightarrow P_1 = 23.5 + \frac{P_2}{4}$$

$$\max_{P_2} \pi_2 = (P_2 - 10) \cdot q_2 - 120 \rightarrow P_2 = 12 + \frac{P_1}{4}$$

$$\begin{cases} P_1 = 23.5 + \frac{P_2}{4}, \\ P_2 = 12 + \frac{P_1}{4}. \end{cases} \leftrightarrow (...) \leftrightarrow \begin{cases} P_1^* = 28.3 \\ P_2^* = 19.1 \end{cases} \rightarrow \begin{cases} \pi_1^* = 37.7 \\ \pi_2^* = 208.8 \end{cases}$$

→ **If both choose B**

$BR_1 : P_1 = 23.5 + \frac{P_2}{4}$

$BR_2 : P_1 = 19.5 + \frac{P_1}{4}$

$$\begin{cases} P_1 = 23.5 + \frac{P_2}{4}, \\ P_2 = 19.5 + \frac{P_1}{4}. \end{cases} \leftrightarrow (...) \leftrightarrow \begin{cases} P_1^* = 30.3 \\ P_2^* = 27.1 \end{cases} \rightarrow \begin{cases} \pi_1^* = 105.8 \\ \pi_2^* = 12.1 \end{cases}$$

	A	B
A	$(\mathbf{301.6}, \mathbf{79.75})$	$(\mathbf{472}, -5)$
B	$(37.7, \mathbf{208.8})$	$(105.8, 12.1)$

Nash Equilibrium: (A,A)

5. (*) Consider the following two period model. In the first period only one firm is present in the market. This monopolist produces 5 units, sold at price $P = 32$. In the second period, a rival firm enters the market with a slightly differentiated product. The two firms will compete in prices, with both having marginal cost equal to zero. The demands for each firm are given by: $q_1 = a - bp_1 + cp_2$ and $q_2 = a - bp_2 + cp_1$.

   (a) Find the equilibrium prices and profits in the second period for $a = 90$, $b = 2$ and $c = 1$.
   Solution:

$$\max_{P_1} \pi_1 = P_1 \cdot q_1 = P_1 \cdot (90 - 2P_1 + P_2)$$

$$\frac{\partial \pi_1}{\partial P_1} = 0 \quad \Leftrightarrow \quad 90 - 4P_1 + P_2 = 0 \Leftrightarrow P_1 = 22.5 + \frac{P_2}{4}$$

By symmetry $\rightarrow P_1 = P_2$

$P_1 = 22.5 + \frac{P_1}{4} \leftrightarrow P_1 = 30 = P_2 \rightarrow \pi_1 = \pi_2 = 1800$

(b) Admit that the firm present in the market in the first period has the possibility to implement the most-favored-costumer clause, i.e., has the possibility to guarantee to each customer in the next period that it would return any difference if the price was smaller in the next period. If this clause is introduced, what will be the equilibrium in the second period? Comment.

Solution:

For firm 2 nothing changes: $P_2 = 22.5 + \frac{P_1}{4}$

For firm 1:

$$\max_{P_1} \pi_1 = P_1 \cdot q_1 - (32 - P_1) \cdot 5$$

$$\frac{\partial \pi_1}{\partial P_1} = 0 \quad \Leftrightarrow \quad 90 - 4P_1 + P_2 + 5 = 0 \Leftrightarrow P_1 = 23.75 + \frac{P_2}{4}$$

$$\begin{cases} P_1 = 23.75 + \frac{P_2}{4}, \\ P_2 = 22.5 + \frac{P_1}{4}. \end{cases} \quad \leftrightarrow (...) \leftrightarrow \quad \begin{cases} P_1^* = 31.3 \\ P_2^* = 30.3 \end{cases} \quad \rightarrow \quad \begin{cases} \pi_1^* \approx 1803 \\ \pi_2^* \approx 1840 \end{cases}$$

6. Two firms compete through prices in a market with negatively sloped demand and both have constant marginal costs. One of them (A) starts a project of R&D, with common knowledge cost, which implies a decrease in it's marginal cost. The other firm (B) does not have access to the project.

(a) Will consumers always win with this procedure?

Sol: No, if firm A remains the most efficient firm, the market equilibrium price will stay at $P = MC - \varepsilon$

(b) Will they never win?

Sol: No, if the market equilibrium price decreases, consumers will benefit. For example, if firm A was not the most efficient firm before the R&D project but becomes so afterward, the market equilibrium price will decline.

(c) Will firm B always lose as a consequence of firm A executing the project? **Sol: No. Firm B will only incur a loss if its profits decrease. For example, if firm A remains the most efficient firm both before and after the project, firm B will not experience any loss.**

(d) Suppose that after the conclusion of the R&D project, firm B has access to it. Would firm B execute it? **Sol: No. Doing so would lead to a Bertrand Paradox, as both firms would have the same cost structure, driving profits to zero. Therefore, firm B has no incentive to execute the R&D project.**

(e) Will this decision be always socially desirable? **Sol: No, because it prevents the occurrence of a Bertrand Paradox, which is socially efficient.**

7. Comment the following statement: "Oranges and apples are substitutes. Yet, the decision variables of firms who produce them may be strategic complements."

Solution: True! Oranges and apples are indeed substitutes. Additionally, two variables are strategic complements if they are positively correlated. In price competition, for example, the best response (BR) function of each firm could be given by $P_1 = a + bP_2$**. Since** $\frac{\partial P_1}{\partial P_2} > 0$**, it follows that** P_1 **and** P_2 **are strategic complements.**

8. Two firms, an old one, denoted O, and a new one, denoted N, compete in prices, which they set simultaneously and independently while serving the market for an homogeneous product whose demand equals $Q = 10 - P$.
   The old firm's constant marginal and average cost, 2, is common knowledge.
   However, the new firm's constant marginal and average cost is only known to itself. On the other hand, the old firm knows that firm N produces at a constant marginal and average cost, which can equal either 3 or 1. Firm O attaches equal probability, 0.5, to firm N's two possible marginal and average costs. This incomplete **but accurate** information held by the old firm is common knowledge.

   (a) What price should the old firm choose? Quantify and explain.
   (b) What price should the new firm choose? Quantify and explain.
   (c) What is the old firm's expected profit? Quantify and explain.
   (d) What are the new firm's possible profits? Quantify and explain.
   (e) Would it be socially better if firm N's constant marginal and average cost was common knowledge? Quantify and explain.

9. Two firms sell imperfectly differentiated products, denoted 1 and 2, whose demand functions are $q_1 = 10 - p_1 + p_2$ and $q_2 = 10 - p_2 + p_1$, respectively. Each produces its product at a constant marginal and average cost of 6, i.e., $c_1 = 6 = c_2$. They compete in prices, which they set simultaneously and independently.

   (a) What price will each firm set? How much will each sell? What profit will each attain? Quantify.

   Firm 1 has embarked on an R&D project that has lowered its constant marginal and average cost to 2.

   (b) What price will each firm set? How much will each sell? What profit will each attain? Quantify.

   Suppose that firm 2 is unaware of firm 1's R&D project. This gives rise to the direct effect. Suppose now that firm 2 becomes aware of the R&D project. This would give rise to additional price changes, which constitute the strategic effect. The two together yield the total effect of the R&D project.

   (c) What is the direct effect of the R&D project on firm 1's decision variable, i.e., its price? And the strategic effect? Quantify and explain.
   (d) What is the direct effect of the R&D project on firm 1's profit? And the strategic effect? Quantify and explain.

5 Market Power

1. Consider a market with a single demand where two firms compete, defining prices simultaneously. The two firms have been tacitly colluding, practicing a monopoly price and sharing the profits generated by their cooperation equally.

   (a) What discount factor must these firms have for cooperation to be maintained in the future?
   Solution:
   Two firms will collude as long as: $\pi^{Collusion} \geq \pi^{Deviation} \leftrightarrow \frac{\pi^M}{2} + \frac{\pi^M}{2}\delta + \frac{\pi^M}{2}\delta^2 + ... \geq \pi^M + 0\delta + 0\delta^2 + ... \leftrightarrow (...) \leftrightarrow \delta \geq \frac{1}{2}$

A change in consumers' preferences has made the market more attractive. As such, it is now expected to grow at a rate g per year.

(b) How does the discount factor required to sustain collusion change?

Solution:

Two firms will collude as long as: $\pi^{Collusion} \geq \pi^{Deviation} \leftrightarrow \frac{\pi^M}{2} + \frac{\pi^M}{2}(1+g)\delta + \frac{\pi^M}{2}(1+g)^2\delta^2 + ... \geq \pi^M + 0\delta + 0\delta^2 + ... \leftrightarrow (...) \leftrightarrow \delta \geq \frac{1}{2(1+g)}$

(c) Study the impact of g on the discount factor and explain the intuition.

Solution: $\Delta^+ g$ implies that the future looks better. Therefore, there is more to lose if one deviates, therefore a lower δ is sufficient to sustain collusion, i.e., it is easier to sustain a collusive agreement.

2. In the homogeneous wallet market, firm 1 and 2 compete in quantities. The demand curve is $P = 100 - 2Q$, where P is price, and Q total demanded quantity. Firm 1 has the following total cost function, $C(q_1) = 20q_1$; while firm 2 has the following cost function, $c(q_2) = 30q_2$.

(a) Compute the equilibrium price, quantities and profits for the one-stage game.

Solution: Usual Cournot setting

First step: Get BR_1

$$\max_{q_1} \pi_1 = P(q_1 + q_2) \cdot q_1 - TC(q_1)$$

$$= [100 - 2(q_1 + q_2)] \cdot q_1 - 20q_1$$

$$\frac{\partial \pi_1}{\partial q_1} = 0 \quad \leftrightarrow \quad 100 - 4q_1 - 2q_2 - 20 = 0 \leftrightarrow q_1 = 20 - \frac{q_2}{2}$$

Second step: Get BR_2

$$\max_{q_2} \pi_2 = P(q_1 + q_2) \cdot q_2 - TC(q_2)$$

$$= [100 - 2(q_1 + q_2)] \cdot q_2 - 30q_2$$

$$\frac{\partial \pi_2}{\partial q_2} = 0 \quad \leftrightarrow \quad 100 - 4q_2 - 2q_1 - 30 = 0 \leftrightarrow q_2 = 17.5 - \frac{q_1}{2}$$

Third step: Find the intersection between the two BR

$$\begin{cases} q_1 = 20 - \frac{q_2}{2}, \\ q_2 = 17.5 - \frac{q_1}{2}. \end{cases} \leftrightarrow (...) \leftrightarrow \begin{cases} q_1^* = 15, \\ q_2^* = 10. \end{cases} \rightarrow \begin{cases} \pi_1^* = 450, \\ \pi_2^* = 200. \end{cases}$$

(b) Let this game allow collusion by becoming an infinitely-repeated game. Which firm will produce as a monopolist now? Why?

Solution: Firm 1, since it is the most efficient one.

(c) What is the efficient cartel solution with lateral payments? What is the smallest share of the profits that firm 1 would be willing to accept?

Solution: As previously discussed, Firm 1 will be the sole producer and will produce the monopoly quantity.

$$\max_{Q} \pi = P(Q) \cdot Q - TC(Q)$$

$$= (100 - 2Q) \cdot Q - 20Q$$

$$\frac{\partial \pi}{\partial Q} = 0 \quad \leftrightarrow \quad 100 - 4Q - 20 = 0 \leftrightarrow Q = 20 \wedge P = 60 \wedge \pi = 800$$

The smallest share of the profit that firm 1 would be willing to accept is $\frac{\pi^{Cournot}}{\pi^{M}} = \frac{450}{800} = 9/16$

(d) For a 75%-25% profit-sharing agreement, which discount factor is needed for it to be sustainable?

Solution:

$\rightarrow$ **Firm 1:** $\pi^{Collusion} \geq \pi^{Deviation} \leftrightarrow 600 + 600\delta + 600\delta^2 + ... \geq 800 + 450\delta + 450\delta^2 + ... \leftrightarrow (...) \leftrightarrow \delta \geq \frac{4}{7}$

$\rightarrow$ **Firm 2: Firm 2 has no incentives to deviate as it would not gain from it**.

Firms will collude as long as $\delta \geq \frac{4}{7}$

(e) What is the efficient and equitable solution (i.e., both firms produce the same)? What are the discount factors needed for this kind of cooperation to hold?

Solution:

$$\max_{q_1,q_2} \pi = P(q_1 + q_2) \cdot q_1 + P(q_1 + q_2) \cdot q_2 - TC(q_1) - TC(q_2)$$

$$= [100 - 2(q_1 + q_2)] \cdot q_1 + [100 - 2(q_1 + q_2)] \cdot q_2 - 20q_1 - 30q_2$$

$$\text{s.t. } q_1 = q_2 = q$$

$$\leftrightarrow$$

$$\max_{q} \pi = (200 - 8q) \cdot q - 50q$$

$$\frac{\partial \pi}{\partial q} = 0 \quad \Leftrightarrow \quad 200 - 16q - 50 = 0 \quad \Leftrightarrow \quad q = 9.375$$

$$Q = 18.75 \quad \wedge \quad P = 62.5 \quad \wedge \quad \pi_1^* = 398.4375 \quad \wedge \quad \pi_2^* = 304.6875$$

3. Two firms, A and B, supply a market whose yearly demand is given by $q = 10 - p$. They produce the good traded in this market at a constant marginal and average cost of 4. Firms compete in quantities, which they choose every year, doing so simultaneously and independently, and expect to do so forever.

Suppose first that A and B are not colluding.

(a) How much will each produce per year? What will their yearly profit be?

Suppose now that A and B are tacitly colluding with the aim of maximizing and equally sharing industry profit.

(b) How much will each produce per year? What will their yearly profit be?

Suppose that if either firm deviates from the collusive agreement, the other resorts to playing forever as if they had never colluded.

(c) What is the condition on the discount factor, δ, that must be obeyed for the two firms to be able to collude?

(d) What would this condition be if the two firms competed in prices instead of competing in quantities?

(e) In which case is it easier to sustain a tacit collusion agreement? Intuitively explain why.

4. Consider two firms interacting in two identical and independent markets. The markets differ in that in market 1 a firm's price at time t is observed at $t+1$, whereas in market 2 it is learnt only at $t+2$. Thus, although each of the markets meets every period, market 2 has longer information lags.

   (a) Derive the set of discount factors such that, in the absence of multimarket contact, collusion in market 2 would be sustainable.

   Solution: Two firms will collude as long as: $\pi^{Collusion} \geq \pi^{Deviation} \leftrightarrow \frac{\pi^M}{2} + \frac{\pi^M}{2}\delta + \frac{\pi^M}{2}\delta^2 + ... \geq \pi^M + \pi^M\delta + 0\delta^2 + 0\delta^3 ... \leftrightarrow (...) \leftrightarrow \delta \geq \frac{\sqrt{2}}{2}$

   (b) Compute the minimum threshold value for the discount factor such that, under multimarket contact, collusion in both markets is sustainable. **Hint:** if firms can use information from one market to detect (and punish) deviation in *all* markets, in which period would it be better to deviate in market 2? And in market 1?

   Solution: In this exercise, we have multi-market contact, therefore, the benefits from collusion are:

$$\frac{\pi_1^M}{2} + \frac{\pi_2^M}{2} + \left[\frac{\pi_1^M}{2} + \frac{\pi_2^M}{2}\right]\delta + \left[\frac{\pi_1^M}{2} + \frac{\pi_2^M}{2}\right]\delta^2 + \cdots = (\frac{\pi_1^M}{2} + \frac{\pi_2^M}{2}) \cdot \frac{1}{1-\delta}.$$

   However, if you want to deviate, the optimal way to do it is to deviate on market 2 in the first and second period and deviate on market 1 only on the second period, and then you get punished in both. You want to do this because you would get punished in both markets anyway. Hence, you get full monopoly profits from market 2 in periods 1 and 2, and from market 1 you get half the monopoly profits in period 1 (you are still colluding) and full monopoly profits in the second period (afterwards you are punished and get zero profits):

$$\frac{\pi_1^M}{2} + \pi_1^M\delta + \pi_2^M + \pi_2^M\delta.$$

   To make collusion sustainable, benefits from collusion should be higher than benefits from deviating:

$$(\frac{\pi_1^M}{2} + \frac{\pi_2^M}{2}) \cdot \frac{1}{1-\delta} \geq \frac{\pi_1^M}{2} + \pi_1^M\delta + \pi_2^M + \pi_2^M\delta.$$

It is also said that the markets are identical, which means $\pi_1^M = \pi_2^M = \pi^M$, therefore we can write the above condition as:

$$\frac{\pi^M}{1-\delta} \geq \frac{\pi^M}{2} + \pi^M + 2\pi^M\delta$$

$$\Rightarrow (...) \Rightarrow \delta \geq 0.64$$

(c) Compare and interpret the results obtained in the two previous items.
Solution: It is easier to sustain a collusive agreement under multi-market contact, as firms have more to gain from colluding and more to lose in the long run if they deviate.

5. Consider a market with demand given by $Q = 40 - P$. There are two firms, with constant marginal costs $c_1 = 10$ and $c_2 = 11$. Compute the Cournot solution and the following Cartel solutions:
Solution: Cournot $q_1 = \frac{31}{3}$**,** $q_2 = \frac{28}{3}$**,** $p = 20.\overline{3}$**,** $\pi_1 = 106.\overline{6}$**,** $\pi_2 = 87.\overline{1}$

- Efficient Cartel with lateral payments
Solution: Efficient cartel → Only the most efficient firm produces and shares profits with the remaining firms

$$\max_Q \pi = (40 - Q) \cdot Q - 10Q$$

$$\frac{\partial \pi}{\partial Q} = 0 \quad \Leftrightarrow \quad 40 - 2Q - 10 = 0 \quad \Leftrightarrow \quad Q = 15 \quad \wedge \quad P = 25 \quad \wedge \quad \pi = 225$$

Lateral payments: $\pi_1 \geq \pi^{Cournot} \rightarrow \pi_1 \geq 106.6 \wedge \pi_2 \geq 87.1$

- Efficient Cartel with half and half division of profits.
Equal division: $\pi_1 = \pi_2 = 112.5$
- Efficient with equal market shares.

$$\max_{q_1, q_2} \pi_1 + \pi_2 = P(q_1 + q_2) \cdot q_1 + P(q_1 + q_2) \cdot q_2 - TC(q_1) - TC(q_2)$$

$$= (40 - q_1 - q_2) \cdot q_1 + (40 - q_1 - q_2) \cdot q_2 - 10q_1 - 11q_2$$

$$\text{s.t. } q_1 = q_2$$

$$\Leftrightarrow \max_q \pi_1 + \pi_2 = (40 - 2q) \cdot q + (40 - 2q) \cdot q - 21q$$

$$\frac{\partial \pi}{\partial q} = 0 \quad \Leftrightarrow \quad 40 - 4q + 40 - 4q - 21 = 0 \quad \Leftrightarrow \quad q^* = 7.375$$

$$Q = 14.75 \quad \wedge \quad P = 25.25$$

$$\pi_1 = (25.25 - 10) \cdot 7.375 = 112.5$$

$$\pi_2 = (25.25 - 11) \cdot 7.375 = 105.1$$

6. In market A there are two firms operating. Firm 1 has a 70% market share and firm 2 has a 30% market share. They have been competing *à* la Bertrand and therefore have no profits in equilibrium. The are considering entering into a collusive agreement, in which firm 1 gets 70% of the monopoly profits and firm 2 gets 30% of the monopoly profits.

   (a) Which discount factors would make this agreement stable? How are these related with each firm's market share?
   Solution:
   Firm 1 will collude if:

$$0.7\pi^M + 0.7\pi^M\delta + 0.7\pi^M\delta^2 + \cdots \geq \pi^M + 0\delta + \cdots \leftrightarrow (\dots) \leftrightarrow \delta \geq 0.3$$

   Firm 2 will collude if:

$$0.3\pi^M + 0.3\pi^M\delta + 0.3\pi^M\delta^2 + \cdots \geq \pi^M + 0\delta + \cdots \leftrightarrow (\dots) \leftrightarrow \delta \geq 0.7$$

   $\Rightarrow$ **Firms will collude if $\delta \geq 0.7$.**

   (b) Generalize for market shares $s_1 = 1 - s_2$. How does the asymmetry in the size of the firms impact the stability of collusive agreements?
   Solution:
   A firm with market share s will collude if:

$$\pi^{Collusion} \geq \pi^{\text{deviation}} \leftrightarrow s\pi^M + s\pi^M\delta + s\pi^M\delta^2 + \cdots \geq \pi^M \leftrightarrow (\dots) \leftrightarrow \delta \geq 1 - s$$

   As previously seen, the larger s is, the easier it is for the firm with a market share of s to engage in a collusive agreement. Conversely, the larger s, the more difficult it becomes for the firm holding a market share of $1 - s$ to be willing to collude. Therefore, market share asymmetry makes collusion more difficult to sustain.

6 Barriers to Entry

1. Firm 1 is the first firm in a given market. Firm 1 can choose one of two technologies, A and B, with respectively the following cost functions:

$$C_A = 60 + 2q_1 \quad C_B = 10 + 8q_1$$

   The inverse demand curve is given by $P = 20 - Q$ where Q is the total output of the industry.

   (a) Which technology would firm 1 choose if its monopoly lasts forever?

   (b) Suppose that firm 2 is considering the possibility of entry in this market and that it can also adopt any of the aforementioned technologies. If firm 2 enters, firms will compete à la Cournot. Knowing this, which technology should firm 1 choose? In this model, what is the welfare effect of the existence of a potential competitor?

2. One firm with constant marginal and average costs equal to 10 is in a market with demand $P = 100 - Q$. Another firm is considering to enter in this market with a technology with constant marginal and average cost equal to 30. The new firm can choose between building a plant with capacity 10 or 100. Assume that firms compete ex post à la Cournot and the possibility that the installed (i.e., incumbent) firm acts aggressively.

(a) What capacity should the new firm choose? Why?
Solution:
$\Rightarrow$ **If it chooses** $k = 100$

$$\max_{q_I} \pi_I = P(q_I + q_E) \cdot q_I - TC(q_I)$$

$$= (100 - q_I - q_E) \cdot q_I - 10q_I$$

$$\frac{\partial \pi_I}{\partial q_I} = 0 \quad \Leftrightarrow \quad 100 - 2q_I - q_E - 10 = 0 \quad \Leftrightarrow q_I = 45 - \frac{q_E}{2}$$

$$\max_{q_E} \pi_I = P(q_I + q_E) \cdot q_E - TC(q_E)$$

$$= (100 - q_I - q_E) \cdot q_E - 30q_E$$

$$\frac{\partial \pi_E}{\partial q_E} = 0 \quad \Leftrightarrow \quad 100 - 2q_E - q_I - 30 = 0 \quad \Leftrightarrow q_E = 35 - \frac{q_I}{2}$$

The incumbent can either:
$\rightarrow$ **Accommodate:**

$$\begin{cases} q_I = 45 - \frac{q_E}{2}, \\ q_E = 35 - \frac{q_I}{2}. \end{cases} \quad \Leftrightarrow (...) \Leftrightarrow \begin{cases} \pi_I = 1344.4 \\ \pi_E = 277.7. \end{cases}$$

$\rightarrow$ **Be aggressive: The incumbent can force the new entrant to produce zero units of output.**
$BR_E : q_E = 35 - \frac{q_I}{2} = 0 \rightarrow q_I = 70 \quad \wedge \quad P = 30 \quad \pi_I = 1400$
As $\pi_{PL} > \pi_{Accommodate}$ **the incumbent will choose to be aggressive and as such** $\pi_E = 0$
$\Rightarrow$ **If it chooses** $k = 10$
The incumbent can again either:
$\rightarrow$ **Accommodate:** $q_E = 10 \rightarrow q_I = 45 - \frac{10}{2} = 40 \quad \wedge \quad P = 50 \quad \wedge \pi_I = 1600 \quad \wedge \quad \pi_E = 300$
$\rightarrow$ **Be aggressive: As previously computed, in this case** $q_I = 70 \quad \wedge \quad \pi_I = 1400 \quad \wedge \quad \pi_E = 0$
In this case, the incumbent will accommodate.
Given this, the new entrant will choose $k = 10$ ***as it allows to have*** $\pi_E > 0$**.**

(b) If the firm that is considering entering has access to a technology equal to the technology of the firm already installed, how do your previous answers change?
Solution:
$\rightarrow$**If** $k = 100$

$$BR_E : q_E = 45 - \frac{q_I}{2} \quad \Rightarrow \quad \text{By symmetry, } q_I = q_E$$

$$\Rightarrow \quad q_I = q_E = 30 \quad \wedge \quad P = 40 \quad \wedge \quad \pi_I = \pi_E = (40 - 10) \cdot 30 = 900$$

When two firms have the same technology, being aggressive is never profitable in a one-shot interaction

$\rightarrow$**If** $k = 10$

$$q_E = 10 \quad \wedge \quad q_I = 45 - \frac{10}{2} = 40 \quad \wedge \quad P = 50 \quad \wedge \quad \pi_E = (50 - 10) \cdot 10 = 500$$

In this case, the incumbent will not adopt a limit pricing strategy, allowing the new entrant to choose $k = 100$***.***

3. Imagine that the demand of some homogeneous product is given by $P = 100 - 2Q$. The total cost is given by $TC = 10Q$. Consider a non-refundable cost of entering the market of $S = 100$. Nowadays the market is covered by only one firm, but there is some potential competitor.

   (a) How much will the first firm produce if it remains a monopolist (without potential competitor)?

   (b) Assuming that the potential competitor enters competing in a Stackelberg fashion, what are the profits for this competitor?

   (c) If the original firm would like to keep the potential competitor out, how much would she need to produce? What would be the resulting price?

   (d) Assuming that the first firm takes a limit price strategy, compute the Lerner index as a function of S. Explain your result.

   (e) What is the value of S such that, for values below it, the first firm would prefer to avoid a limit price strategy?

7 Bibliography

Several of the exercises you can find here are from the following textbooks. You may use these texts to find even more exercises that are not contained in this document. Furthermore, the vast database of past Midterms and Exams will help you prepare for the assessments.

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