In-Class Lecture

Time Value of Money
Interest rates and Cash-flows

Advanced Financial Management

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Key takeaways

03 **Apply the perpetuity and annuity formulas.**

04 **Convert quoted interest rates in the form provided into effective interest rates which are appropriate to use in the valuation of a stream of cash flows.**

Perpetuity and Annuities: summary

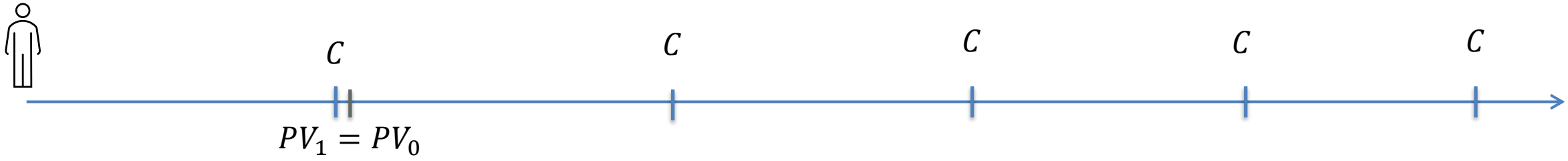
	With constant cash-flows	With cash-flows growing at a constant rate
Stream of perpetual cash-flows	$PV\ of\ Perpetuity = \frac{CF_1}{r}$	$PV\ of\ growing\ Perpetuity = \frac{CF_1}{r-g}$ — When there is growth in cash-flows, the cash-flow that must be used in the formulas is the first one in the stream
Stream of cash-flows for a limited number of periods	$PV\ of\ Annuity = \frac{CF_1}{r} \times \left(1 - \frac{1}{(1+r)^n}\right)$ — These formulas provide the present value represented in the period right before the first payment, not necessarily today	$PV\ of\ growing\ Annuity = \frac{CF_1}{r-g} \times \left(1 - \frac{(1+g)^n}{(1+r)^n}\right)$

How to apply the formulas correctly

Computation

- Formulas assume the first cash-flow accrues <u>at the end</u> of the first period!
  - Remember to apply the formulas taking this into account
- Use effective rates
- Consistency is key!
  - Frequency of payments, discount rates and growth rates should be the same (e.g. with monthly payments, use monthly discount and growth rates).
    - **Note:** the frequency of payments dictates what rates you should use
  - Discount nominal cash-flows with nominal interest rates and discount real cash-flows with real interest rates!

Proof of perpetuity formula



$$PV_0 = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \cdots$$

$$PV_1 = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \cdots$$

$$PV_0 = \frac{C + PV_1}{1+r} \quad \Rightarrow PV_0 = \frac{C + PV_0}{1+r} \quad \Rightarrow PV_0 = \frac{C}{r}$$

Annuities in real life with mortgages

A mortgage loan is an annuity with PV equal to the loan amount and C the monthly payment.

Mortgage loans are typically amortizing: each monthly payment C contains both interest payment on the outstanding balance as well as part of the balance itself.

Example

You are buying a small apartment with a 30-year mortgage loan of €100,000. Payments are due monthly at an APR of 6% compounded monthly.

a. What is the value of the monthly payment?
b. What is the loan balance at the end of the first year? (the amount you would need to pay back to the bank in full at the end of the first year to pay off the mortgage)
c. Split the total payment in the first year into interest payments and amortization

Example: amortizing loan

a. What is the value of the monthly payment?

Effective monthly interest rate: $r_m = \frac{6\%}{12} = 0.5\%$

The monthly payment solves the equation:

$$100000 = \frac{C}{0.005}\left(1 - \left(\frac{1}{1+0.005}\right)^{360}\right) \rightarrow C = 599.6$$

b. What is the loan balance at the end of the first year? (the amount you would need to pay back to the bank in full at the end of the first year to pay off the mortgage)

Loan balance at the end of the first year is the PV of all future payments at that point in time.

$$\text{loan balance} = \frac{599.6}{0.005}\left(1 - \left(\frac{1}{1+0.005}\right)^{348}\right) = 98772$$

Example: amortizing loan

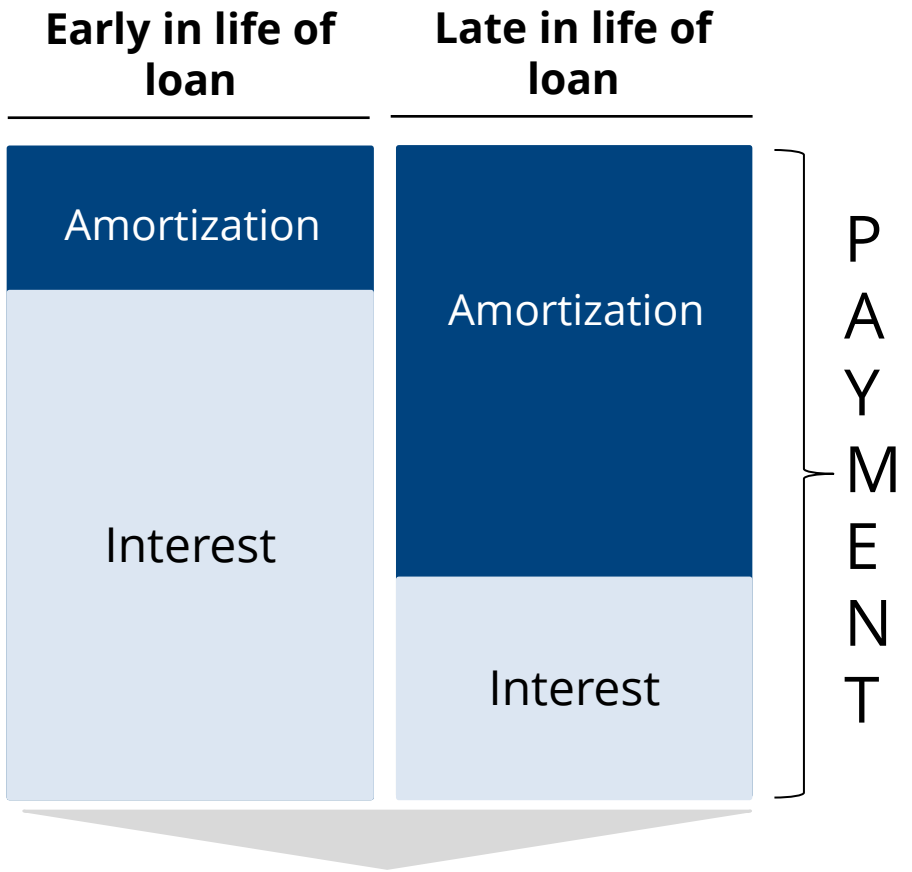
c. Split the total payment in the first year into interest payments and amortization

Total payment in first year:

12 * 599.6 = 7,194.6

Amortization: 100000-98772 = 1228

Interest payment: 7194.6 – 1228 = 5966.6

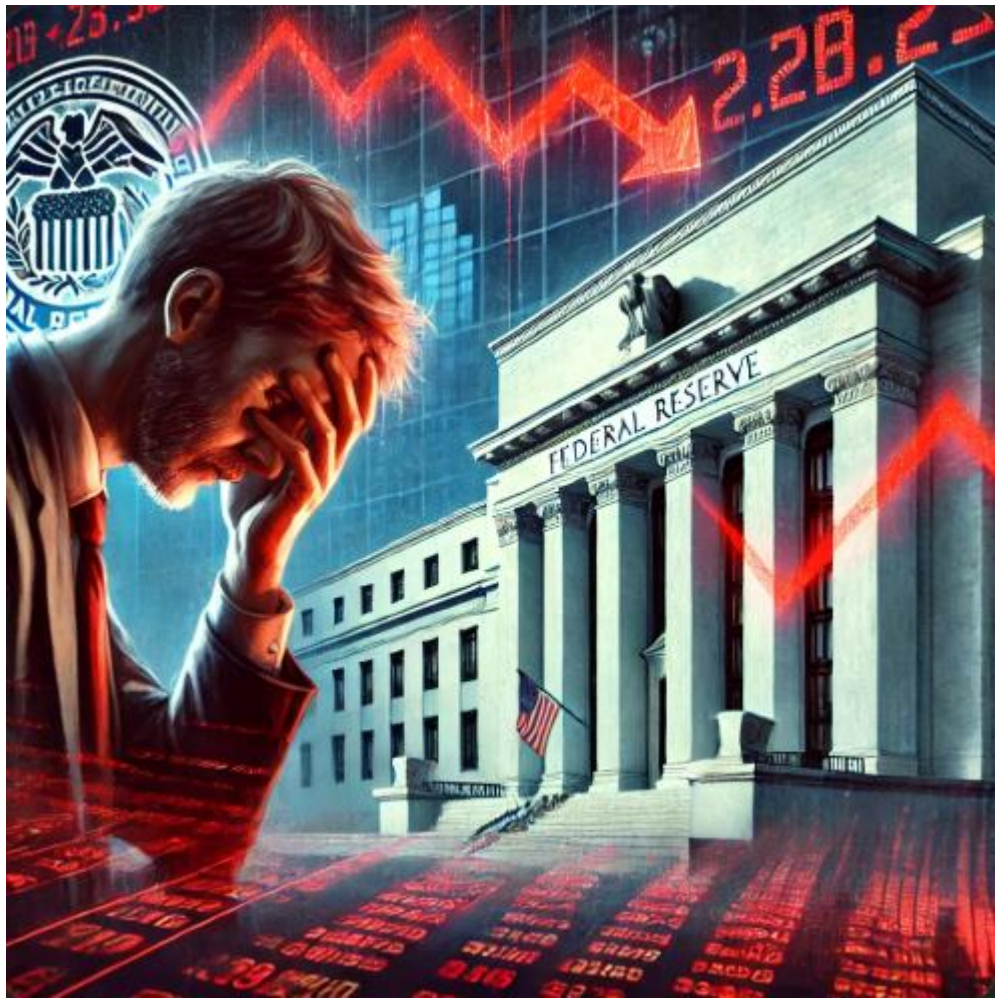


Exercise 1

Emma is 30 years old and her salary next year will be €40,000. Emma forecasts that her salary will increase at a constant rate of 5% per year until her retirement at age of 60.

a. If the EAR is 8%, what is the PV of these future salary payments?
b. If Emma saves 5% of her salary each year and invests these savings at an interest rate of 8%, how much will she have saved by age 60?
c. If Emma plans to spend these savings in even amounts over her 20 years of retirement, how much can she spend each year?


NEWS


FEDERAL RESERVE

News discussion

1. Why should we care about Central Banks rate changes?

2. What happened to the time value of money on March 3rd 2020 in the USA?

Suppose you are living in the US and you currently have a mortgage. You pay $1000 every month and there are 48 payments remaining. The article mentions the following: “The Fed's move should lower the cost of borrowing, including for credit cards, auto loans and mortgages”. If the FED rate cut decreases the monthly interest rate, then surely the PV of your loan has increased.

3. Comment on the accuracy of the previous statement.

4) Why does the article mention that declines in indexes occurred despite the interest rate cut? That is, why should we have expected an increase in the indexes (holding everything else constant)?

Interest rates

	Without compounding	With compounding
Calculating the annual rate from rate for the period *n*	*Stated annual rate* $APR = n \times r_n$	*Effective annual rate* $EAR = (1+r_n)^n-1$ ▶ $EAR = (1+\frac{APR}{n})^n-1$
Calculating the rate for the period *n* from the annual rate	*Proportional rate* $r_n = \frac{APR}{n}$	*Equivalent rate* $r_n = (1+EAR)^{1/n}-1$

Exercise 2

A firm is considering buying a new telephone system which will last 4 years. The system has an upfront cost of €150,000. Alternatively the firm can lease the system from the manufacturer for €4,000 paid at the end of each month.

a. If the firm can borrow or lend at a 5% stated annual interest rate with semi-annual compounding, should it purchase or lease the system?
b. If instead the firm can borrow or lend at a 15% stated annual interest rate with monthly compounding, does the firm decision change?

Quiz

Through Moodle, complete the two quizzes:

- Summative quiz 01
- Summative quiz 02

Review

1. We use the effective rate
2. The rate and the horizon must be in the same units
3. We only compare cashflows at the same time period
4. We only undertake positive-NPV projects and if we need to choose, we always get the highest NPV