Applied Corporate Finance

Financial and Real Options

Rui Silva

Financial Options Outline

- What are financial options? What kind of options are traded?
- What are the Payoffs From an Option?
- What Determines Option Values?

What are options?
What kind of options are traded?

- The first standardized options contracts were traded in 1973 at the Chicago Board Options Exchange (CBOE). These contracts were an instant success. Today, there is active trade in options on stocks, stock indices, exchange rates, commodities, and interest rates.

- Options exist primarily as **CALL** and **PUT** options, and an option can be either **European** or **American**. Buying an option is called "going long" or "taking the long position"; selling an option is called "going short" or "taking the short position" or "writing an option". In general, the long position holder does not know the identity of the short position holder. The exchange does all the "match-making."

What are options?
What kind of options are traded?

- Call Option: The long position holder has the **right** to purchase an underlying asset for a specified price (the exercise or strike price) at a specified time (the maturity date).

  Put Option: The long position holder has the **right** to sell the underlying asset for the strike price at the maturity date.

- European Option: The right to buy or sell can only be exercised on the maturity date.

  American Option: The right to buy or sell can be exercised on or before the maturity date.

  An American option can not be less valuable than its European counterpart. (WHY?)

What are options?
What kind of options are traded?

- An option is said to be "in the money" if immediate exercise (even if it is not actually done) would be of positive value. This happens if the exercise price is less (for a call option) or greater (for a put) than the current price of the underlying asset.

- Otherwise the option is called "out of the money."

- If the exercise price is equal to the current market price, the option is "at the money."

What are the Payoffs From an Option?

- Payoffs and Profits of options at maturity: (stock price at T = S_T, exercise price = X, put price paid = P, call price paid = C)

- Mathematically, the **payoffs** to the long position at maturity are as follows:

  Call: Max [S_T - X ; 0]

  Put: Max [X - S_T ; 0]

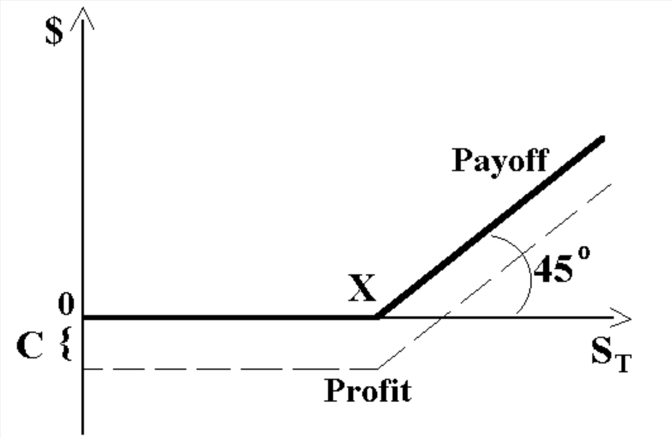
- The payoffs to the short position are the mirror image:

  A short call pays: - Max [S_T - X ; 0] = Min [0, X - S_T]

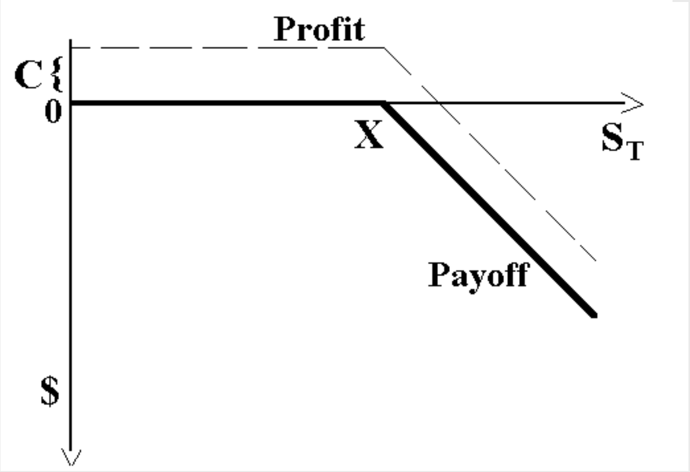
  A short put pays: - Max [X - S_T ; 0] = Min [0, S_T - X]

What are the Payoffs From an Option?

- Long position for a call:

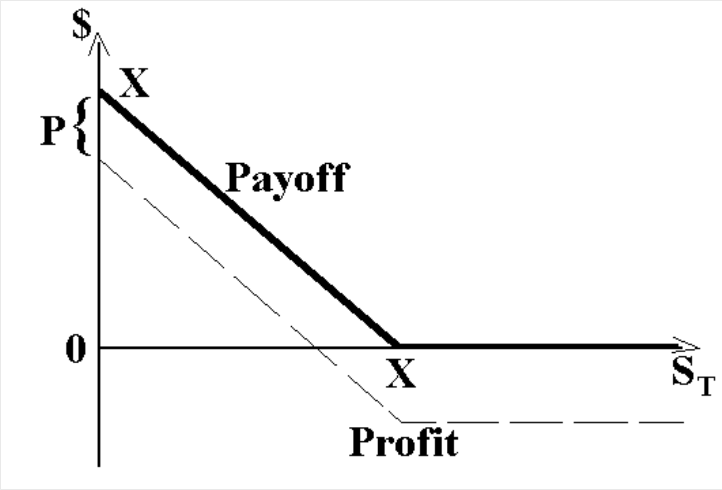


- Short position for a call:

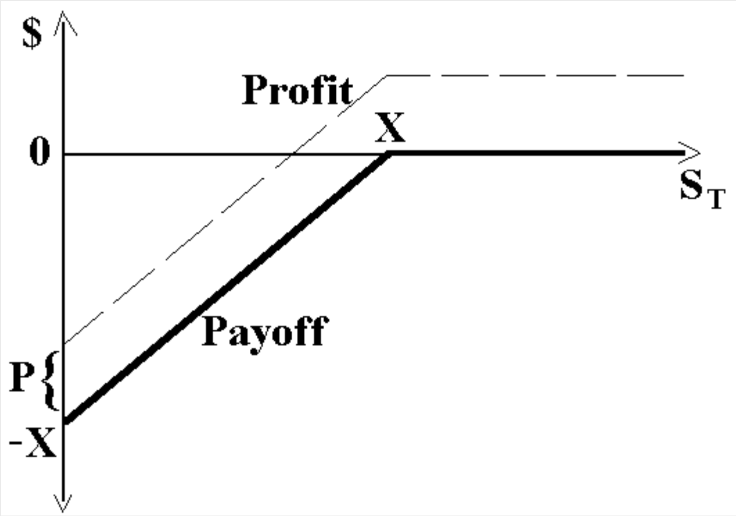


What are the Payoffs From an Option?

- Long position for a put:



- Short position for a put:



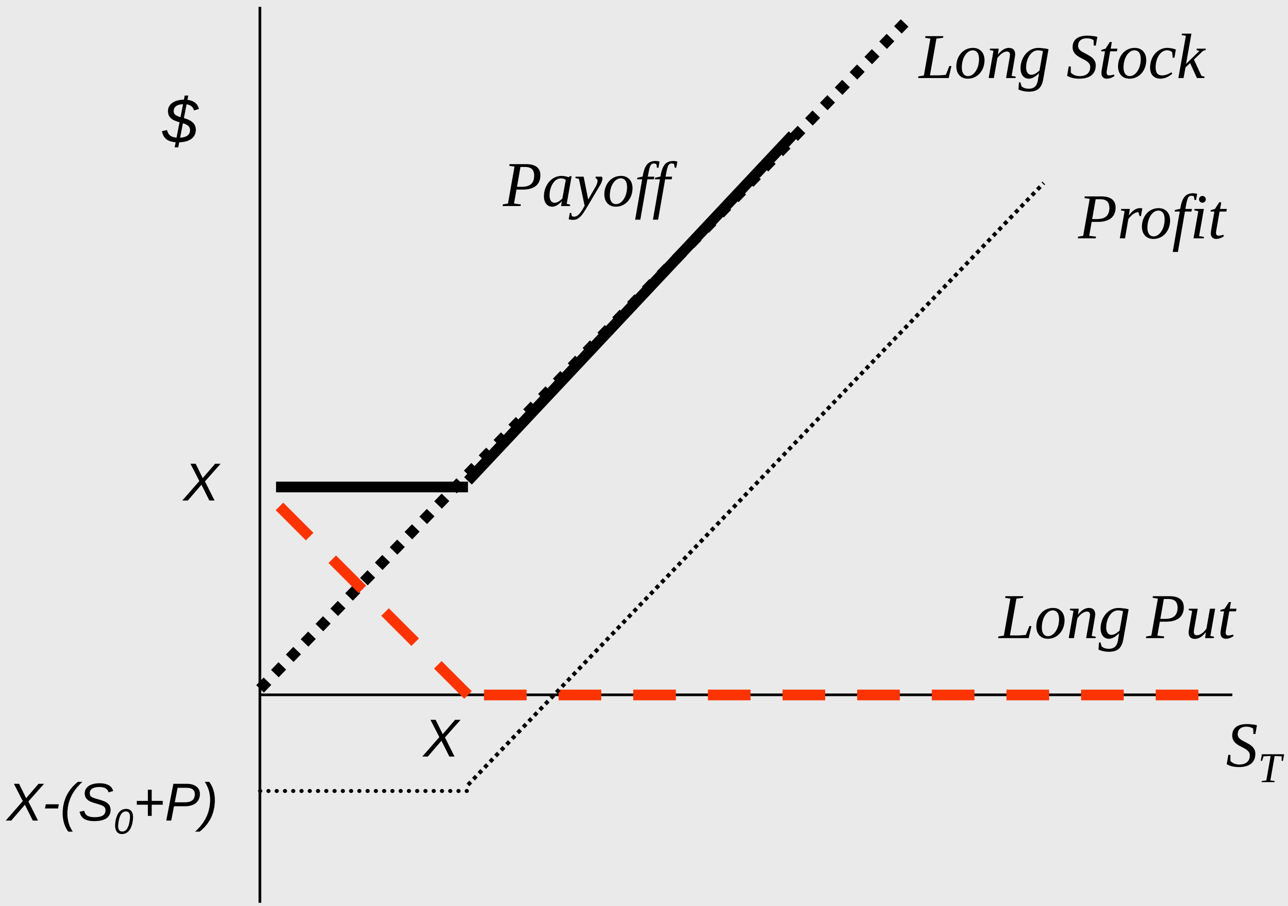
What are the Payoffs From an Option?

- Puts, calls, the stock itself and riskless borrowing/lending can be combined in various ways to create strategies that are appropriate for an investor's individual needs.

- For example: protective put, straddle, etc.

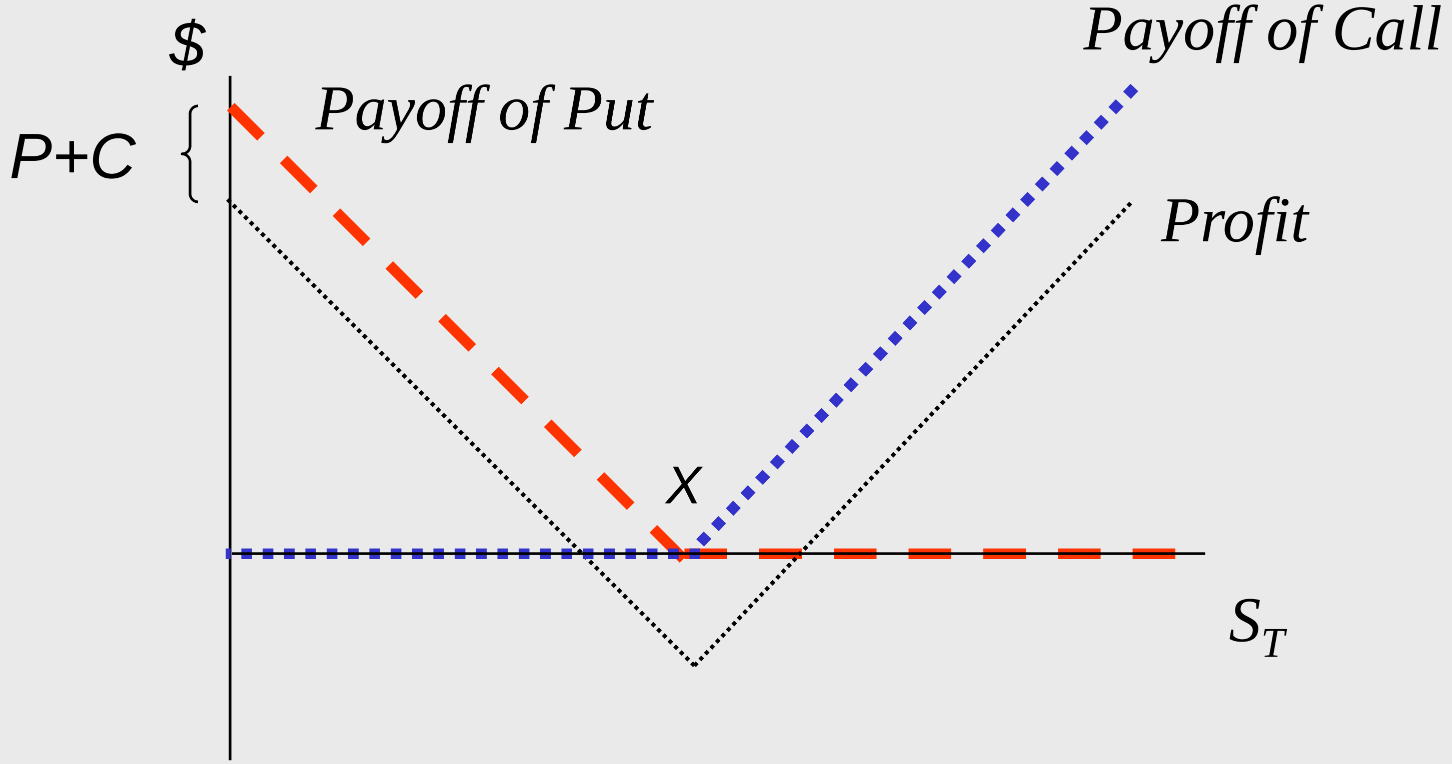
Protective Put.

Long stock and long put with strike X.



Straddle.

Long call and long put both with strike X - Strategy for profiting from high volatility.

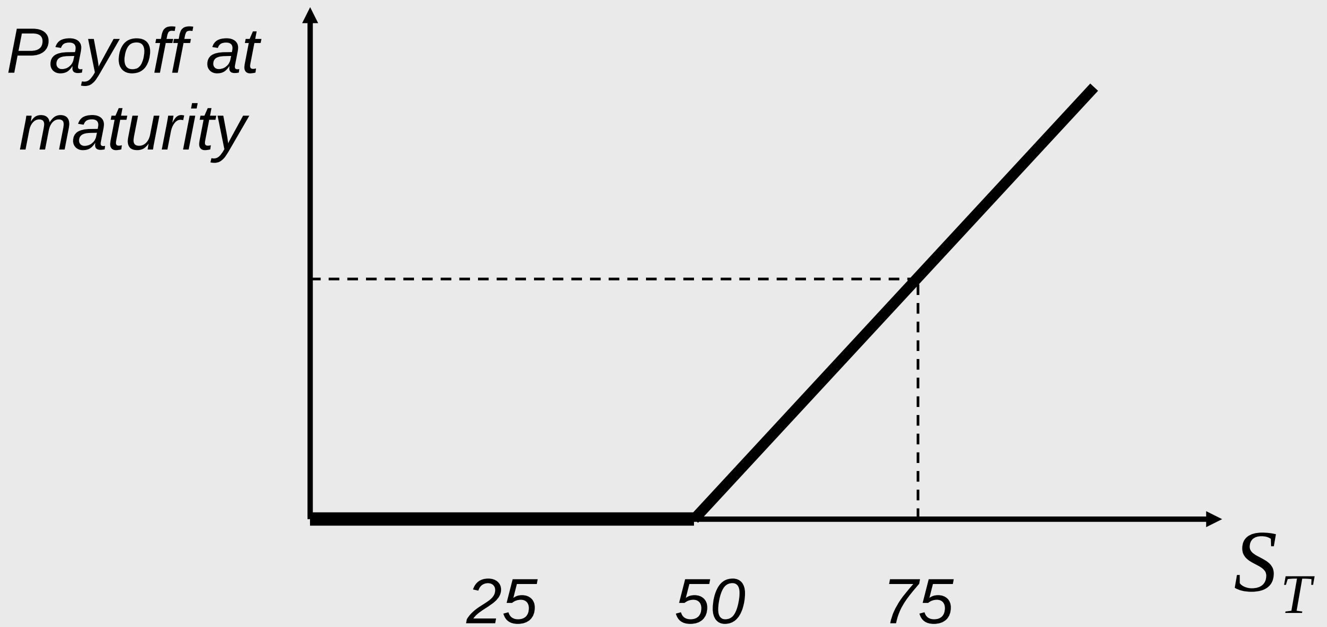


What Determines Option Values?

- *If we know how to value options, we can value stocks, corporate bonds, and many other corporate securities that have option features. Examples include callable bonds, convertible bonds, callable-convertible bonds, warrants, many types of executive compensation packages, the ability of managers to change some features of investment projects in the future, and many more.*

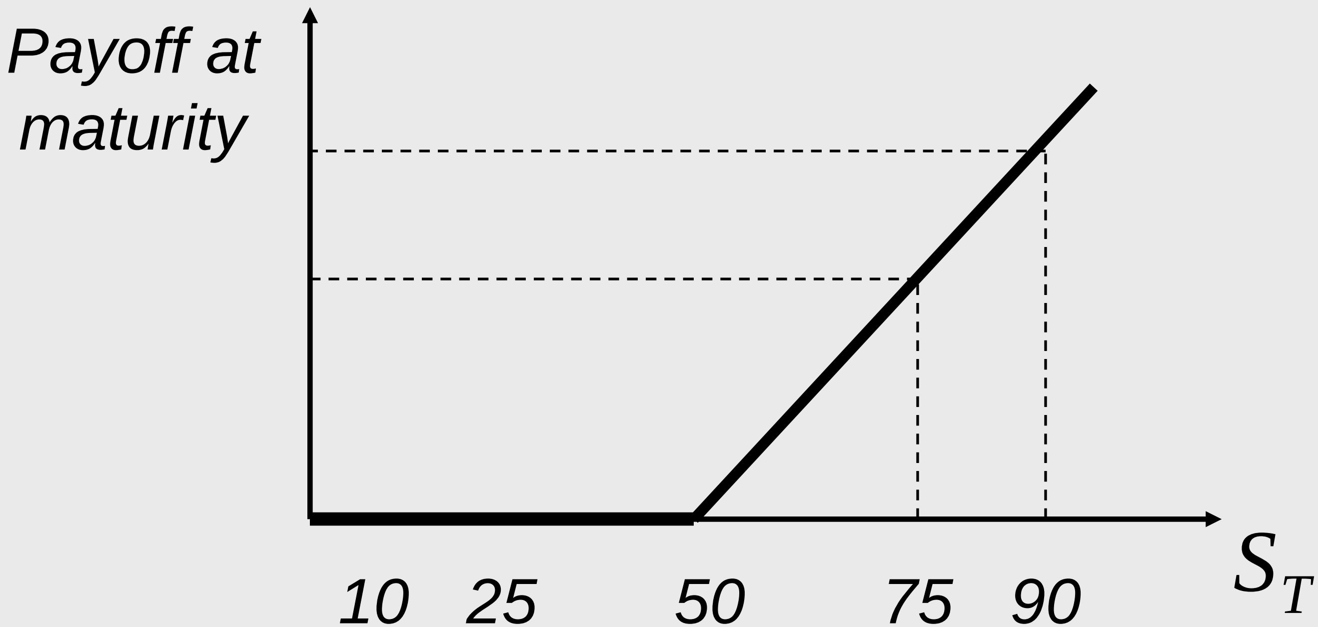
What Determines Option Values?

- An Example of a European Call Option (C). Assume that: S = 50; T = 1year; X = 50.
- Assume that the stock price at maturity can either be 75 or 25.



What Determines Option Values?

- Now assume that the stock price at maturity can either be 10 or 90, i.e. larger volatility. Do not lose on the downside, but gain on the upside. The value of the option increases with volatility.



What Determines Option Values?

- In the same example of European call C, what do you think happens to the value of the option if you change the strike price?

- What about the maturity? For example, two years instead of 1?

Real Options

Plan of Attack

- What are real options?
- Conditions for a Project to have Real Option value
- Some examples of real options and how incorporating them affects valuation.
- How to value real options:
  - Decision Trees
    - Application: Option to Open or Close a Mine
  - Arbitrage Pricing – Binomial Trees
    - Application: Option to Delay an Investment
  - Black-Scholes
    - Application: Option to Delay an Investment
- Methodological Issues

What are Options?

- Financial Options
  - Calls (puts) give the right, but not the obligation, to buy (sell) shares at a given price at a given date.

- Real Options
  - The right to make/adjust business decisions in response to changing conditions or new information.
  - Examples:
    - Option to take follow-on investment,
    - Option to wait,
    - Option to abandon,
    - Option to expand.

Example: Mark I and Mark II (1)

- We are considering whether to invest \$12M in a marketing campaign for our Mark I product.

- The campaign is either a success (and generate sales of \$22M), or a failure (and generates no sales). There is a 50/50 probability.

- Discount rate is 10%.

- What is the NPV of Mark I?
  – NPV= -12 + 11/1.1 = -2

Example: Mark I and Mark II (2)

- Let's say we could also invest \$24M in a marketing campaign for our Mark II product.
- Mark II either generates sales of \$44M, or \$0 with 50/50 probability.
- Discount rate is still 10%.
- What is the NPV of Mark II?
  - NPV= -24 + 22/1.1 = -4

Example: Mark I and Mark II (3)

- Now suppose we can
  1. Do the campaign for Mark I
  2. Observe if campaign is successful
  3. Then decide whether to campaign for Mark II.
- Suppose also that we know that if campaign for Mark I succeeds so will the campaign for Mark II.

- What are the expected Cash Flows now?
  - Time 0: -$12M
  - Time 1: 1/2 (22-24) + 1/2 (0) = -$1M
  - Time 2: 1/2 (44) + 1/2 (0) = $22M
- NPV= -$12M + (-$1M/1.1) + ($22M/1.1^2)= $5.3M.
- We can seemingly string two unprofitable projects and get a profitable one. How?

Example: Mark I and Mark II (4)

- Mark I has an option value because it informs us about the outcome of Mark II.

- We have flexibility to adjust our investment in Mark II accordingly and avoid investing in the case it would fail.

- Note:
  - if we could learn about Mark II without investing in Mark I (i.e., by simply waiting), then Mark I would lose its real option value.

Real Options Conditions

- Three main conditions must be met for a current project to have real option value:
  1. Learning: New information must arrive
     - prices change or we get new information about customer demand, technology, product, etc.
  2. Adaptability: There is the option to adjust the project in response to new information
     - expand, close, make follow up investment, etc.
  3. Either the new information or the option to adjust requires the investment in the current project.

Some Examples of Real Options

- Natural Resource Extraction
  - When oil prices increase can increase extraction.
- Movie Sequels
  - Value of a movie includes the real option to make sequel (ex: Toy Story 3)
- Investment in new technology
  - VCs make investment in new technology with option to continue investing if technology is successful
  - Similar to R&D investments
- Land Development
  - When real estate market changes different types of housing may be more profitable.

Some Examples of Real Options

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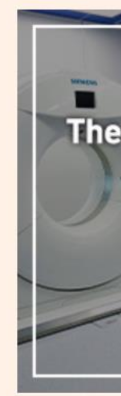

Some Examples of Real Options

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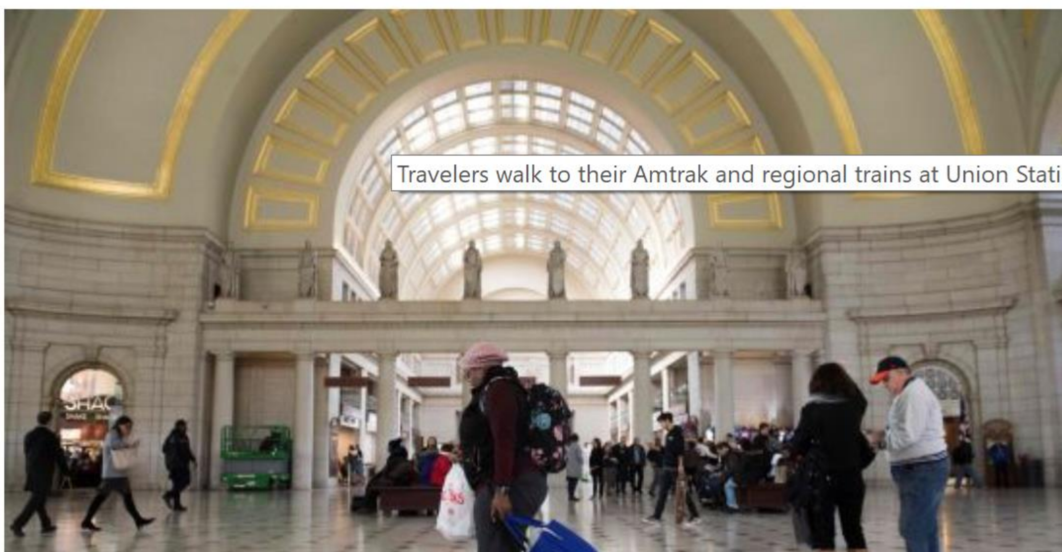

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Anxious Investors Seek Refuge in Gold

Metal pushes higher as shifting political scene unnerves some; an unusual divergence

By Ira Iosebashvili

Updated Feb. 6, 2017 8:45 p.m. ET

Gold prices rose to their highest level in nearly three months on Monday, reflecting investors' anxiety over a rapidly evolving global political landscape.

U.S. gold futures rose $11.50, or 0.9%, to $1,230 a troy ounce after

Valuation of Real Options

- Valuing an option just requires the calculation of its NPV (as always)

- In practice, computing the true NPV is difficult because we must account for contingent future cash flows and risk.
  - This is what makes option pricing complicated

- Two valuation methods:
  - Decision tree valuation (simpler, but difficult to account for risk)
  - Arbitrage pricing (complex, but more accurate treatment of risk)
    - Binomial Trees
    - Black-Scholes

Valuing Real Options with Decision Trees

Mine Example: Open or Close?

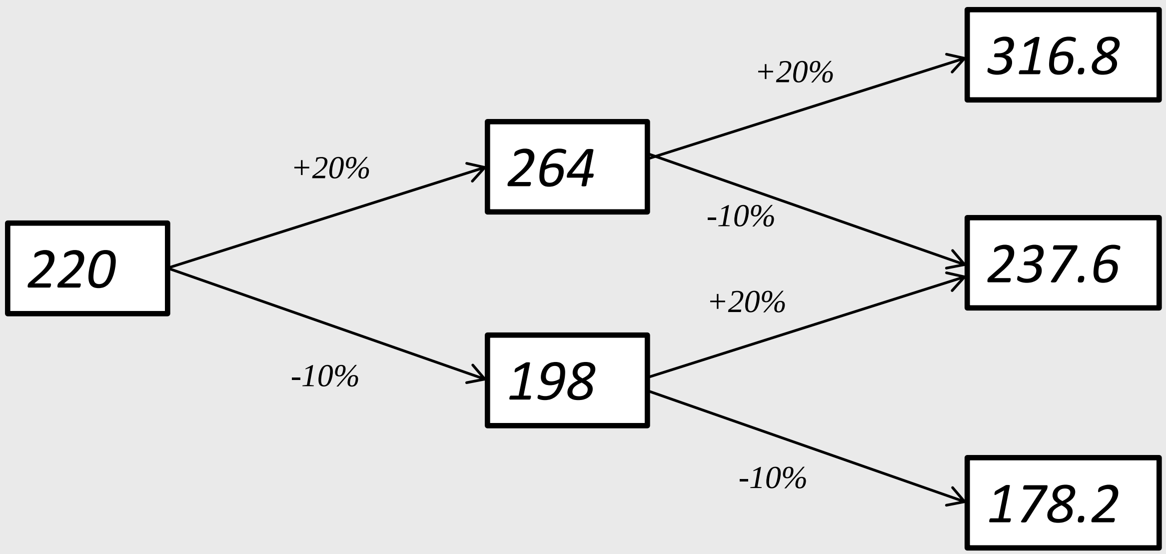
- Mine produces 50K ounces of gold per year and will operate for 3 years starting this year

- The price of gold:
  - Is currently $220
  - Expected to rise 20% or fall 10% with equal probability in each of the next 2 years
  - Uncorrelated with the market

- Extraction costs:
  - Expected to equal $230/ounce
  - Any risk is uncorrelated with the market
  - No fixed costs of running the mine

- Risk Free Rate is 5%

PV of always operating the mine

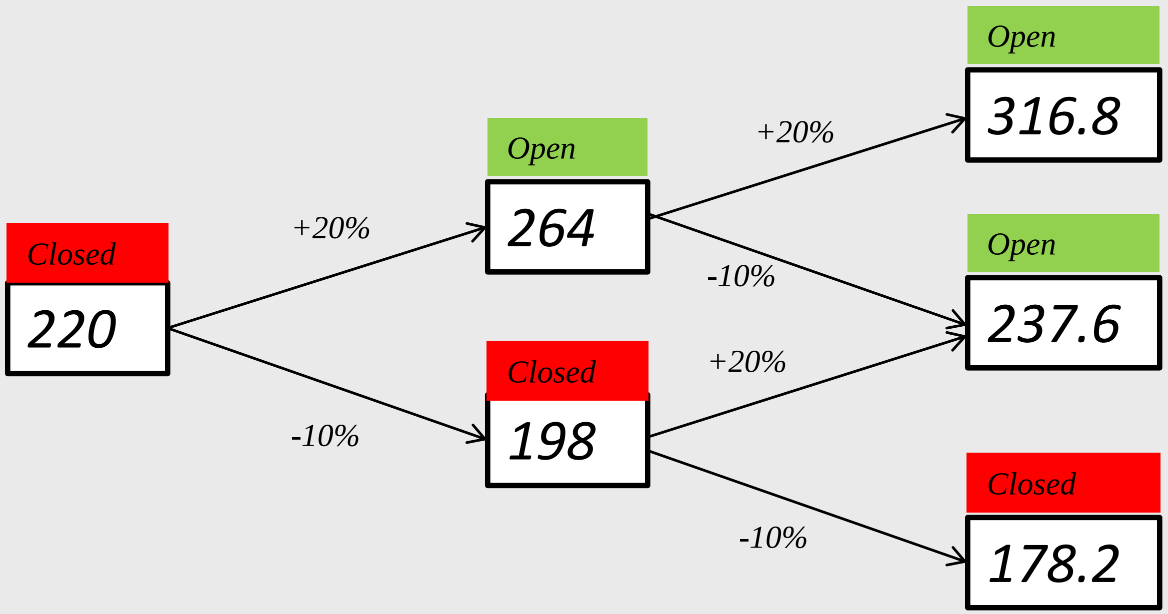


- Present Value:

$$PV = 50 * (220 - 230) + \frac{50 * (231 - 230)}{1.05} + \frac{50 * (242.6 - 230)}{1.05^2}$$
$$= 119$$

- If we had flexibility to open/close the mine contingent on price, when would we operate mine?

PV with option to open and close



- Present Value:

$$PV = 0.5 * \frac{[50*(264-230)]}{1.05} + 0.25 * \left[\frac{50*(316.8-230)}{1.05^2}\right] +$$

$$+2 * 0.25 * \left[\frac{50*(237.6-230)}{1.05^2}\right] = 1966$$

Mine Example: Conclusion

- The option to open or shut the mine depending on the price of gold increased the value of the project tremendously.

- As new information about the price arrives we change our decision

- The static NPV approach only compares open mine to closed mine.
  – misses the dynamics of operating the mine.

- We used traditional discounting to calculate the value of the project:
  – We assumed the beta of the underlying asset (gold) is zero;
  – What if this assumption is wrong?

Valuing Real Options with Arbitrage Pricing

Arbitrage Pricing

- So far we have ignored systematic risk.
  – If systematic risk is important, our valuation in the last example is wrong.

- If the underlying asset is traded and market prices are available, we can price option using arbitrage pricing.

- Arbitrage pricing
  – Law of One Price
  – Two portfolios with identical payoffs in all states of the world (all branches of the decision tree) must have the same price today.

Condo Example: The Option to Wait

- You own a plot of land and have the choice:
  - build 4 condos on the plot
  - build 6 condos on the plot

- Cost per unit (same costs now and later):
  - $140K if 4 are built
  - $180K if 6 are built

- Sale Prices
  - Today: $200K per unit
  - Next year: $270K or $170K per unit
    - (the underlying asset is traded)
  - The risk free rate is 10%.

Condo Example: PV of Acting Today

- If you build and sell today you have two options:
  - Build 4 condos: 4 x ($200-$140) = $240
  - Build 6 condos: 6 x ($200-$180) = $120

- $240 > $120, so if you must build now, you choose the 4-unit project with an NPV of $240.

- What is the option to wait?

Condo Example: Optimal Action Given the State

- How many units should we build in each state?

Good outcome: Build ___ units and realize $_____.

Today

Bad outcome: Build ___ units and realize $_____.

Condo Example: Pricing the Option to Wait

	Condo Value	Option Payoff	Risk Free
Good State			0.1
Bad state			0.1

- Now consider the replicating portfolio: 4.2 condos and a $540 loan at risk free rate. What are the payoffs:
  - In good state:
  - In bad state:
  - Today's price of the portfolio
- Arbitrage pricing

Condo Example: Decision

- The PV of option to wait exceeds the PV of building and selling today.

- The cost is that we receive cash flow later (time value of money). The benefit is that we can respond to market conditions

- Question: What are the probabilities of each state?

- Question: For what kind of investments may the “wait and see” strategy make sense?

Replicating Portfolios

- Where did we get 4.2 (condos) and $540 (loan amount)?
- Recall from option pricing that the option payoff can be replicated with Δ units of the underlying asset and B in the risk-free bond.

$$C = \Delta S + B$$

- Where

$$\Delta = \frac{\text{Spread in Option Payoffs}}{\text{Spread in underlying Payoffs}}$$

- For our example

$$\Delta = \frac{540 - 120}{270 - 170} = \frac{420}{100} = 4.2$$

- We can substitute into the good (or bad) state payoffs and solve for B.

$$540 = \Delta S + (1 + 0.10) * B = 4.2 * 270 + 1.1B$$

$$B = -540$$

General Formula: Risk-Neutral Pricing (1)

- Movement of the underlying:
  - u: % change for up movement of underlying
  - d: % change for down movement of underlying
  - r: risk-free interest rate

- Value of option payoff next period
  - C_u after up-tick
  - C_d after down-tick

- Present value of option today (value of the replicating portfolio consisting of underlying and risk-free)

$$PV = \frac{1}{1+r}\left(\frac{r-d}{u-d}C_u + \frac{u-r}{u-d}C_d\right)$$

$$= \frac{1}{1+r}\left(qC_u + [1-q]C_d\right)$$

General Formula: Risk-Neutral Pricing (2)

1. Compute risk-neutral probabilities
   – Identify the current price of the underlying and its possible (up and down) payoffs

$$
\begin{array}{ccccc}
 & & \underline{\text{Underlying}} & & \underline{\text{Derivative}} \\
 & \overset{q}{\nearrow} & S_u & & C_u \\
S_0 & & & & \\
 & \underset{1-q}{\searrow} & S_d & & C_d
\end{array}
$$

   – Compute the risk-neutral probability q.

$$(1+r_f)S_0 = q \times S_u + (1-q) \times S_d$$

$$(1+r_f)\left(\frac{S_0}{S_0}\right) = q \times \left(\frac{S_u}{S_0}\right) + (1-q) \times \left(\frac{S_d}{S_0}\right)$$

$$(1+r_f) = q \times (1+u) + (1-q) \times (1+d)$$

$$q = \frac{r_f - d}{u - d}$$

General Formula: Risk-Neutral Pricing (3)

2. Use the risk-neutral probabilities to compute payoff

$$C_T = q \times C_u + (1-q) \times C_d$$

3. Discount using the risk-free rate:

$$C_0 = \frac{C_T}{\left(1 + r_f\right)^T}$$

Detour (1): Two Approaches to Valuing Cash Flows

Risky Cash Flows

Adjust discount rate for risk.

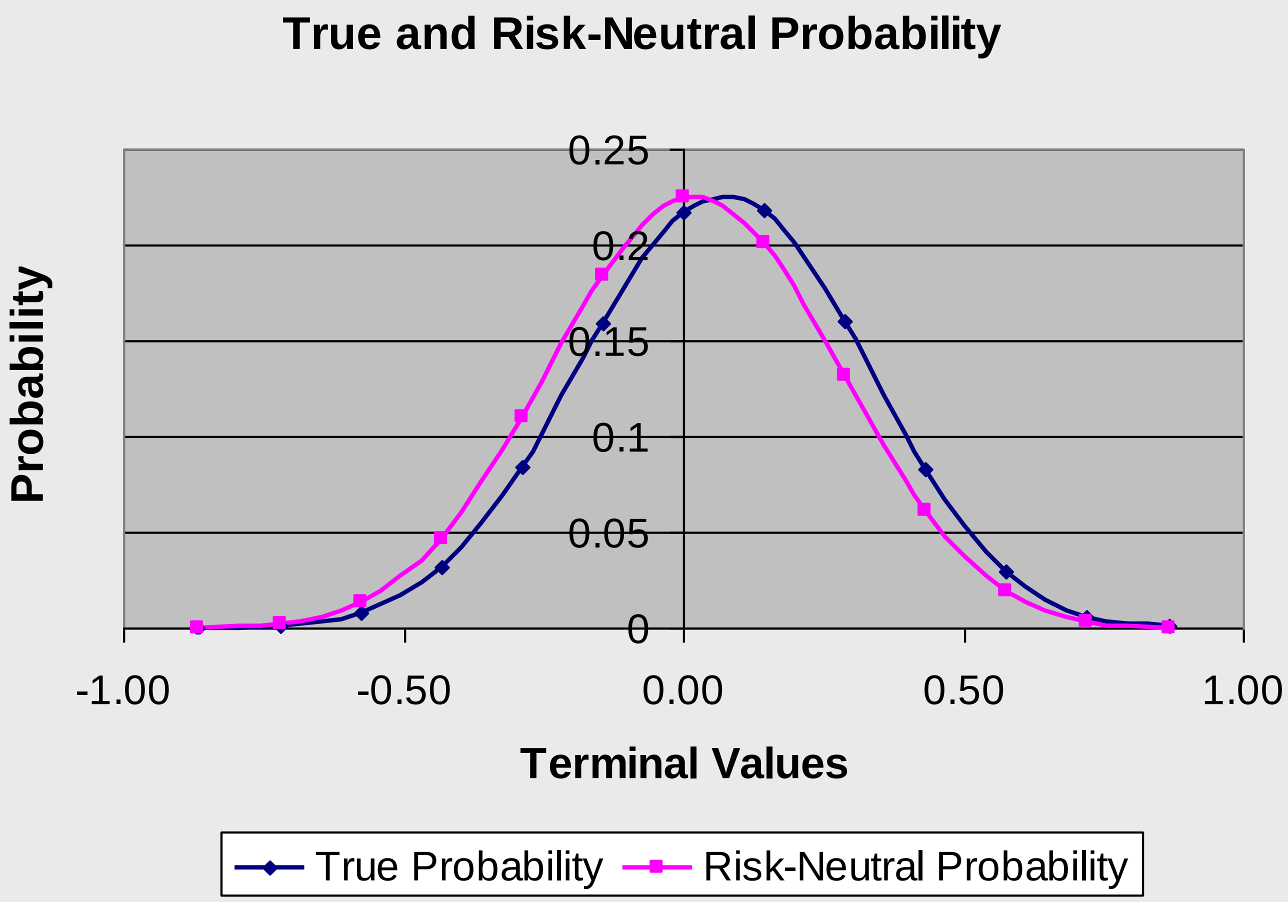
Adjust cash flows for risk.

$$V = \frac{E[CF_t]}{1 + r_{risk-adjusted}}$$

$$V = \frac{E[CF_t] - Risk\ Adjustment}{1 + r_f}$$

$$V = \frac{E^*[CF_t]}{1 + r_f} \text{ or } \frac{E^Q[CF_t]}{1 + r_f}$$

Detour (2): True and Risk-Neutral Probabilities



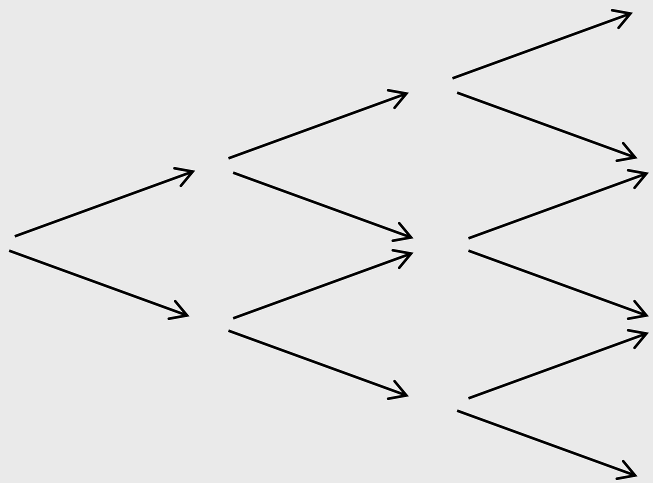
Applying the General Formula: Condo Example

- u = 270/200 = 1.35 = +35%
- d = 170/200 = -0.85 = -15%
- r = 1.10 = +10%
- C_u = 540
- C_d = 120

$$PV = \frac{1}{1+10\%}\left(\frac{10\% - (-15\%)}{35\% - (-15\%)}540 + \frac{35\% - 10\%}{35\% - (-15\%)}120\right)$$

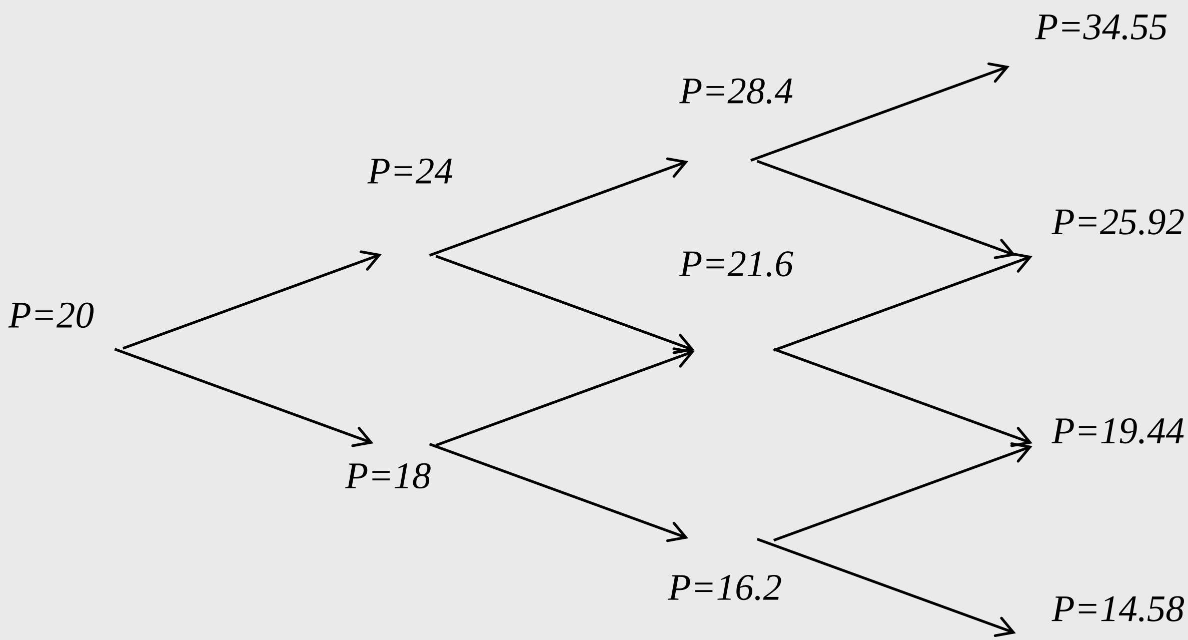
= 300

Pricing options over multiple periods (1)



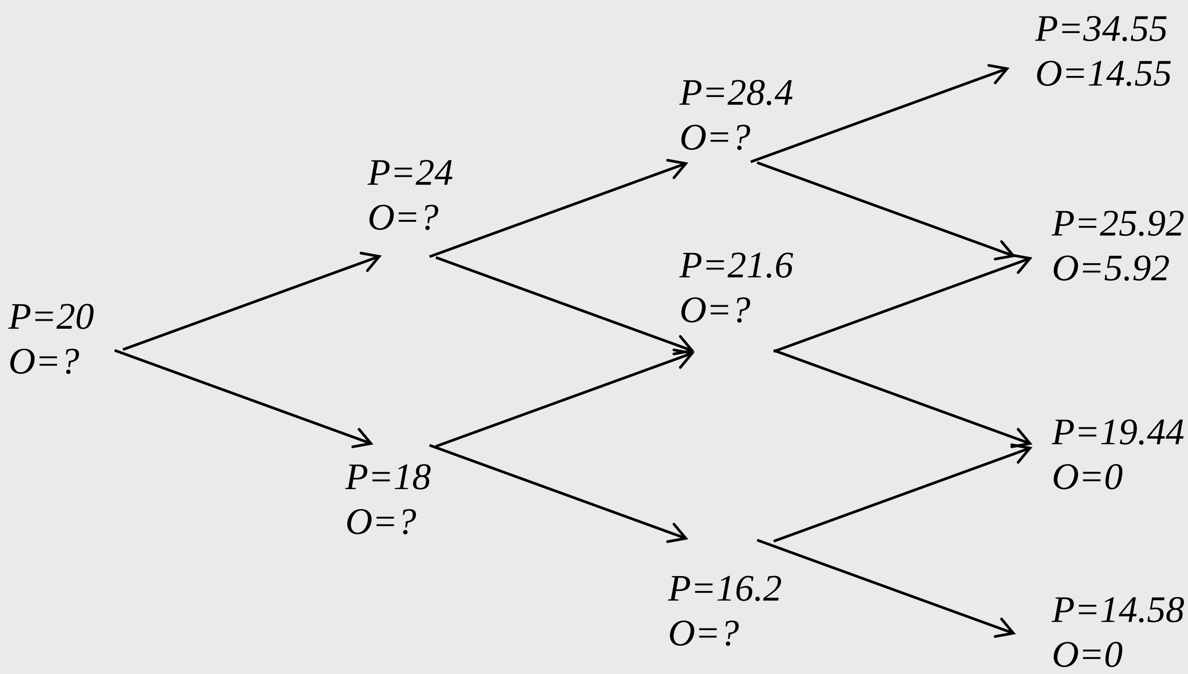
- Recipe:
    1. Determine the value of the underlying asset at each point
    2. Determine the value of the option at end point
    3. Price option backwards starting at the end point

Pricing options over multiple periods (2)



- Value of the underlying (stock or project) given by *P.*
  - Here, it either goes up by 20% or down by 10% each period
- Suppose we have a call option on *P* with a strike price of $20…

Pricing options over multiple periods (3)



1. Determine value of option at endpoints
2. Price option at second-to-last period, using the general formula from before

$$PV = \frac{1}{1+0.1}\left(\frac{0.1-(-0.1)}{0.2-(-0.1)}14.55 + \frac{0.2-0.1}{0.2-(-0.1)}5.92\right) = 10.61$$

3. Continue backwards to today

Moving to Continuous Time: Black-Scholes

- We can expand model by increasing the number of intermediate periods (each period represents shorter and shorter time)
- Recall the Black-Scholes Formula:

$$C = S^x * N(d_1) - PV(K) * N(d_2)$$

where:

- $S^x = S - PV(Dividends)$
- $d_1 = \frac{\ln[S^x / PV(K)]}{\sigma\sqrt{T}} + \frac{\sigma\sqrt{T}}{2}$
- $d_2 = d_1 - \sigma\sqrt{T}$
- $N(\cdot)$ =Normal distribution CDF
  - =NORMSDIST(d) in Excel

Mapping between Real and Financial Options

Financial Option	Variable	Real Option
Stock Price	S	Current Market Value of Asset
Strike Price	K	Upfront Investment Required
Expiration Date	T	Final Decision Date
Risk-Free Rate	r_f	Risk-Free Rate
Volatility of Stock	σ	Volatility of Asset Value
Dividend	$Dividend$	FCF Lost from Delay

Source: Berk & DeMarzo

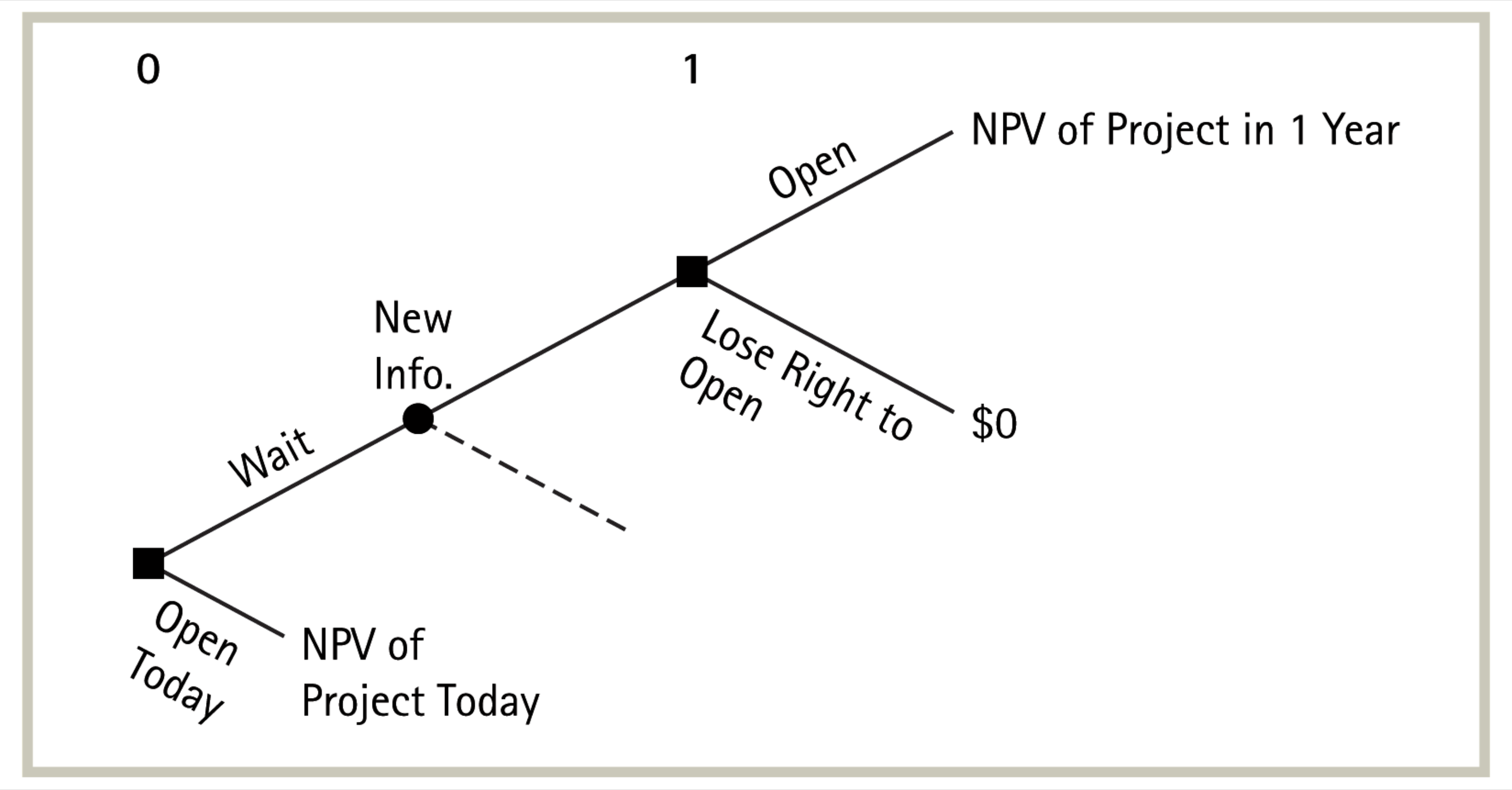
Restaurant Example: Investment as a Call Option

- Suppose you have negotiated a deal with a major restaurant chain to open one of its restaurants in your hometown.
  - The terms of the contract specify that you must open the restaurant either immediately or in exactly one year.
    - If you do neither, you lose the right to open the restaurant at all.

- How does the decision tree look?

Source: Berk & DeMarzo

Restaurant Example: Decision Tree



Restaurant Example: Valuation (1)

- How much you should pay for this opportunity?
  - It will cost $5 million to open the restaurant, whether you open it now or in one year.
  - If you open the restaurant immediately, you expect it to generate $600,000 in free cash flow the first year.
    - Future cash flows are expected to grow at a rate of 2% per year.
  - The cost of capital for this investment is 12%.

Restaurant Example: Valuation (2)

- If the restaurant were to open today, its value would be:

$$V = \frac{\$600{,}000}{12\% - 2\%} = \$6\,million$$

- This would give an NPV of $1 million.
  - $6 million – $5 million = $1 million

- When should you open the restaurant?
- Let’s map this into our Real Options framework.

Restaurant Example: Valuation (3)

- The payoff if you delay is equivalent to the payoff of a one-year European call option on the restaurant with a strike price of $5 million.
- Assume:
  - The risk-free interest rate is 5%.
  - The volatility is 40%.
  - If you wait to open the restaurant you have an opportunity cost of $600,000 (the free cash flow in the first year).
    - In terms of a financial option, the free cash flow is equivalent to a dividend paid by a stock.
    - The holder of a call option does not receive the dividend until the option is exercised.

Restaurant Example: Valuation (4)

Mapping between Real and Financial Options

Variable	Real Option	Example
S	Current Market Value of Asset	\$6 million
K	Upfront Investment Required	\$5 million
T	Final Decision Date	1 year
r_f	Risk-Free Rate	5%
σ	Volatility of Asset Value	40%
$Dividend$	FCF Lost from Delay	\$0.6 million

Restaurant Example: Valuation (5)

- The current value of the asset without the “dividends” that will be missed is:

$$S^x = S - PV(dividends) = \$6MM\ - \frac{\$0.6\ MM}{1.12} = \$5.46\ MM$$

- The present value of the cost to open the restaurant in one year is:

$$PV(K) = \frac{\$5\ MM}{1.05} = \$4.76\ MM$$

- Now we compute $d_1$and d_2:

$$d_1 = \frac{\ln[S^x/PV(K)]}{\sigma\sqrt{T}} + \frac{\sigma\sqrt{T}}{2} = \frac{\ln[5.46/4.76]}{0.40} + \frac{0.40}{2} = 0.543$$

$$d_2 = d_1 - \sigma\sqrt{T} = 0.543 - 0.40 = 0.143$$

Restaurant Example: Valuation (6)

- With all the inputs gathered, recall Black-Scholes Formula:

$$C = S^x * N(d_1) - PV(K) * N(d_2)$$

- The current value of the call option to open the restaurant is:

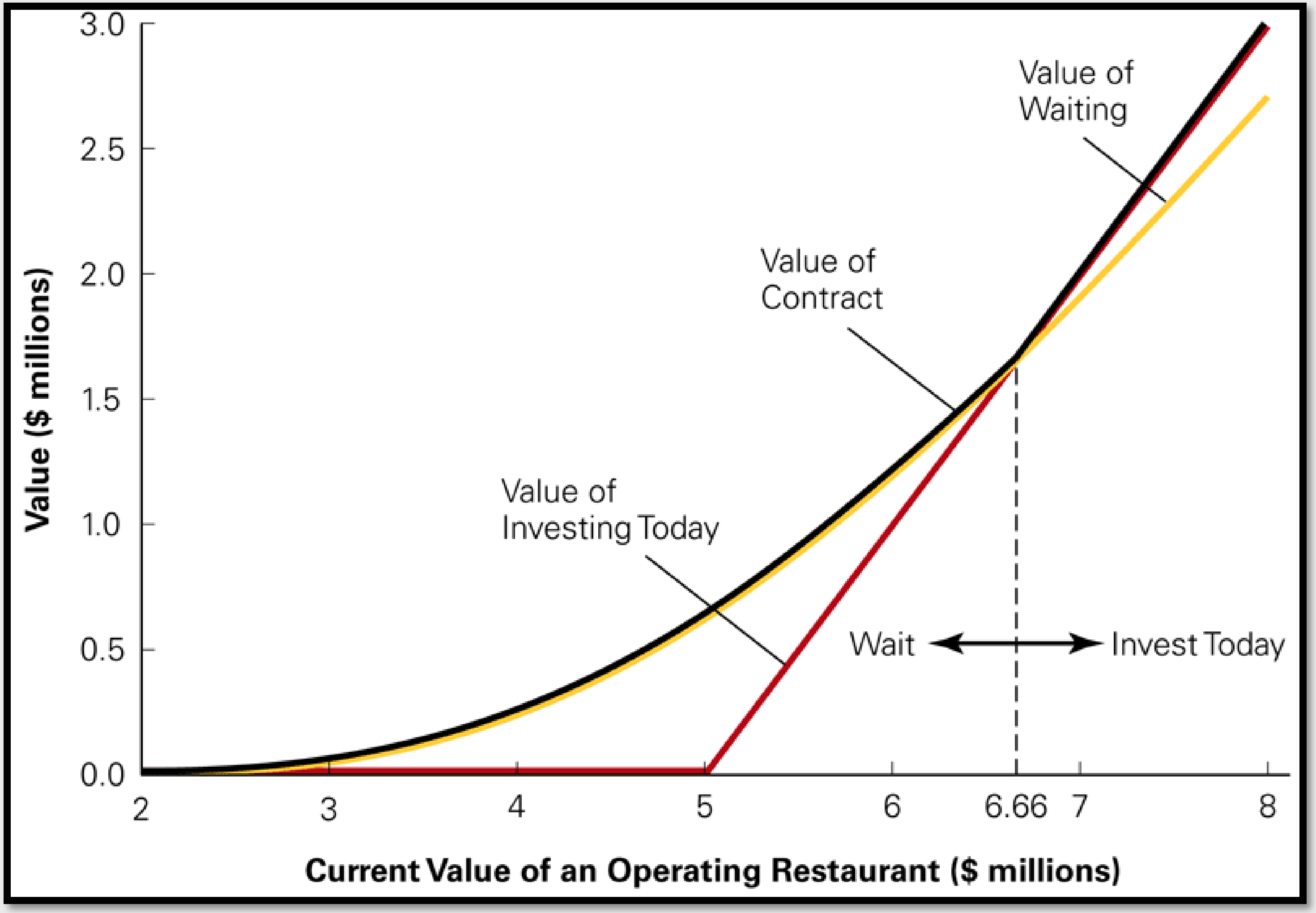
$$\begin{aligned} C &= S^x * N(d_1) - PV(K) * N(d_2) \\ &= (\$5.46) * (0.706) - (\$4.76) * (0.557) \\ &= \$1.20\ MM \end{aligned}$$

- The value today from waiting to invest in the restaurant next year (and only opening it if it is profitable to do so) is $1.20 million.
  - This exceeds the NPV of $1 million from opening the restaurant today. Thus, you are better off waiting to invest.

Restaurant Example: Valuation (7)

- What is the advantage of waiting in this case?
  - If you wait, you will *learn* more about the likely success of the business.
  - Because the investment in the restaurant is not yet committed, you can cancel your plans if the popularity of the restaurant should decline.
    - By opening the restaurant today, you give up this option to "*walk away.*"
  - Whether it is optimal to invest today or in one year will depend on the magnitude of any lost profits from the first year, compared to the benefit of preserving your right to change your decision.

Restaurant Example: Valuation (8)



Factors Affecting the Timing of Investment (1)

- When you have the option of deciding when to invest, it is usually optimal to invest only when the NPV is substantially greater than zero.
  - You should invest today only if the NPV of investing today exceeds the value of the option of waiting.
  - Adding the option to wait can make positive an investment that currently has negative NPV

Factors Affecting the Timing of Investment (2)

- Other factors affecting the decision to wait
  - Product Market Competition
    - The option to wait can be offset by competitive considerations.
  - Volatility
    - The option to wait is most valuable when there is a great deal of uncertainty.
  - Dividends
    - Absent dividends, it is not optimal to exercise a call option early.
    - In the real option context, it is always better to wait unless there is a cost to doing so. The greater the cost, the less attractive the option to delay becomes.

Restaurant Example: Sensitivity Analysis (1)

<u>**Case 1**</u>: Suppose your current estimate of the restaurant's value is $6 million. What would be the value of the restaurant contract if the volatility of the restaurant's value were 25% instead of 40%?

<u>**Case 2**</u>: Alternatively, suppose the volatility is 40%, but waiting would lead competitors to expand and reduce the future free cash flows of the restaurant by 10%. What is the value of the contract in this case?

Restaurant Example: Sensitivity Analysis (2)

Case 1: Lower Volatility

- With a lower volatility of 25%, we have

$$d_1 = \frac{\ln[S^x / PV(K)]}{\sigma\sqrt{T}} + \frac{\sigma\sqrt{T}}{2} = \frac{\ln(5.46/4.76)}{0.25} + 0.125 = 0.674$$

$$d_2 = d1 - \sigma\sqrt{T} = 0.674 - 0.25 = 0.424$$

- The value of the call option is

$$C = S^x N(d_1) - PV(K)N(d_2)$$
$$= (5.46)(0.750) - (4.76)(0.664)$$
$$= \$0.93 \text{ million}$$

$\Rightarrow$ In this case, it is better to invest immediately and get an NPV of $1 million. With the lower volatility, the value of waiting for new information is not high enough to justify waiting

Restaurant Example: Sensitivity Analysis (3)

Case 2: Competition

- Competitors expand in response to a delay in investment:

$$S^x = S - PV(\text{First - Year FCF}) - PV(\text{Lost FCF from Competition})$$
$$= \left(\$6\text{ million} - \frac{\$0.6\text{ million}}{1.12}\right) \times (1 - 0.10) = \$4.92\text{ million}$$

- Now, $d_1 = \dfrac{\ln[S^x / PV(K)]}{\sigma\sqrt{T}} + \dfrac{\sigma\sqrt{T}}{2} = \dfrac{\ln(4.92/4.76)}{0.40} + 0.20 = 0.283$

$$d_2 = d1 - \sigma\sqrt{T} = 0.283 - 0.40 = -0.117$$

- The value of the call option is

$$C = S^x N(d_1) - PV(K)N(d_2)$$
$$= (4.92)(0.611) - (4.76)(0.453)$$
$$= \$0.85\text{ million}$$

$\Rightarrow$Despite information to be gained, it is too costly to wait.

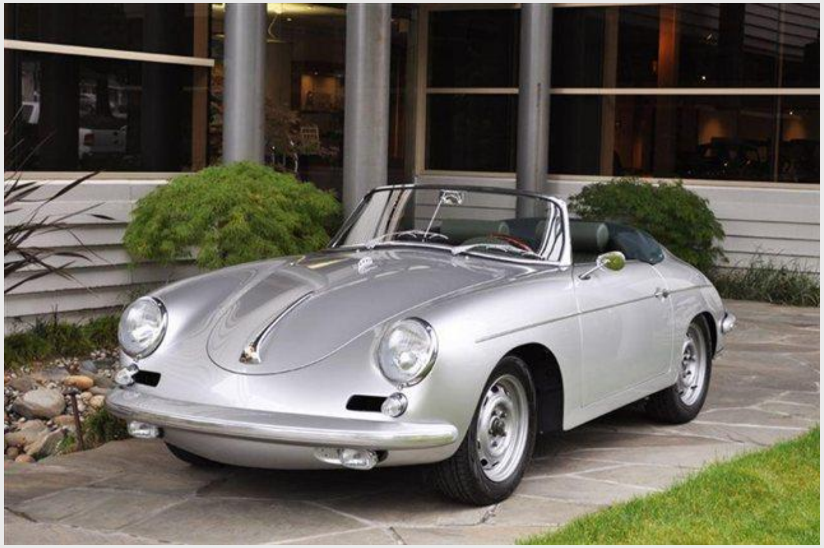
Building and Using the Binomial Tree

Black-Scholes and Binomial Trees

- We build binomial trees to describe what the future may look like. The result of a binomial tree is a probability distribution.
  - This allows us to explicitly evaluate the uncertainty inherent in our forecasts.

- Suppose we own a classic automobile that is currently worth £100,000. The collectible car market is notoriously volatile – prices are very sensitive to global economic conditions. So the price of the car can go up or down.

- We don't know what will happen, but we have some idea about what is more or less likely. We use probabilities to capture and describe our uncertainty.



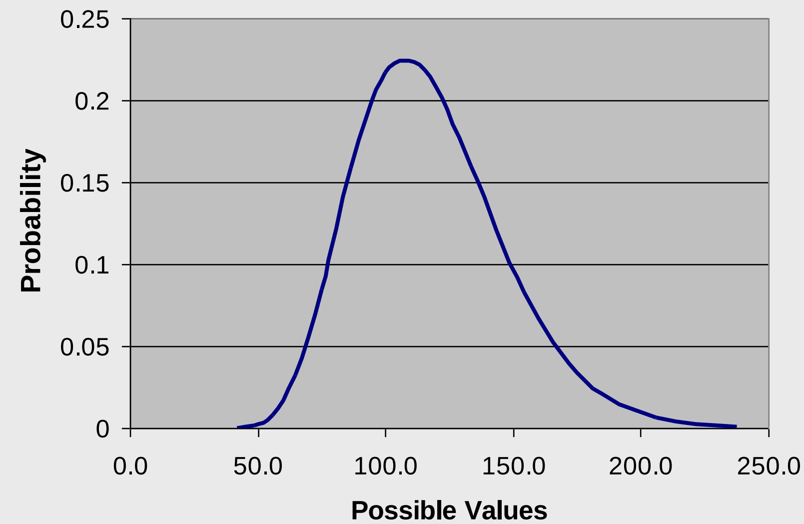
Projecting the Distribution (1)

Today

In One Year

Car Value = £100k

Car Value = ?



- We use the binomial model, along with our subjective assessments of how the car's value may change, to generate this picture.
- This is a *lognormal* distribution.

Projecting the Distribution (2)

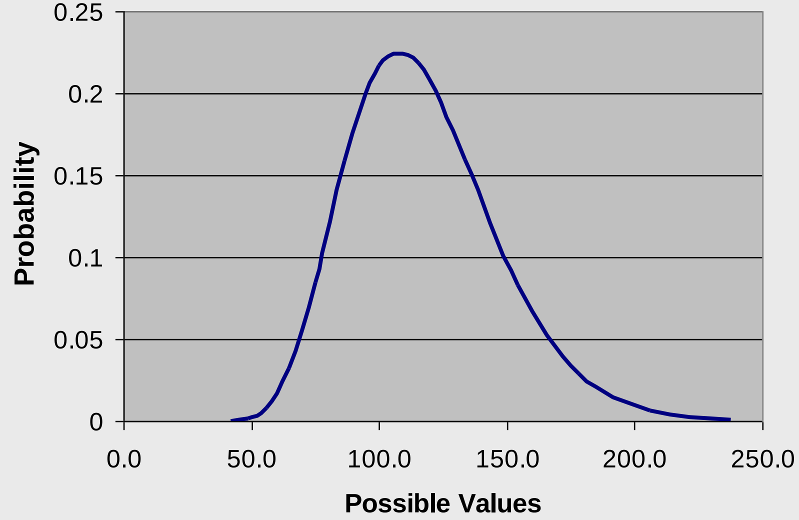
Today   In One Year

Car Value = £100k

Assume

Expected Return = 10%

σ of Returns = 25%



If random changes in car prices are described by a *normal* distribution with mean value 10% and standard deviation 25%, then random car prices are described by the *lognormal* distribution graphed above. The binomial model generates this lognormal distribution for us.

Building the Binomial Tree (1)

- We'll build our one-year binomial tree (or lattice) using the following parameters:
  - The starting value of the underlying (the rare car) is £100
  - The expected return on rare cars is 10%
    - Recall: not really needed.
  - The standard deviation of returns on rare cars is 25%.
  - We will do 12 steps over the year, so Δt = 1/12 year = .0833 year.

Building the Binomial Tree (2)

- The up step size in a *recombinant* binomial tree is:

$$U = e^{\sigma\sqrt{\Delta t}}$$

- The down step size is:

$$D = \frac{1}{U}$$

- So, using our assumptions
  - $U = e^{0.25\sqrt{0.0833}} = 1.075$
  - $D = \frac{1}{1.075} = 0.930$

Building the Binomial Tree (3)

Today

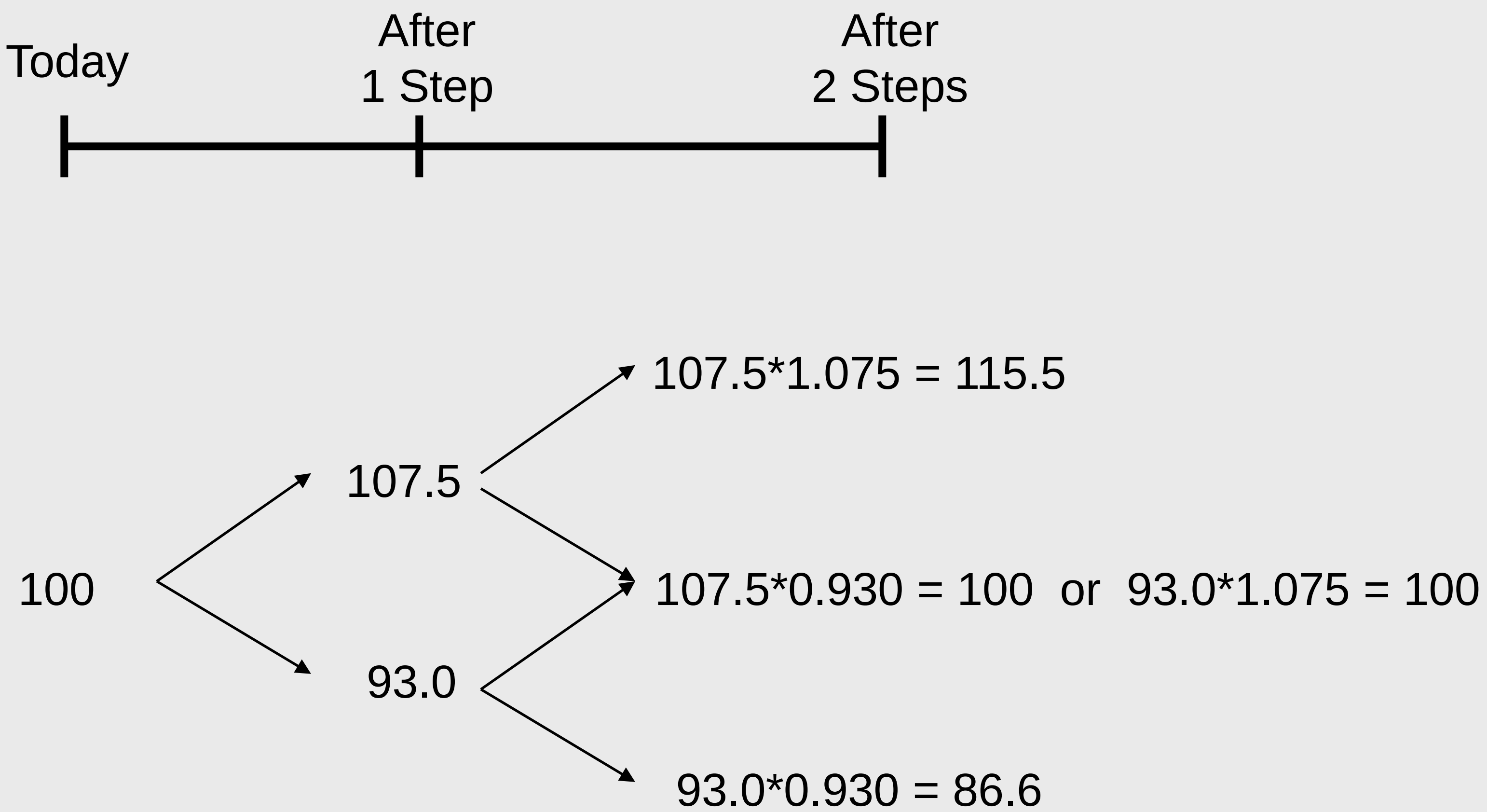
After
1 Step

100*1.075 = 107.5

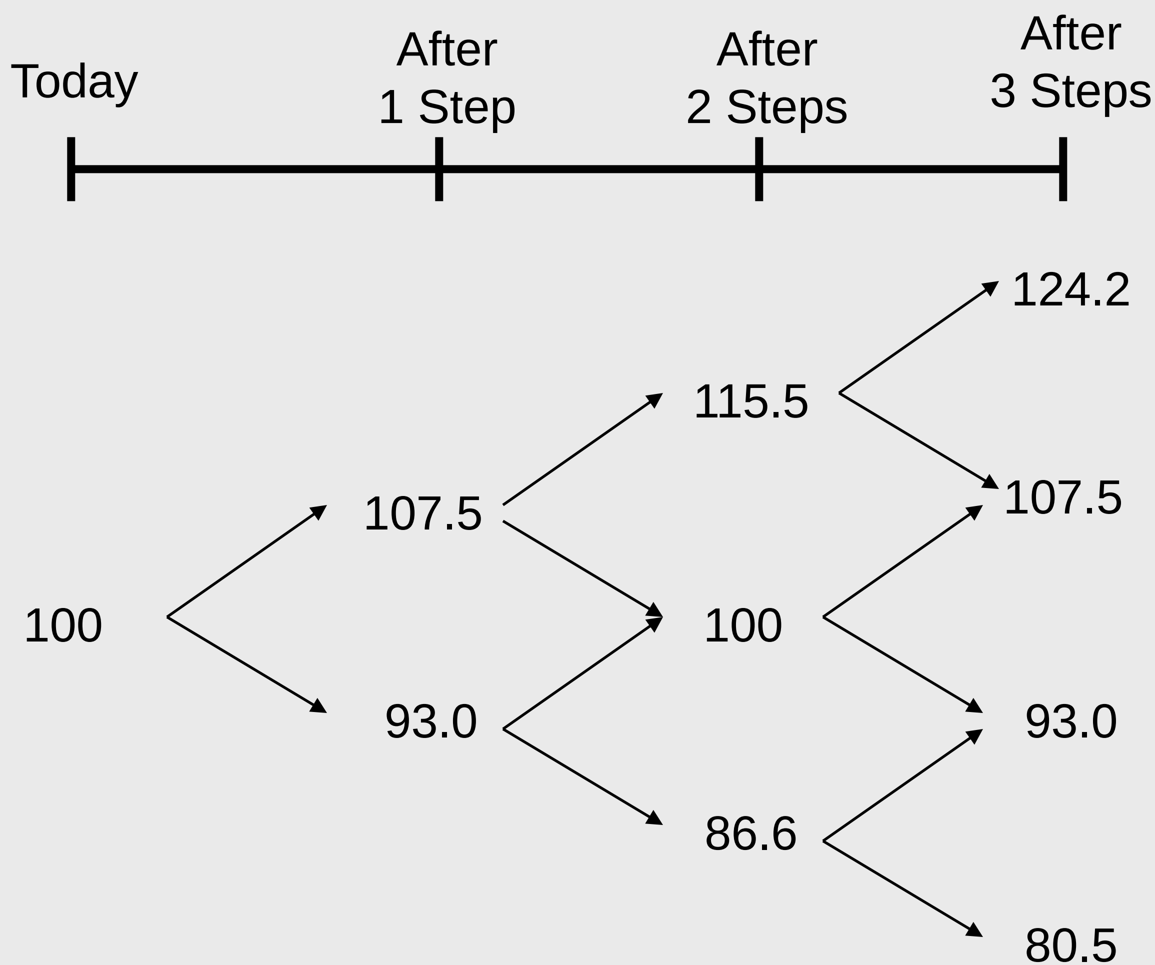
100

100*0.930 = 93.0

Building the Binomial Tree (4)



Building the Binomial Tree (5)



Building the Binomial Tree (6)

		1	2	3	4	5	6	7	8	9	10	11	12
													237.7
												221.2	
											205.8		205.8
sigma		0.25								191.5		191.5	
t		0.0833							178.1		178.1		178.1
u		1.0748						165.7		165.7		165.7	
d		0.9304					154.2		154.2		154.2		154.2
						143.5		143.5		143.5		143.5	
					133.5		133.5		133.5		133.5		133.5
				124.2		124.2		124.2		124.2		124.2	
			115.5		115.5		115.5		115.5		115.5		115.5
		107.5		107.5		107.5		107.5		107.5		107.5	
	100.0		100.0		100.0		100.0		100.0		100.0		100.0
		93.0		93.0		93.0		93.0		93.0		93.0	
			86.6		86.6		86.6		86.6		86.6		86.6
				80.5		80.5		80.5		80.5		80.5	
					74.9		74.9		74.9		74.9		74.9
						69.7		69.7		69.7		69.7	
							64.9		64.9		64.9		64.9
								60.3		60.3		60.3	
									56.1		56.1		56.1
										52.2		52.2	
											48.6		48.6
												45.2	
													42.1

Valuing Derivatives With the Tree

1. Build the binomial tree
2. Write down the payoffs on the derivative at the end of the tree in each state.
3. Work backwards, step by step, using the risk-neutral probability (q) and the risk-free discount rate:

$$q = \frac{e^{r\Delta t} - D}{U - D}$$

$$\frac{1}{e^{r\Delta t}} = e^{-r\Delta t}$$

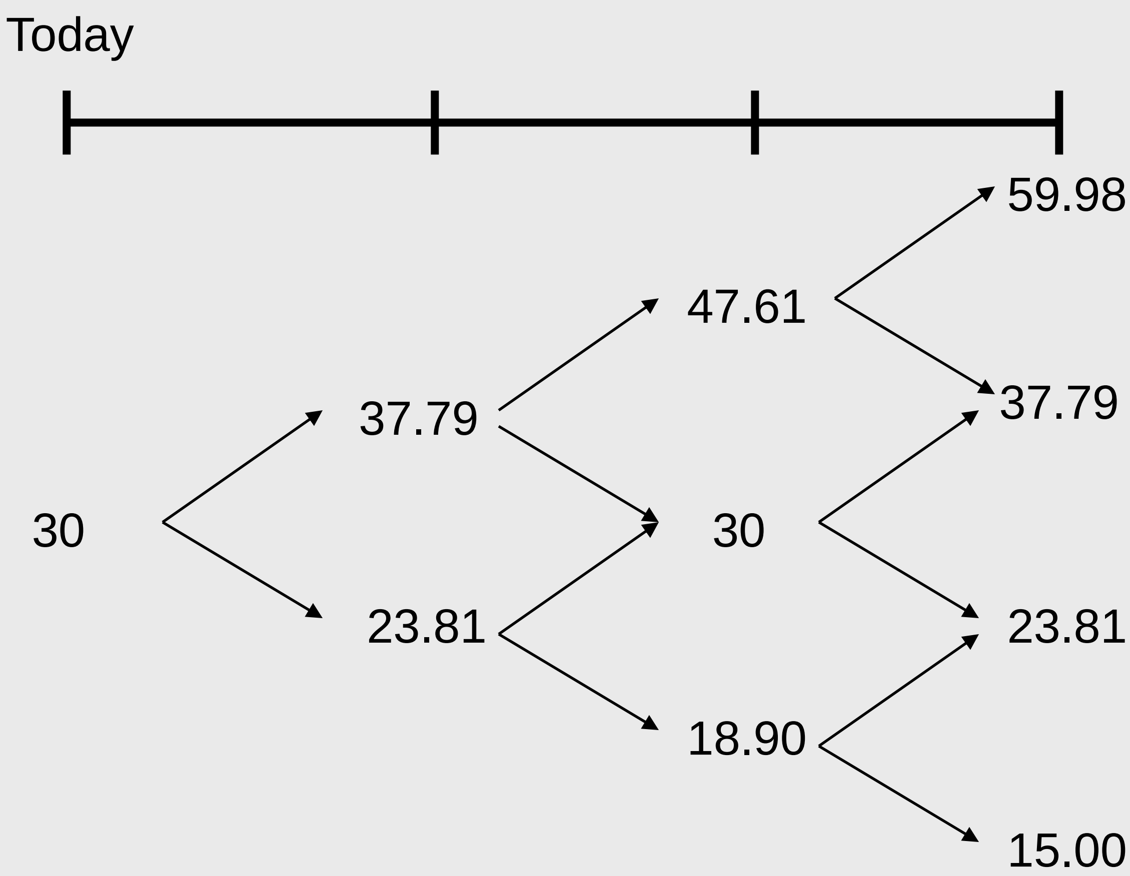
Example: Equity Call Option

- Current price of the underlying stock: $30.
- Volatility: 40%
- Risk-free Rate: 10%
- Call option: Strike price = $35. 1 year to expiration.

- Value the call option in a binomial tree with three steps (Δt = 1/3).
  - $U = e^{\sigma\sqrt{\Delta t}} = e^{0.40\sqrt{0.333}} = 1.2598$
  - $D = \frac{1}{1.2598} = 0.7938$
  - $q = \frac{e^{r\Delta t} - D}{U - D} = \frac{e^{0.10*0.333} - 0.7938}{1.2598 - 0.7938} = 0.5153$
  - $\frac{1}{e^{r\Delta t}} = e^{-r\Delta t} = 0.9672$

Example: Equity Call Option (Step 1)

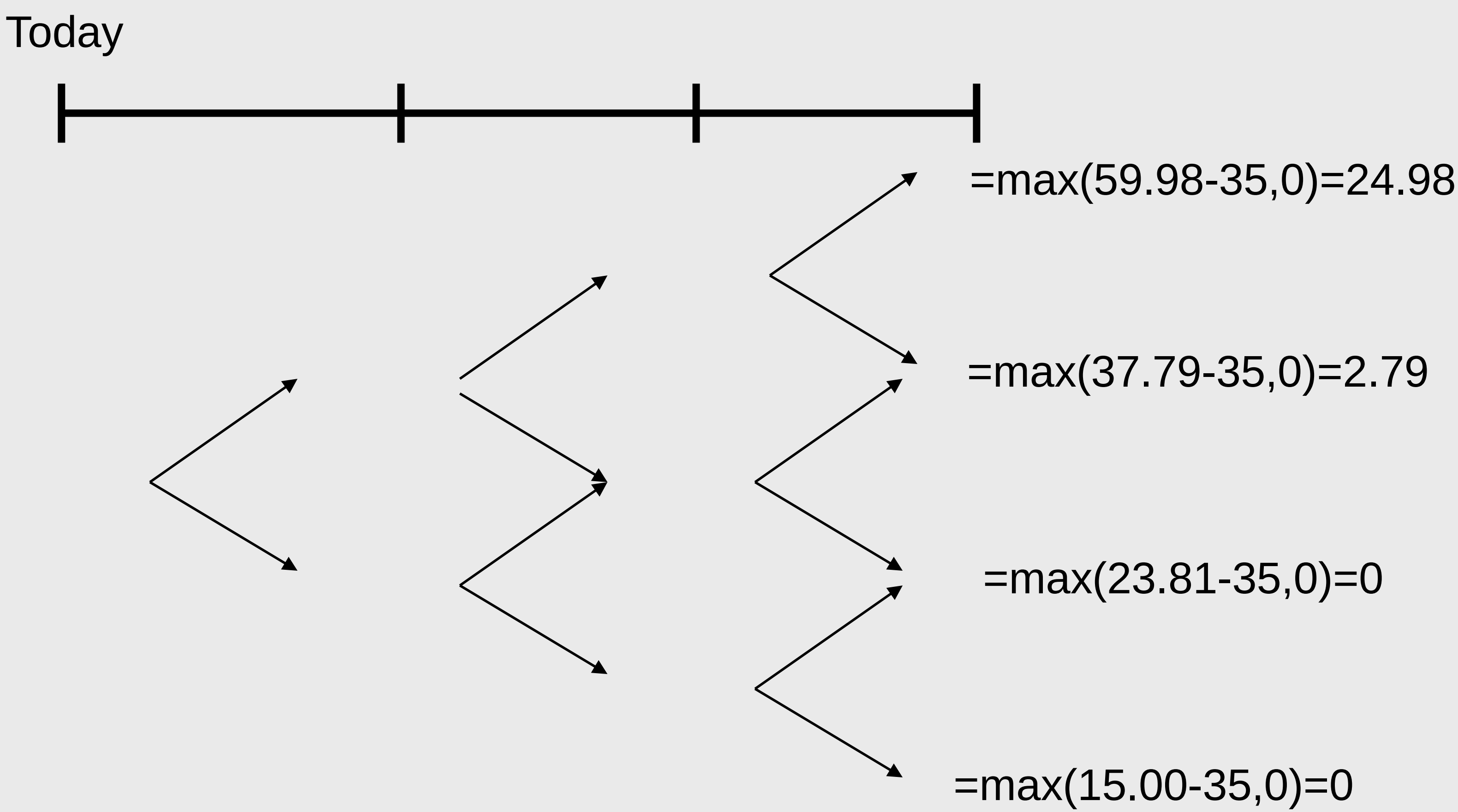
Building the Tree using U and D.



Example: Equity Call Option (Step 2)

Compute the payoffs at the end of the tree.

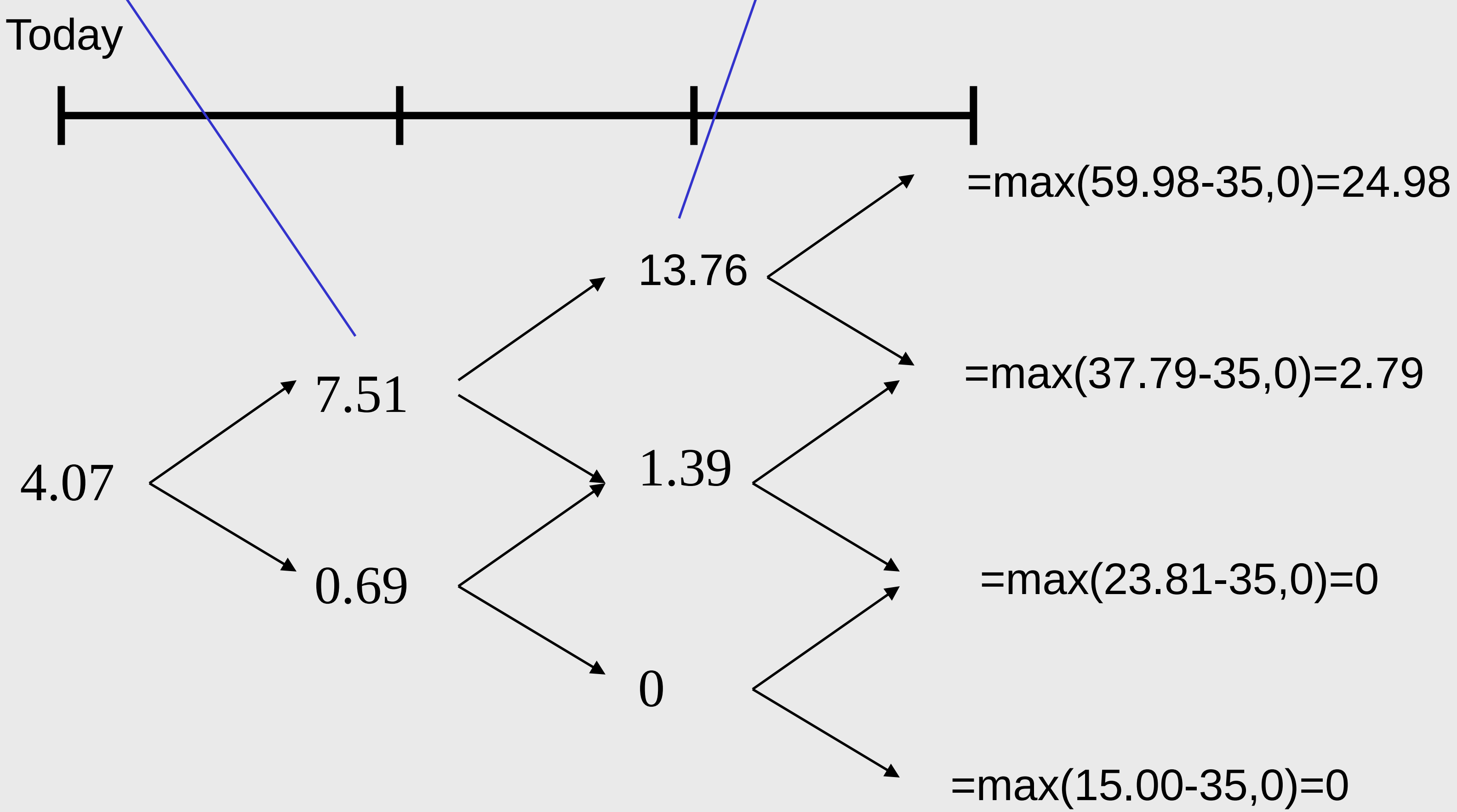
- Recall K=35



Example: Equity Call Option (Step 3)

$$7.51 = \frac{0.515 \times 13.76 + (1 - 0.515) \times 1.39}{e^{0.1*(0.3333)}}$$

$$13.76 = \frac{0.515 \times 24.98 + (1 - 0.515) \times 2.79}{e^{0.1*(0.3333)}}$$



Summary of the Process

1. Compute the cash flows at each node. Identify and incorporate managerial flexibility at each point in the binomial tree. At each stage, modify the values to reflect optimal decision making. Be careful to include the cost of additional investment, if necessary.
2. Calculate the option value at the end point
3. Work backwards and value the real option using risk-neutral probabilities and discounting by the risk-free rate.

Example: Abandonment Option (1)

- Suppose you are considering building a manufacturing facility. It will cost $11 million today to build the facility and a DCF analysis estimates that the expected present value of cash inflows is $12 million. Thus, the static NPV is $1 million.

- Management has the option to stop production and sell the facility to a third party for $10 million at any time in the next year.

- Assumptions: Expected volatility of cash flows = 45%; Facility pays off cash flows at the end of the year (no dividends). Yield on 1 year US Treasury is 4.90%.

- What is the value of this abandonment option?

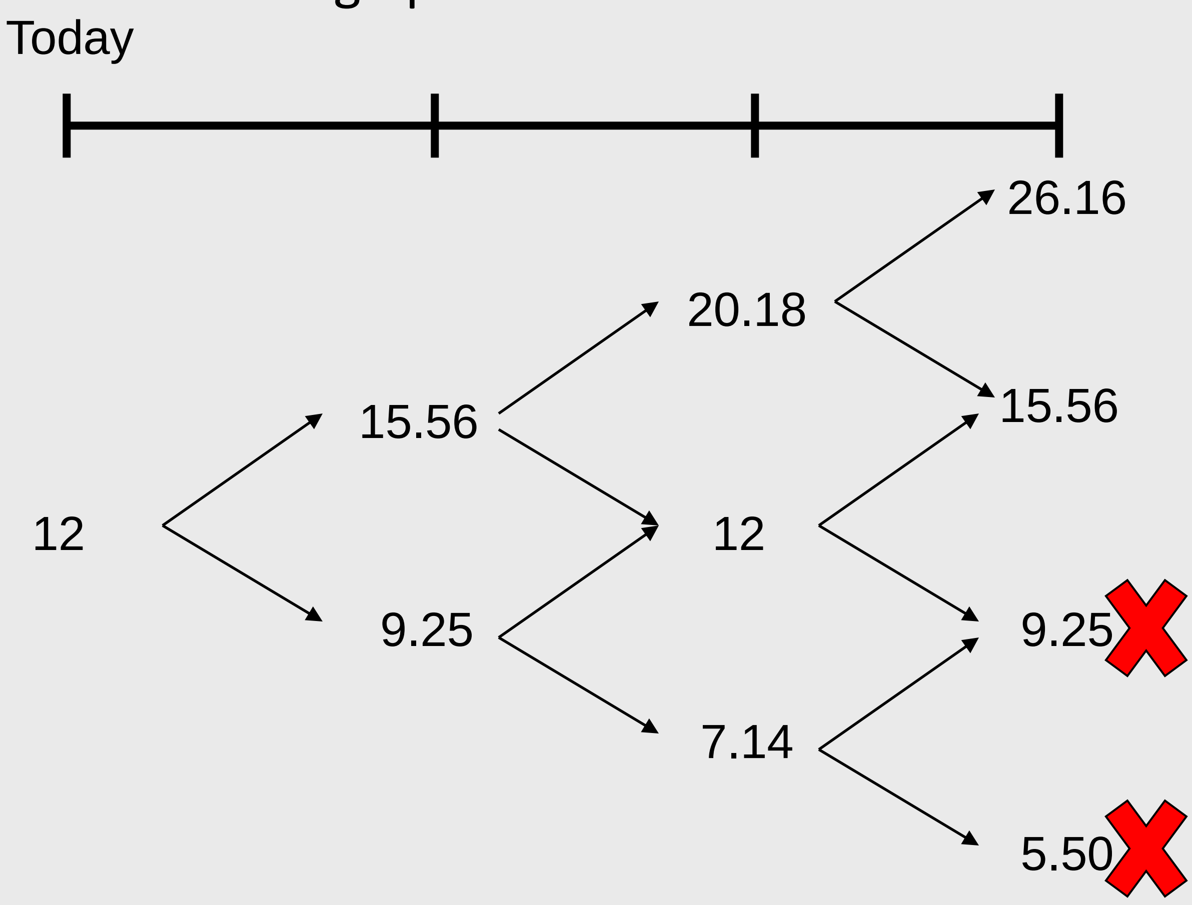
Example: Abandonment Option (2)

- PV of cash inflows: \$12.
- Volatility: 45%
- Risk-free Rate: 4.9%
- Option to abandon for price = \$10. 1 year to expiration.

- Value the call option in a binomial tree with three steps (Δt = 1/3).
  - $U = e^{\sigma\sqrt{\Delta t}} = e^{0.45\sqrt{0.333}} = 1.2967$
  - $D = \frac{1}{1.2967} = 0.7712$
  - $q = \frac{e^{r\Delta t} - D}{U - D} = \frac{e^{0.049*0.333} - 0.7712}{1.2967 - 0.7712} = 0.4667$
  - $\frac{1}{e^{r\Delta t}} = e^{-r\Delta t} = 0.9838$

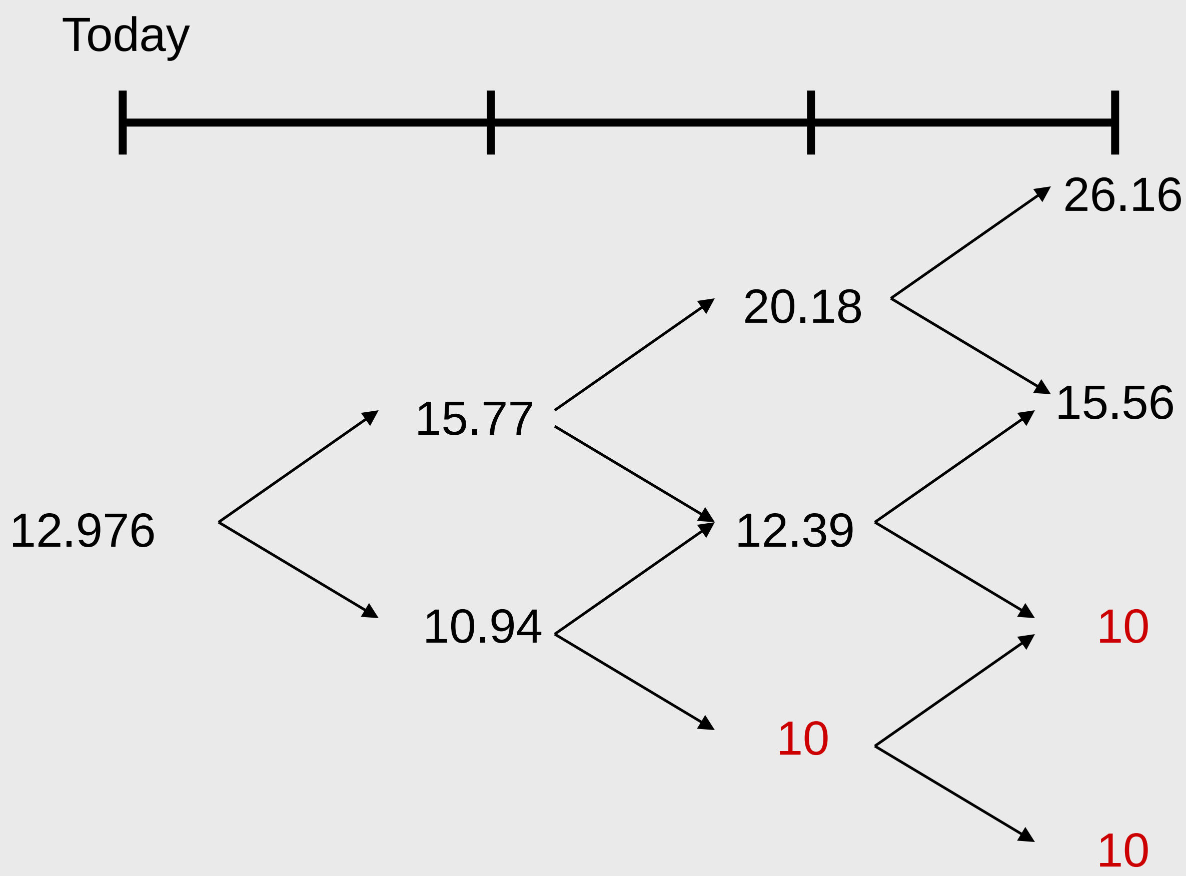
Example: Abandonment Option (3)

- What is the optimal decision at each stage?
- In which states would it better to sell the facility rather than continuing operations?



Example: Abandonment Option (4)

⇒ Value of Abandonment Option = **$0.976 million**



The value comes from the fact that we've limited our losses in bad states of the world.

Other Types of Real Options

- Compound Options:
  - Options on options. Phased investment is an example (design phase, engineering phase, construction). You can stop or defer at the end of each phase – so each phase is an option that is contingent on the exercise of earlier options.

- Switching Options:
  - Portfolios of options that allow the owner to switch at a fixed cost (or costs) between two or more modes of operation. Peak load generating equipment is an example – gas fired turbines are switched on when electricity prices go up and switched of then they go down.

- Rainbow Options:
  - More than one type of uncertainty affects the option value. For example, pharmaceutical R&D faces uncertainty about FDA approval, technology risk, and pricing risk.

Methodological Issues

1. The underlying assets are not traded. Replicating portfolio?
2. The value of the underlying assets might not follow a continuous-time diffusion process – BS will underestimate the value of deep out-of-the money options in this case.
3. The variance is not really known and probably isn't constant over time.
4. Exercise is not instantaneous – it takes time to build a manufacturing plant!
5. Model inputs are measured with greater error compared to financial options (revenues, costs, etc… compared to observing a market price).

Key Points from Real Options (1)

- Out-of-the-money real options have value
  - Even if an investment has a negative NPV, if there is a chance it could be positive in the future, the opportunity is worth something today.
- In-the-money real options need not be exercised immediately
  - The option to delay may be worth more than the NPV of undertaking the investment immediately.
- Waiting can be valuable
  - By waiting for uncertainty to resolve you can make better decisions.
- Delay investment expenses as much as possible
  - Committing capital before it is absolutely necessary gives up the option to make a better decision once uncertainty is resolved.
- Create value by exploiting real options
  - The firm must continually re-evaluate its investment opportunities, including the options to delay or abandon projects, as well as to create or grow them.

Key Points from Real Options (2)

- Three primary conditions for a project to have real option value:
  1. Learning: New information must arrive
  2. Adaptability: There is the option to adjust the project in response to new information
  3. Investing in current project is required to either receive the new information or the option to adjust the project
- Decision trees help analyse situations with option value
  - Keep track of the value of the project at each point
  - In complex models, you may need to start at the end of project and work your way back to the beginning
- Arbitrage pricing used when systematic risk is important to accurately capture risk-return trade-off
  - The binomial method is a powerful tool for valuing options in a two-stage, up-down model of the world
  - In the limit, valuation of a binomial tree for a call option gives the same value as the Black-Scholes formula.