**Industrial Organization**

**Resit Exam Fall 2024 – Solution Topics**

**1.** False. The entry of a new firm may destroy tacit collusion even if the new entrant is as efficient as the incumbents.

In general, in a market with $n$ firms engaging in Bertrand competition, tacit collusion is sustainable as long as the present value of profits under collusion exceeds the present value of profits from deviation. Mathematically, this condition can be expressed as:

$$\frac{\pi^M}{N} + \delta\frac{\pi^M}{N} + \delta^2\frac{\pi^M}{N} + \cdots > \pi^M + 0\delta + 0\delta^2 + \cdots \rightarrow \ \delta > 1 - \frac{1}{N}$$

Assume an initial scenario with two symmetric firms. In this case, collusion is sustainable if $\delta > \frac{1}{2}$. When a new symmetric competitor enters the market—i.e., a firm that is as efficient as the incumbents—the condition for collusion changes. With three symmetric firms, collusion is sustainable only if $\delta > \frac{2}{3}$.

If the discount factor ($\delta$) of each firm lies between $\delta \in \left]\frac{1}{2}, \frac{2}{3}\right[$, the dynamics of collusion shift. Before the new entrant's arrival, the two incumbents were able to sustain collusion since $\delta > \frac{1}{2}$. However, after the entrant joins, collusion becomes unsustainable because $\delta < \frac{2}{3}$.

This demonstrates how the entry of a new firm can undermine tacit collusion even in the absence of differences in efficiency.

**2.** True.

In a market where symmetric firms compete à la Bertrand, with constant marginal costs and negligible fixed costs, the outcome will typically result in only one firm operating. If both firms were to enter the market, they would engage in price competition, driving prices down to marginal cost. In this scenario, each firm's profit would be negative, as they would incur their fixed costs without earning sufficient revenue, creating incentives for one or both firms to exit the market.

This reasoning can be illustrated with the following payoff matrix:

| **1/2** | **Enter** | **Not Enter** |
|---|---|---|
| **Enter** | $(-F, -F)$ | $(\boldsymbol{\pi^M - F, 0})$ |
| **Not Enter** | $(\boldsymbol{0, \pi^M - F})$ | $(0,0)$ |

- If both firms enter, they compete by charging their marginal cost, leading to negative profits equal to their fixed costs ($-F$) for each firm.

- If only one firm enters, the entrant earns the monopoly profit minus fixed costs, while the other firm, by staying out of the market, earns zero profit.
- If neither firm enters, both firms earn zero profit.

In this setup, there are two Nash equilibria: in each equilibrium, only one firm enters the market while the other stays out.[1] This highlights the unsustainability of both firms operating under Bertrand competition with these cost structures.

**3.**

**(i)**

As explained in the exercise, *I*'s demand function is composed by two parts: (i) stranded consumers – represented by $C$; and (ii) new consumers - represented by $10 - p_I + p_E$.

**(ii)**

Since $\frac{dq_i}{dp_j} > 0$, the apps supplied by the two firms are imperfect substitutes.

**(iii)**

Betrand model with differentiated products.

Firm *I*'s profit-maximization problem:

$$\max_{p_I} \pi_I = (p_I - 2)(C + 10 - p_I + p_E)$$

$$FOC: \frac{d\pi_I}{dp_I} = 0 \Leftrightarrow C + 10 - 2p_I + p_E + 2 = 0 \Leftrightarrow \boldsymbol{p_I^*} = \boldsymbol{\frac{12 + C + p_E}{2}}$$

Firm *E*'s profit-maximization problem:

$$\max_{p_E} \pi_E = (p_E - 2)(10 - p_E + p_I)$$

$$FOC: \frac{d\pi_E}{dp_E} = 0 \Leftrightarrow 10 - 2p_E + p_I + 2 = 0 \Leftrightarrow \boldsymbol{p_E^*} = \boldsymbol{6 + \frac{p_I}{2}}$$

[1] Assuming that $\pi^M - F > 0$.

**(iv)**

In equilibrium:

$$\begin{cases} p_I^* = \dfrac{12 + C + p_E}{2} \\ p_E^* = 6 + \dfrac{p_I}{2} \end{cases} \leftrightarrow \begin{cases} p_E^* = 6 + \dfrac{12 + C + p_E}{4} \end{cases} \leftrightarrow \begin{cases} \dfrac{3}{4} p_E^* = 9 + \dfrac{C}{4} \end{cases} \leftrightarrow \begin{cases} p_E^* = 12 + \dfrac{C}{3} \end{cases}$$

$$\leftrightarrow \begin{cases} p_I^* = 6 + \dfrac{C}{2} + 6 + \dfrac{C}{6} \\ p_E^* = 12 + \dfrac{C}{3} \end{cases} \leftrightarrow \begin{cases} p_I^* = 12 + \dfrac{2C}{3} \\ p_E^* = 12 + \dfrac{C}{3} \end{cases}$$

Firm *I* will be able to charge a higher price – $12 + \frac{2C}{3} > 12 + \frac{C}{3}$ – because some consumers are already locked into its app. This consumer lock-in increases Firm I's market power, enabling it to set higher prices compared to Firm E, which does not have stranded consumers.

**(v)**

Since both firms have been in the market for the same amount of time and have each captured half of the initial demand ($C$), the demand functions for each firm are as follows:

$$q_I = \frac{C}{2} + 10 - p_I + p_E$$
$$q_E = \frac{C}{2} + 10 - p_E + p_I$$

**(vi)**

Betrand model with differentiated products.

Firm *I*'s profit-maximization problem:

$$\max_{p_I} \pi_I = (p_I - 2)\left(\frac{C}{2} + 10 - p_I + p_E\right)$$

$$FOC: \frac{d\pi_I}{dp_I} = 0 \Leftrightarrow \frac{C}{2} + 10 - 2p_I + p_E + 2 = 0 \Leftrightarrow \boldsymbol{p_I^* = 6 + \frac{C}{4} + \frac{p_E}{2}}$$

By symmetry: $p_I = p_E \rightarrow p_I = 6 + \frac{C}{4} + \frac{p_I}{2} \leftrightarrow p_I = 12 + \frac{C}{2} = p_E$

**(vii)**

Compared to question (iv), Firm *I*'s price has decreased, while Firm *E*'s price has increased. This outcome arises because market power—i.e., the ability to charge higher prices—stems from having stranded consumers who are willing to pay more due to being locked into an app. In this case, Firm *I* now has fewer stranded consumers, which reduces its ability to charge higher prices. Conversely, Firm *E* has gained more stranded consumers, enabling it to charge a higher price.

**(viii)**

No, Firm I's consumers are now better off, while Firm E's consumers are worse off.

**4.**

**(i)**

Firms will collude as long as the present value of profits under collusion is higher than the present value of profits under deviation. In this case, that happens when each firm's discount factor is higher than ½:

$$\frac{\pi^M}{2} + \delta\frac{\pi^M}{2} + \delta^2\frac{\pi^M}{2} + \cdots > \pi^M \Leftrightarrow \cdots \Leftrightarrow \delta > \frac{1}{2}$$

**(ii)**

The optimal collusion price is the monopoly price:

$$\max_P \pi^M = (P-6)(10-P)$$

$$FOC: \frac{d\pi^M}{dP} = 0 \Leftrightarrow 10 - 2P + 6 = 0 \Leftrightarrow \boldsymbol{P^* = 8 \wedge Q^* = 2 \wedge \pi^* = (8-6) \times 2 = 4}$$

Each firm will earn a profit of $\frac{\pi^M}{2} = 2$

**(iii)**

Firm 3 will collude at the pre-entry optimal tacit collusive price only if the present value of profits under collusion exceeds the present value of profits under deviation. This condition can be expressed as:

$$\frac{8}{3} + \frac{8}{3}\delta + \frac{8}{3}\delta^2 + \cdots > 9 + 8\delta + 8\delta^2 + \cdots$$

Under collusion, Firm 3 charges a price of $8$ and sells a quantity of $\frac{2}{3}$. In this case, Firm 3's profit is:

$$\pi_3 = (8-4) \times \frac{2}{3} = \frac{8}{3}.$$

If Firm 3 deviates, it charges its monopoly price of $7$ in the first period, earning a profit of $9$. From the second period onwards, Firm 3 must undercut Firms 1 and 2 by charging $6 - \varepsilon$. At this price, Firm 3 earns a profit of $8$ in each subsequent period.

Given that the present value of profits under deviation is always higher than the present value of profits under collusion, Firm 3 will not collude with the incumbents.

**(iv)**

Yes, it benefits because the price decreases from 8 to 6, leading to an increase in Total Surplus. Initially, Total Surplus is 6, calculated as the sum of profits for Firms 1 and 2 and consumer surplus:

$$\pi_1 + \pi_2 + CS = 2 + 2 + 2 = 6$$

After the price decreases to 6, Total Surplus rises to 16, comprising the profit of Firm 3 and consumer surplus:

$$\pi_3 + CS = 8 + 8 = 16$$

**(v)**

No, Firm 3, despite becoming the sole producer in the market, will not be able to charge the monopoly price of 7. Instead, it will charge a lower price of $6 - \varepsilon$.