Market Structure

Paulo Fagandini*

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1 Introduction

We will analyze the market structure by studying the different measures of market concentration, looking at each firm's market share, and the indexes that are constructed over those. Through the whole of this document, I will assume that N represents the number of firms that are present in the market.

2 Market Shares

The market share is the percentage of the market that some supplier serves. For example, if we measure sales, the market share of firm i would be $s_i := \frac{sales_i}{\sum_{j=1}^N sales_j}$, or in words, the sales of firm i over the total amount of sales in that market.

We will assume that there are no negative sales, and, that selling 0 means that the firm does not participate in the market.

Fact 1. *The market shares are defined in the interval* $(0, 1]$.

Proof. If there is only one firm in the market, say, firm 1, its market share is $s_1 = \frac{sales_1}{\sum_{i=1}^1 sales_i} = 1$, which is trivial.

If there are 2 or more firms in the market, then the market share of firm i is $\frac{sales_i}{\sum_{j=1}^N sales_j}$. However, the denominator is $sales_i$ plus the sales of the other firms, and as by assumption all those sales are positive, the denominator must be greater than $sales_i$, and therefore the division must be smaller than 1.

It can never be zero, as if it would be zero, it means that the firm is not selling, and therefore it is out of the market, or that there are infinite sales, which also cannot happen. In the same way, as we only consider positive market shares, and only summation among them, numerator and denominator are to be always positive. □

Fact 2. *The summation of the market shares is equal to 1.*

Proof. Note that:

$$\sum_{i=1}^N s_i = \sum_{i=1}^N \left(\frac{sales_i}{\sum_{j=1}^N sales_j} \right)$$

Let the total of sales $\left(\sum_{j=1}^N sales_j\right)$ be equal to S. As S does not depend on i (sub-index), then it can be moved out of the summation as a constant.

$$\sum_{i=1}^N \frac{sales_i}{\sum_{j=1}^N sales_j} = \sum_{i=1}^N \frac{sales_i}{S} = \frac{1}{S} \sum_{i=1}^N sales_i$$

But $\sum_{i=1}^N sales_j = S$, therefore:

$$\frac{1}{S} \sum_{i=1}^N sales_i = \frac{1}{S} S = 1$$

□

Fact 3. *The average of the market shares is equal to* $1/N$

Proof. The average of the market shares is defined as:

$$\overline{s} = \frac{\sum_{k=1}^N s_k}{N}$$

But, by Fact 2, we know that $\sum_{k=1}^N s_k = 1$, therefore:

$$\overline{s} = \frac{\sum_{k=1}^N s_k}{N} = \frac{1}{N}$$

□

*This is a non revised document, so when you find a typo, please let me know at paulo.ruiz@novasbe.pt. Any error in this document is only mine, and it does not necessarily represents the Instructor's or other TA's views. I thank the students who pointed to previously existing typos.

3 Market Concentration

In this section it will be defined some market concentration measures, and explained their main characteristics, along with their advantages and disadvantages for their purpose.

I will also show if they satisfy the conditions that Alexis Jacquemin defined as necessary to be considered a good market concentration measure:

- **Non-ambiguous character (NAC)** Given 2 markets, it should be possible to say without any doubt which one is more concentrated.
- **Scale invariance (SI)** The measure should depend only upon the relative dimension of each firm.
- **Transference (T)** The measure must increase if we reduce the market share of a small firm in favor of a bigger firm.
- **Monotonicity in the # of firms (MNF)** If the N firms have identical markets shares, then the measure should be decreasing in N.
- **Cardinality (Ca)** If each firm is divided into Z equal firms, the measure should decrease in the same proportion.

Consider as well for all this section, that the market has N firms, which are ranked according to their market shares, that is firm 1 is the largest firm, firm 2 is the second largest, and so on and so forth. That means that $s_1 \geq s_2 \geq s_3 ... \geq s_N$.

3.1 C_k Index

The C_k index is the summation of the market shares of the biggest k firms in the market. It only makes sense if $k < N$, otherwise it will only tell you that the number of firms in the market is lower or equal than k.

$$C_k := \sum_{i=1}^{k} s_i$$

The C_k index presents some problems when trying to compare markets, as it does not describe the concentration of the whole market, just the biggest firms. It is however, a commonly used index because it is very practical, as it is much more practical to estimate the market shares of the biggest firms, and usually it can give quite a decent idea of how concentrated the market is.

NAC <u>No</u>, C_k does not satisfy this condition. Consider a market with more than 2 firms and that $k = 1$. If we assume $s_2 = s_3$ or $s_2 > s_3$ we obtain the same value for C_1, however the second case would give us a market with higher concentration.[1]

SI <u>Yes</u>, because the market shares do not depend on the units in which they are measured.

T <u>No</u>. Consider the example provided for **NAC**. If firm 3 transfer some market share to firm 2, the index C_1 would not change at all.

MNF <u>Yes</u>. If all the firms are equal, then the market share of each firm is $1/N$. The C_k is $\sum_{i=1}^{N} s_i = \sum_{i=1}^{N} \frac{1}{N} = \frac{k}{N}$. $C_k = \frac{k}{N}$ is decreasing in N, and therefore MNF is satisfied.

Ca <u>No</u>. Consider a market with $s_1 = 0.40$, $s_2 = 0.30$, $s_3 = 0.20$, and $s_4 = 0.10$. Consider $Z = 2$ and $k = 2$. The original C_2 would be $C_2^0 = 0.60$, while after splitting each firm in 2 it would be $C_2^1 = 0.40$. $\frac{C_2^1}{C_2^0} = \frac{2}{3} \neq \frac{1}{2}$. The firms were divided by 2, but the C_k changed only a 33%. Therefore the C_k index does not satisfy the **Ca** condition.

3.2 H Index

The Herfindahl index H, also targets to measure how concentrated a market is. The main difference with the C_k index is that the H index considers the whole market.

$$H := \sum_{i=1}^{N} (s_i)^2$$

The H index is the weighted average of the market shares, where the weight of each market share, is the market share itself. The advantage of doing this, is that bigger shares are over-represented while smaller shares are under-represented. This leads to higher H indexes where the market is highly concentrated, and a lower H index when there is a low market concentration.

The H index is recognized as one of the indexes that best describes the market concentration, however, it is very difficult to gather all the information that is necessary to compute it.

Fact 4. *The H index can be written as $H = \frac{1}{N} + N\sigma_s^2$, where σ_s^2 is the variance of the market shares.*

Proof. Let $\overline{s}$ be the average of the market shares.

[1] It is enough to provide a counter example when we want to show that something does not hold. On the other hand, if we want to show that something *does* hold, we need to show that it holds always, with a generic case.

$$
\begin{aligned}
H &= \sum_{i=1}^{N}(s_i)^2 = \sum_{i=1}^{N}(s_i+0)^2 = \sum_{i=1}^{N}(s_i+\overline{s}-\overline{s})^2 \\
&= \sum_{i=1}^{N}[(s_i-\overline{s})+\overline{s}]^2 \\
&= \sum_{i=1}^{N}[(s_i-\overline{s})^2+2\overline{s}(s_i-\overline{s})+\overline{s}^2]
\end{aligned}
$$

$$
= \sum_{i=1}^{N}(s_i-\overline{s})^2+\sum_{i=1}^{N}2\overline{s}(s_i-\overline{s})+\sum_{i=1}^{N}\overline{s}^2
$$

Using the fact that we can always multiply by 1, we create a $1=\frac{N}{N}$,

$$
\begin{aligned}
&= \sum_{i=1}^{N}(s_i-\overline{s})^2+\sum_{i=1}^{N}2\overline{s}(s_i-\overline{s})+\sum_{i=1}^{N}\overline{s}^2 \\
&= \frac{N}{N}\sum_{i=1}^{N}(s_i-\overline{s})^2+\sum_{i=1}^{N}2\overline{s}(s_i-\overline{s})+\sum_{i=1}^{N}\overline{s}^2
\end{aligned}
$$

And realize that $\frac{\sum_{i=1}^{N}(s_i-\overline{s})^2}{N}=\sigma_s^2$,

$$
\begin{aligned}
&= N\sigma_s^2+\sum_{i=1}^{N}2\overline{s}(s_i-\overline{s})+\sum_{i=1}^{N}\overline{s}^2 \\
&= N\sigma_s^2+2\overline{s}\sum_{i=1}^{N}(s_i-\overline{s})+\sum_{i=1}^{N}\overline{s}^2 \\
&= N\sigma_s^2+2\overline{s}\left(\sum_{i=1}^{N}s_i-\sum_{i=1}^{N}\overline{s}\right)+\sum_{i=1}^{N}\overline{s}^2 \\
&= N\sigma_s^2+2\overline{s}\left(\sum_{i=1}^{N}s_i-\overline{s}\sum_{i=1}^{N}1\right)+\overline{s}^2\sum_{i=1}^{N}1
\end{aligned}
$$

Realizing that $\sum_{i=1}^{N}1=N$, as it is summing up N times one, we obtain:

$$
= N\sigma_s^2+2\overline{s}\left(\sum_{i=1}^{N}s_i-\overline{s}N\right)+\overline{s}^2N
$$

Using Fact 2 and 3, to replace $\sum_{i=1}^{N}s_i=1$ and $\overline{s}=\frac{1}{N}$,

$$
\begin{aligned}
&= N\sigma_s^2+2\overline{s}\,(1-1)+\frac{1}{N^2}N \\
&= \frac{1}{N}+N\sigma_S^2
\end{aligned}
$$

□

This means that the higher the variance of the market shares, then the higher the market concentration, and this is even more pronounced if N is large. Now, as $\frac{\partial H}{\partial N}=\sigma_s^2-\frac{1}{N^2}$, we obtain that H is actually convex in the number of firms, decreasing at first, and then increasing (note that if $N\to\infty$, then $1/N^2\to 0$ and $H\to\sigma_s^2$).[2] H reaches its minimum when $N=\frac{1}{\sigma_s}$, so the higher the variance (or standard deviation, or dispersion, or heterogeneity in market shares) the sooner H becomes increasing in the number of firms. Take a moment to think about it. If σ_s^2 is small, then all the firms are relatively similar to start with, however if we want to increase the number of firms, and keep the dispersion constant, we must allow for firms that become outliers from above, that mean, to start having higher market shares, in order to balance the, every time smaller, new firms, increasing the measure of market concentration.

Definition 1 (Adelman's Equivalent Number)**.** *EN is a function of H, that represents the number of firms with the same market share that would lead to a market concentration of H.*

$$
EN := \frac{1}{H}
$$

Note that from Fact 4, if we consider that all the firms have the same market share, we would have $\sigma_s^2=0$, and therefore $H=\frac{1}{N}$. From there it is trivial to get the Adelman's Equivalent Number. This number can help to understand the level of concentration between two markets, in relatively simple terms. How many firms of the same size would replicate this market concentration? The higher the number, the less concentrated the market is (as each firm has less and less market share).

NAC Yes. As the highest market shares are weighted more in the computation of this index, a higher concentration will always lead to higher values for the H index.

SI Yes. Again, as it uses the market shares, it does not depend on the units measures to compute those.

T Yes. The market shares are $s_1\geq s_2\geq s_3...\geq s_N$. Consider firm s_{j+k} (where $j+k\leq N$) transfers y from her share to s_j (which by assumption must be bigger than firm $j+k$. The old H index is $\sum_{i=1}^{N}s_i^2$, while the new is $\sum_{i=1}^{N}s_i^2-s_j^2-s_{j+k}^2+(s_j+y)^2+(s_{j+k}-y)^2$. Note that all the other firms are left untouched, so from the summation we need to remove s_j and s_{j+k}, because they did change. Then we add the new

[2] Here $\to$ is used as "converge", or in a more informal way "goes to".

market shares squared to the H index in the last two terms. Let's look at the terms after the summation:

$$\begin{aligned}
& -s_j^2 - s_{j+k}^2 + (s_j + y)^2 + (s_{j+k} - y)^2 \\
&= \cancel{-s_j^2} \cancel{-s_{j+k}^2} + \cancel{s_j^2} + 2ys_j + y^2 + \cancel{s_{j+k}^2} - 2ys_{j+k} + y^2 \\
&= 2y^2 + 2y(s_j - s_{j+k})
\end{aligned}$$

But $s_j \geq s_{j+k}$, and therefore the last term is strictly positive. This means that the new Herfindahl index is greater than before (we have the same old H index, plus something positive, then it has to be larger), and therefore the market is more concentrated.

MNF Yes. $H = \sum_{1=1}^{N} s_i^2$. If firms are equal, then $s_i = \frac{1}{N}$. $H = \sum_{i=1}^{N} \frac{1}{N^2} = \frac{1}{N^2} \sum_{i=1}^{N} 1 = \frac{1}{N^2} N = \frac{1}{N}$ and therefore it is decreasing in N.

Ca Yes. $H = \sum_{i=1}^{N} s_i^2$. If we divide each firm by z, we now have zN firms, and the new H index is $\sum_{i=1}^{N} \left(\sum_{j=1}^{z} \left(\frac{s_i}{z}\right)^2 \right)$, so for each old firm (indexed by i) we need to add the square of the market shares of the firms that were born from it (indexed by j). Note however, that nothing is really indexed by j, as the market share of the new firms are $\frac{s_i}{z}$, so they can go out of the sum: $\sum_{i=1}^{N} \left(\left(\frac{s_i}{z}\right)^2 \sum_{j=1}^{z} 1 \right) = \sum_{i=1}^{N} \left(\left(\frac{s_i}{z}\right)^2 z \right) = \sum_{i=1}^{N} \frac{s_i^2}{z} = \frac{1}{z} \sum_{i=1}^{N} s_i^2 = \frac{1}{z} H$. In other words, the H index has adjusted just as each firm did, dividing itself by z.

4 I Index

The I index measures the stability of the market shares. It aims to measure how much do market shares change in time. It is defined as:

$$I := \frac{1}{2} \sum_{i=1}^{N} |s_{i,1} - s_{i,0}|$$

Where $s_{i,t}$ means the market share of firm i in period t. The logic behind this index is that, even when only a few firms hold a big share of the market, if those few firms are different firms every period, then this represent less of a concern than if the same firms hold that big share of the market permanently. The change in the market shares could reflect for example competition among firms.

5 Missing information about the market shares

In this section we discuss what you can do when the information with respect to some firms is missing. In general, we will not be able to obtain the market shares of the whole set of firms in the market. We will probably obtain information about "some" of them, and usually the biggest ones.[3].

In these cases, you will try to find an interval in which the index could be. For example: *The H index should be in* $[H_{min}, H_{max}]$.

The H index is increasing in market concentration, so the H index should be maximum when we assume that whatever is in "others" lead to the highest possible concentration in that market. This would be the case if "others" would be a single firm.[4] Suppose that you know only M firms, and you have the market share of "others" as s_{others}. The Herfindahl index in this situation would be $H_{max} = \sum_{i=1}^{M} (s_i)^2 + s_{others}^2$. Now if we are trying to find the H_{min} we need to look for the minimum possible concentration, given the information we already have about $\{s_i\}_{i=1}^{M}$. For that, we need that to think on how "others" would give us the lowest concentration possible. This happens if others is composed by equal firms (so you cannot say where the market share of others is concentrated). This implies that *a firm* in others will have the following market share $\hat{s} = \frac{1}{K}$ where I have assumed there are K firms in others. This would add to our H index $K \frac{1}{K^2} = \frac{1}{K}$, which can be minimized if $K \to \infty$, so $\frac{1}{K} \to 0$. The H_{min} is then $\sum_{i=1}^{M} s_i^2 + 0$. Summarizing, if you know the firms from 1 to M, the interval for the H index is $\left[\sum_{i=1}^{M} s_i^2, \sum_{i=1}^{M} s_i^2 + s_{others}^2 \right]$.

For the I index we need to consider the cases in which we have maximum and minimum volatility. Again, for the firms for which we do have information, we cannot speculate, and we just compute their variation. Let's assume again that you know M firm's market share in both periods, and you have again, "others" firms with a market share for each period. The part over which we cannot speculate is $I_{known} = \frac{1}{2} \sum_{i=1}^{M} |s_{i,1} - s_{i,0}|$. However, we do not know exactly what is going on with "others". In the worst case (lowest variation), this is just a single firm, and we would need to add $\frac{1}{2} |s_{others,1} - s_{others,0}|$ to

[3]However, you cannot make this assumption in an evaluation. This information should be explicit in the question in order for you to consider that "others" contains smaller firms than the ones for which you have information.

[4]In the case we are told that the firms for which we do have information are the biggest, then others, in the worst case, would contain firms as the smallest of the known firms, and a residual as a single firm.

I_{known}. Then $I_{min} = I_{known} + \frac{1}{2}|s_{others,1} - s_{others,0}|$ which represents the case with less variation (worst case). For the best case, we want maximum variation, that is going to be obtained only if we assume that in the first period all the "other" firms left the market, and the new "other" are only new firms. This would imply that the maximum variation would be $I_{known} + \frac{1}{2}|0 - s_{others,0}| + \frac{1}{2}|s_{others,1} - 0|$. The first term is all the firms that left the market, while the second reflects all the new firms that entered the market. The interval is going to be

$$\left[\frac{1}{2}\left(\sum_{i=1}^{M}|s_{i,1} - s_{i,0}| + |s_{others,1} - s_{others,0}|\right), \frac{1}{2}\left(\sum_{i=1}^{M}|s_{i,1} - s_{i,0}| + |0 - s_{others,0}| + |s_{others,1} - 0|\right)\right]$$