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Econometrics II

Exam
1hour 30 min

March 17, 2017

Q1: Use the output in Figure 1 to explain the log wage.

a) [1.0] Which estimators can we use to estimate a fixed effects model?

b) [1.5] Provide a meaningful interpretation of the slope coefficient estimate in the output headed FE.

c) [1.0] Write the estimating equation for log wage and provide the necessary transformation to obtain the within estimator via OLS.

d) [1.5] Explain how to perform an appropriate test in order to find which model is preferred: FE or RE.

e) [1.0] Discuss the strengths and weaknesses of the within estimator.

Figure 1

```
. describe ln_wage tenure
              storage  display     value
variable name   type   format      label      variable label
-------------------------------------------------------------------------------
ln_wage         float  %9.0g                  ln(wage/GNP deflator)
tenure          float  %9.0g                  job tenure, in years

. xtsum ln_wage tenure
Variable         |      Mean   Std. Dev.       Min        Max |    Observations
-----------------+--------------------------------------------+----------------
ln_wage  overall |  1.914642   .5016681    .137109   4.005049 |     N =    2460
         between |             .4592862   .6776364   3.259719 |     n =     820
         within  |             .2022341   .7843202   3.599762 |     T =       3
                 |                                            |
tenure   overall |  6.661958   5.741345          0   25.91667 |     N =    2460
         between |             5.524363          0   24.30556 |     n =     820
         within  |             1.571394  -4.171375   16.80085 |     T =       3

. quietly xtreg ln_wage tenure, fe
. estimates store FE
. quietly xtreg ln_wage tenure, re
. estimates store RE

. estimates table FE RE, stats(N) b(%7.4f) se(%7.4f)  stfmt(%8.2f)
------------------------------------
    Variable |    FE         RE
-------------+----------------------
      tenure |  0.0179      0.0216
             |  0.0031      0.0021
       _cons |  1.7957      1.7706
             |  0.0215      0.0207
-------------+----------------------
           N |    2460        2460
------------------------------------
                        legend: b/se
```

Q2: Consider a first-order autoregressive model with individual and time effects of the form

$$y_{it} = \alpha_i + \delta_t + \rho y_{i,t-1} + v_{it}, \;\; i = 1, ..., N; \;\; t = 1, ..., T \tag{1}$$

with $E(v_{it}|y_{i0}, ..., y_{i,t-1}, \; \delta_0, ..., \delta_t, \alpha_i) = 0$. Suppose that $T = 3$ so that for each individual we observe $y_{i0,} \; y_{i1}, \; y_{i2}, y_{i3}$.

a) [2.0] Obtain the first-difference estimate of ρ and discuss its properties.

b) [2.0] Propose a consistent estimator of ρ for large N.

c) Consider now the heterogeneous autoregressive panel with large N and T,

$$y_{it} = \rho_i y_{i,t-1} + v_{it}, \;\; i = 1, ..., N; \;\; t = 1, ..., T \tag{2}$$

with $\rho_i \sim iidN(0, \sigma_w^2)$ and ρ_i independently distributed with y_{it} and v_{it} for all t.

c.i) [2.0] Indicate why the within estimator is inconsistent in the context of (2).

c.ii) [1.5] Why would the mean group estimator be a suitable procedure in (2)?

Q3: Consider the following simple panel data model

$$y_{it} = x_{it}\beta + \alpha_i^* + v_{it},\ \ i = 1, ..., N;\ \ t = 1, ..., T$$

where β is one dimensional and where it is assumed that

$$\alpha_i^* = \overline{x}_i\lambda + \alpha_i$$

with $\alpha_i \sim NID(0, \sigma_\alpha^2)$ and $v_{it} \sim NID(0, \sigma_v^2)$ mutually independent and independent of all $x_{it}s$, where $\overline{x}_i = \sum_{t=1}^T x_{it}$. The parameter β can be estimated by the fixed effects (or within) estimator given by

$$\widehat{\beta}_{FE} = \frac{\sum\limits_{n=1}^{N}\sum\limits_{t=1}^{T}(y_{it} - \overline{y}_i)(x_{it} - \overline{x}_i)}{\sum\limits_{n=1}^{N}\sum\limits_{t=1}^{T}(x_{it} - \overline{x}_i)^2}.$$

As an alternative, the correlation between the error term $\alpha_i^* + v_{it}$ and x_{it} can be handled by instrumental variables.

a) [1.5] Give an expression for the $\widehat{\beta}_{IV}$ using $(x_{it} - \overline{x}_i)$ as an instrument for x_{it}. Show that $\widehat{\beta}_{IV}$ and $\widehat{\beta}_{FE}$ are identical.

b) [2.5] Another way to eliminate the individual effects α_i^* from the model is doing the following transformation:

$$y_{it} - \overline{y}_i = (x_{it} - \overline{x}_i)\beta + (v_{it} - \overline{v}_i),\ \ i = 1, ..., N;\ \ t = 1, ..., T$$

Which is the OLS estimator $(\widehat{\beta})$ based on this model? In which conditions is $\widehat{\beta}$ a consistent estimator of β?

c) [2.5] Consider the between estimator $\widehat{\beta}_B$ for β. Give an expression for $\widehat{\beta}_B$ and show that it is unbiased for $\beta + \lambda$.