# Microeconomic Theory II Spring 2024 Final Exam

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#### You have a total of 120 minutes (2 hours) to solve the exam. Identify each sheet with your Student Number and Name. Good luck!

I (4.5 points)

Consider a market with a monopolist that produces a good of variable quality  $q \ge 0$  at cost C(q) per unit of good (priced at p). Each customer wants to buy one unit of the good and is characterized by her taste for quality  $\theta$ , where  $\theta \in \{\theta_L, \theta_H\}$  with  $\theta_L < \theta_H$ . An agent's utility will therefore be  $\theta.q - p$ . Each customer knows her  $\theta$ , but the monopolist only knows that the proportion of agents with  $\theta = \theta_H$  is equal to  $\lambda$ .

- a. Determine the optimal contracts when the monopolist is able to observe types.
- b. Under asymmetric information, and assuming that the monopolist must serve both types of customers, what are the second best contracts? Explain which type receives an informational rent and calculate it.
- c. Might it be better to exclude one type of consumer in this case?

# II (5.5 points)

The Government wants to implement the efficient provision of a public good that is valued by I agents. Let x denote the amount of public good, let  $t_i$  denote the (possibly negative) transfer from the Government to each agent and let each agent's utility function be given by  $u_i(x, t_i, \theta_i) = \theta_i v(x) + t_i$ , where  $\theta_i$  is private information,  $\theta_i$  has a distribution  $P_i$  on  $\left[\underline{\theta_i}, \overline{\theta_i}\right]$  that is common knowledge, and v is such that v' > 0 and v'' < 0. Assume that the cost of providing the good is c(x) = cx.

- a. Assuming that the Government has no budgetary concerns and can legally require and enforce participation, describe the mechanism that achieves dominant-strategy implementation of the efficient public good provision. Prove that dominant-strategy incentive compatibility is satisfied.
- b. If each agent can refuse to participate, can the mechanism derived in a. still implement the efficient provision in dominant strategies?
- c. Assume now that the Government must balance its budget but participation is compulsory. Describe the mechanism that achieves the Bayesian implementation of the efficient public good provision.

#### III (6 points)

Consider a bilateral trade setting in which the buyer's and seller's valuations are drawn independently from the uniform distribution on [0,2]. The profits for both the buyer and the seller equal the respective net gains from trade.

- a. Show that if *f* is a Bayesian incentive compatible and interim individually rational social choice function that is ex post efficient, the sum of the buyer's and seller's expected profits under *f* cannot be less than 2/3.
- b. Show that, in fact, there is no social choice function that is ex post efficient and satisfies budget balance such that, under truth-telling, the sum of the buyer's and seller's expected profits exceeds 1/3.

### IV (4 points)

Consider the following voting scheme: each agent submits a total preference ordering, along with additional information of that agent's utility for each outcome. (Obviously, the preference ordering reflects the ordering of the outcomes by utility, with the highest utility outcome being most preferred.) Let the utility for each outcome be an element of [0; 100]. The social welfare function orders each outcome by the sum of the utilities of that outcome for each agent. In the case of ties, the outcome with the earlier lexicographic ordering is preferred.

- a. Is this Pareto optimal?
- b. Is this independent of irrelevant utilities? (In this setting, define IIU as the proposition that the social ordering of a and b does not change as long as agents do not change their utilities for a and b.).
- c. Is this non dictatorial?
- d. Are your answers to the previous questions compatible with Arrow's theorem? Explain why or why not.