Microeconomic Theory II Spring 2023 Final Exam

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You have a total of 120 minutes (2 hours) to solve the exam.	
Identify each sheet with your Student Number and Name.	
Good luck!	

I (5 points)

The owner of a firm puts a manager in charge of a project. The project can give rise to two different results, measured in terms of the owner's profits: π_1 and π_2 , where $\pi_1 < \pi_2$. The probability of each result depends on the manager's effort.

Now assume that there are only two possible effort levels; high efort, e_H and low effort $e_L = 0$. Under high effort the probability of getting the good result π_2 is p, and so the probability of getting result π_1 is 1 - p. On the other hand, under low effort the result is always π_1 . Exerting high effort implies a disutility to the manager of v, while low effort has no disutility.

The manager's utility function is $U(w, e_H) = \sqrt{w} - v$; or $U(w, 0) = \sqrt{w}$, depending on whether he exerts high or low effort. If the manager decides not to work on the project, he is guaranteed a reservation utility of $\underline{U} = \sqrt{\underline{w}}$, where \underline{w} is his reservation wage.

- a. Assume that the manager's effort is observable and verifiable. Describe the optimal contract for the firm's owner if she prefers low effort and if she prefers high effort. When will the owner demand each effort level?
- b. Now assume that the manager's effort is not verifiable, and so the only variable that can appear in the contract is the final result obtained.
 - i. What is the optimal contract that the owner offers when she would like the manager to exert low effort?
 - ii. If she would like the manager to exert high effort, what is the constrained maximization problem that the owner must solve to calculate the optimal contract? Describe the optimal contract in this case.
 - iii. What are the owner's profits for each effort level? Describe what the owner must compare in order to decide which effort level to demand.

II (6.5 points)

An indivisible good is worth v to the Government. There are two potential suppliers whose production costs are θ_1 and θ_2 . These costs are private information, but their distributions are common knowledge: each has the same cdf F on $[\underline{\theta}, \overline{\theta}]$, which has an associated strictly positive density function f. The Government wants to design a mechanism (transfers and a decision function) that maximizes the expected value of its net gain from trade, but it cannot force the suppliers to sell (the potential suppliers have the outside option of getting 0 profits). However, the Government need not worry about budget balance.

- a. Specify the problem and characterise each supplier's profit under incentive compatibility in terms of an integral equation and a monotonicity constraint.
- b. Assuming that $\frac{F(x)}{f(x)}$ is strictly increasing, describe the optimal mechanism.
- c. Can you think of a simple real-life mechanism that would meet all the requirements you derived in b.?

III (4.5 points)

Assume that 2 regions value a bridge and that it would cost c to build it. Each individual valuation μ_i for the bridge follows a uniform distribution on [0,2]. Assuming that participation can be compulsory, suggest a mechanism that ensures the efficient provision of the public good, showing that it satisfies Bayesian incentive compatibility and budget balance.

IV (4 points)

There are 100 people living in Chicutzimi, Quebec. The residents of Chicutzimi are divided into three groups, A with 31 members, B with 49 members and C with 20 members. The residents of Chicutzimi want to decide the level of English taught in the local Chicutzimi high school. The options are to teach all courses in the English language (let us denote this option by AE), teach all courses in French but have an English as a second language course (let us denote this option by SE) or have no English taught at school (let us denote this option by NE). It is well known what the different groups think about the choice at hand:

	A (31)	B (49)	C (20)
Most preferred	AE	NE	SE
Second best	SE	SE	AE
Least preferred	NE	AE	NE

In a town meeting on Monday three offers are brought up as to how to aggregate the preferences: (i) Majority rule (ii) Borda Rule: Each voter will rank the three options. For his top choice the voter will give a score of 3, for the second best the voter will give a score of 1 and for the least preferred option the voter will give zero. The social preference will rank alternatives according to their aggregate score in the entire population. (iii) Dictatorship: Mr. Dubois who practically owns more than half of the Chicutzimi real estate and businesses will determine the social preference according to his own personal preference. Mr. Dubois is a member of group A.

(a) Which aggregation rule will each group prefer?

(b) Is the social preference that results from the Majority rule aggregation method a rational preference relation? Does your answer contradict Arrow's theorem?

(c) Show that if the social choice function f(.) maximizes the social welfare functional determined by the Borda rule, then f(.) is not strategy-proof.