1. Let the walrasian equilibrium be denoted by $\left[p, (x^h)_{h\in H}, (y^j)_{j\in J}\right]$.

Assume, towards a contradiction, that it the allocation does not have the core property. Then, there is a coalition S and an allocation and production plan $(x^{h\prime})_{h\in H}, (y^{j\prime})_{j\in J}$ such that for all $h \in S$ we have $u^h(x^{h\prime}) > u^h(x^h)$ and such that $\sum_{h\in S} x^{h'} = \sum_{h\in S} \omega^h + \sum_j y^{j'}$.

In equilibrium, due to the CRS technology, profits must be zero and $p \cdot y^j = 0$ for all j.

Moreover, due to utility maximization, it must be that $p \cdot x^{h'} > p \cdot x^h = p \cdot \omega^h$ (due to zero profits and LNS) for all $h \in S$.

But then, summing over $h \in S, p \cdot \sum x^{h'} > p \cdot \sum \omega^h$ and , $p \cdot \left(\sum_{h\in S} \omega^h + \sum_j y^{j'}\right) > p \cdot \sum_{h\in S} \omega^h$ i.e. $p \cdot \sum_j y^{j'} > 0 = p \cdot \sum_j y^j$, contradicting profit maximization in equilibrium.

2. a) The set of Pareto efficient points will be $x_1 \in [0,3], y_1 = 2$.

b) Individual demands for 1 are $y_1 = 3 - \frac{1}{4p}, x_1 = \frac{1}{4p^2}$, if prices are positive.

2 will consume all wealth on good x if prices are positive. But then no equilibrium exists with positive prices. If the price of y is 0, then the price ratio is $+\infty$. But then 1 will want to consume infinite amounts of y.

c) For a price ratio of $+\infty$, agent 1 will want to consume infinite amounts of good y. Therefore, even though the initial endowment satisfies the expenditure minimization problem, it does not satisfy the utility maximization problem; therefore, a quasiequilibrium with transfers need not be an equilibrium with transfers. Note that the assumptions of the second welfare theorem are not met.

3.

CRRA utilities instead of CAPM model

$$U^h(x_0, x_1, \dots, x_S) = \frac{x_0^{1-\rho}}{1-\rho} + \beta \sum_{s=1}^{S} \gamma_s \frac{x_s^{1-\rho}}{1-\rho}$$

$p_0 = 1$; $\omega = (\omega_0, \tilde\omega) = \sum_{h=1}^{4} \omega^h$

1) State prices

$$\frac{x_0^{h\,-\rho}}{\beta \gamma_s x_s^{h\,-\rho}} = \frac{1}{p_s} \iff p_s = \beta\gamma_s\left(\frac{x_0^h}{x_s^h}\right)^{\rho} \iff x_s^h = x_0^h\left(\frac{\beta\gamma_s}{p_s}\right)^{1/\rho}$$

Aggregating $\sum_h x_s^h = \sum_h x_0^h \cdot \left(\frac{\beta\gamma_s}{p_s}\right)^{1/\rho}$

$$\omega_s = \omega_0\left(\frac{\beta\gamma_s}{p_s}\right)^{1/\rho} \iff \boxed{p_s = \beta\gamma_s\left(\frac{\omega_0}{\omega_s}\right)^{\rho}}$$

2) $R = \frac{1}{\sum_s p_s} = \frac{1}{\beta\sum \gamma_s\left(\frac{\omega_0}{\omega_s}\right)^{\rho}}$

3) Show that in equilibrium, price of an asset $\tilde x = (x_1, \dots, x_S)$ is of the form $\pi(\tilde x) = \frac{E\tilde x}{R} - \mathrm{cov}\left[f\left(\frac{\tilde\omega}{\omega_0}\right), \tilde x\right]$ (f increasing)

Positive linear pricing rule $\pi(\tilde x) = \sum p_s x_s = \beta\sum \gamma_s\left(\frac{\omega_0}{\omega_s}\right)^{\rho} x_s = \beta\sum \gamma_s x_s\left(\frac{\omega_0}{\omega_s}\right)^{\rho}$ → $E(\tilde x \tilde\lambda)$

Taking $\lambda_s = \left(\frac{\omega_0}{\omega_s}\right)^{\rho}$, $\tilde\lambda = \left(\frac{\omega_0}{\tilde\omega}\right)^{\rho}$

$$\pi(\tilde x) = \beta E[\tilde x \tilde\lambda] = \beta\left(E\tilde x \cdot E\tilde\lambda + \mathrm{cov}(\tilde x, \tilde\lambda)\right)$$

$$E(\tilde\lambda) = \sum_s \gamma_s\left(\frac{\omega_0}{\omega_s}\right)^{\rho} = \sum_s \frac{p_s}{\beta} = \frac{1}{R\beta}$$

So $\pi(\tilde x) = \frac{\beta}{R\beta} \cdot E\tilde x + \beta\,\mathrm{cov}(\tilde x, \tilde\lambda) = \frac{E(\tilde x)}{R} + \underbrace{\beta\,\mathrm{cov}(\tilde x, \tilde\lambda)}_{= -\mathrm{cov}\left(\tilde x, f\left(\frac{\tilde\omega}{\omega_0}\right)\right)}$

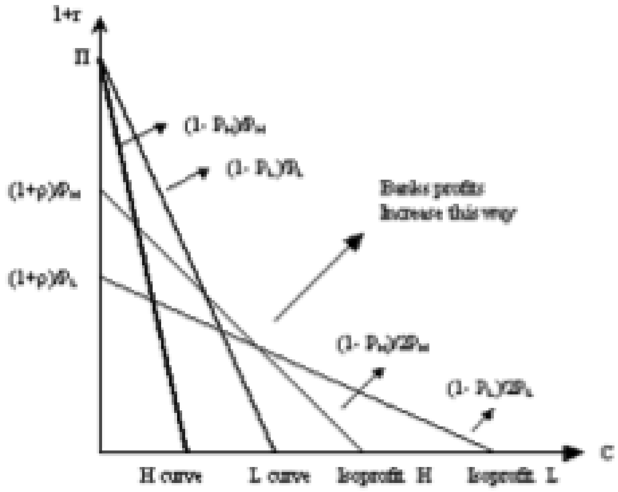
$$\pi(\tilde x) = \frac{E\tilde x}{R} - \mathrm{cov}(\tilde x, -\beta\tilde\lambda)$$

$f\left(\frac{\tilde\omega}{\omega_0}\right) = -\beta\left(\frac{\omega_0}{\tilde\omega}\right)^{\rho} = -\beta\left(\frac{\tilde\omega}{\omega_0}\right)^{-\rho}$ which is increasing in $\frac{\tilde\omega}{\omega_0}$

$(f(x) = -\beta x^{-\rho})$

4. a) Efficiency would require all cars to be traded (1.2q>q for all q). Adverse selection causes inefficiency. If all sellers took their cars to the market, the WTP would be 0.6 and that would lead sellers of cars with q>0.6 to leave the market. The process continues (the WTP would always be 0.6q for the q fraction that remains in the market), until no car remains.

   b) (See solutions for MWG 13.C.1)



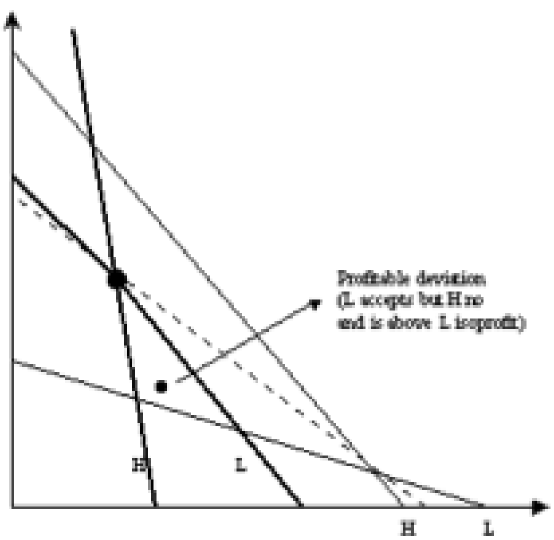
a) Show that no pooling equilibrium can exist.

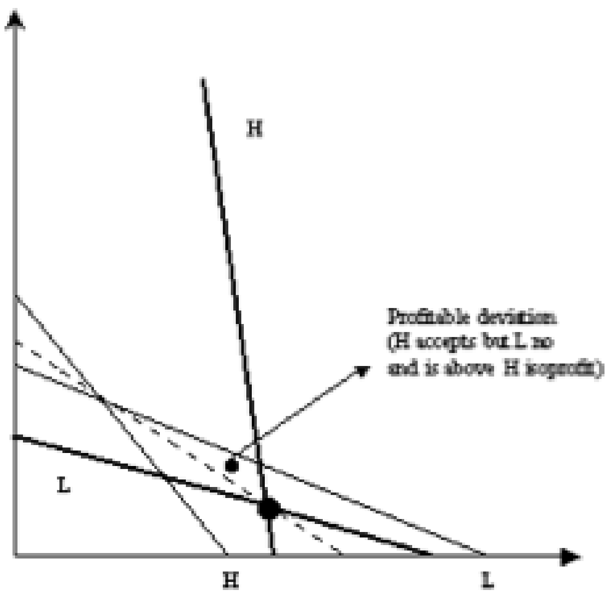
Lemma: In any equilibrium, banks make zero profits.

Proof: Suppose not: (r_L, C_L) and (r_H, C_H) are the equilibrium contracts with aggregate profits Π. The other bank could deviate and get $(r_L - \varepsilon, C_L)$ and $(r_H - \varepsilon, C_H)$. If (r_L, C_L) and (r_H, C_H) are incentive compatible, so are $(r_L - \varepsilon, C_L)$ and $(r_H - \varepsilon, C_H)$. Deviating bank makes profits close to Π.

Lemma: No pooling equilibria can exist.

Proof: Suppose not.





$\lambda \left[P_H (1+r) + (1-P_H) \frac{C}{2} \right] + (1-\lambda) \left[P_L (1+r) + (1-P_L) \frac{C}{2} \right] = 1+\rho.$

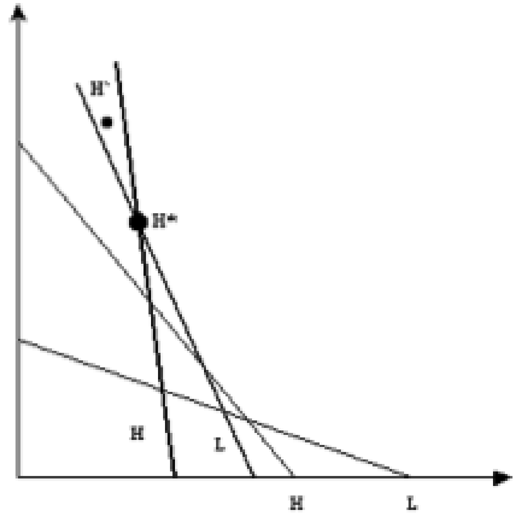
With $\overline{P} = \lambda P_H + (1-\lambda) P_L$ comes $\overline{P}(1+r) + (1-\overline{P})\frac{C}{2} = 1+\rho$ (pooling break-even line).

b) What does a separating equilibrium look like if it exists?

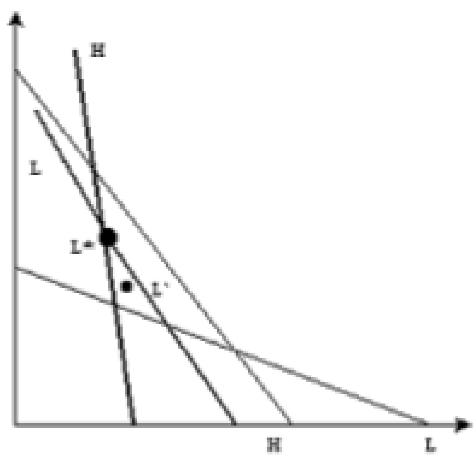
Lemma: In a separating equilibrium banks make zero profits from each type.

Proof: Assume not:

- Making positive profits from high type: if the initial contract for H is above the zero profit line, we can always draw a contract H' such that the H type would rather take it, but the L type will choose her initial contract. Then, we can capture all the H-types and with H' close enough to H, profits will be positive.

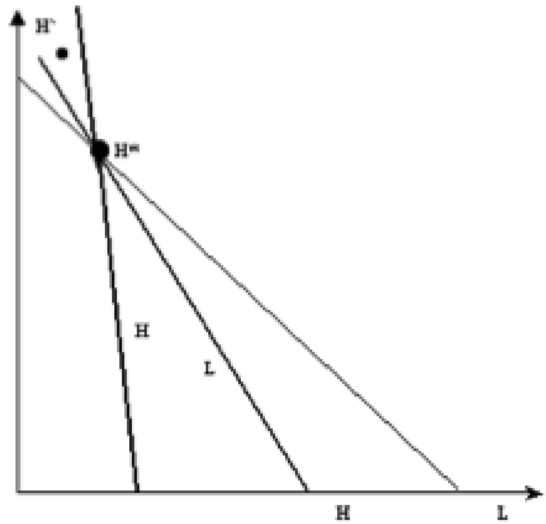


- Making profits from low type: symmetric reasoning applies. A bank can offer L' and capture all L and make positive profits.



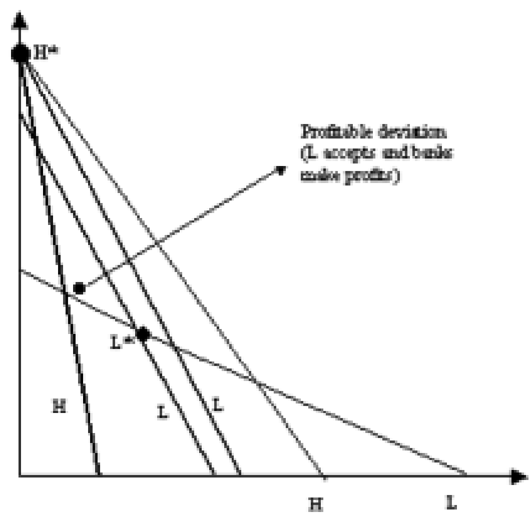
Lemma: High type allocation is $\left(\frac{1+\rho}{P_H}, 0\right)$ - distortion at the top.

Proof: Suppose not. We know that H contract must be on zero profit line. Suppose $C_H > 0$. Then, if H is the contract with $C_H > 0$, there is a profitable deviation to H' (clearly the L contract must be below U_L and therefore the L type will keep choosing her contract).

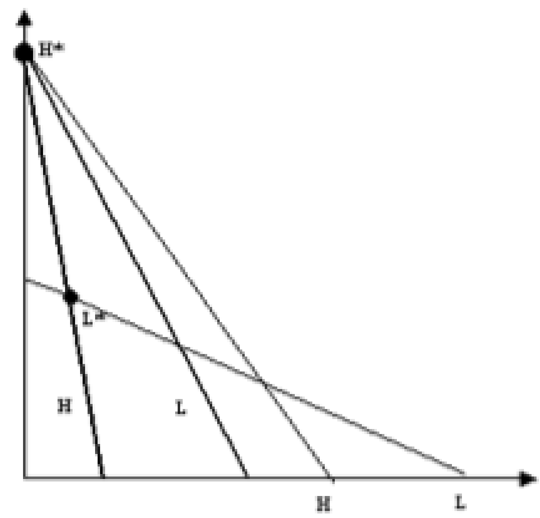


Lemma: H type will be indifferent.

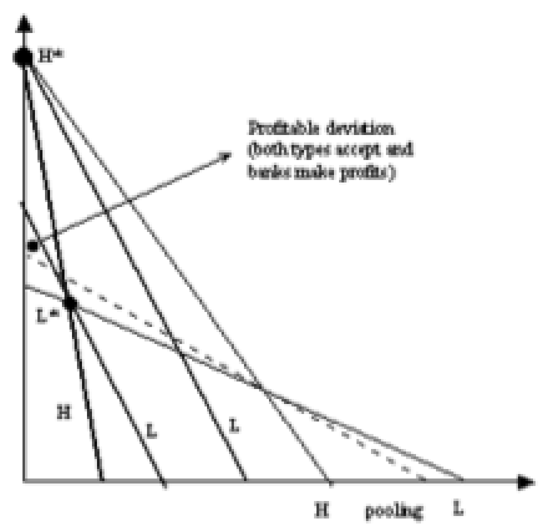
Proof: Suppose not. L^* must be on the zero profit line such that H doesn´t prefer L^* and L doesn´t prefer H^*. Then, if L were such that type H were not indifferent between H^* and L^*, we could find a profitable deviation:



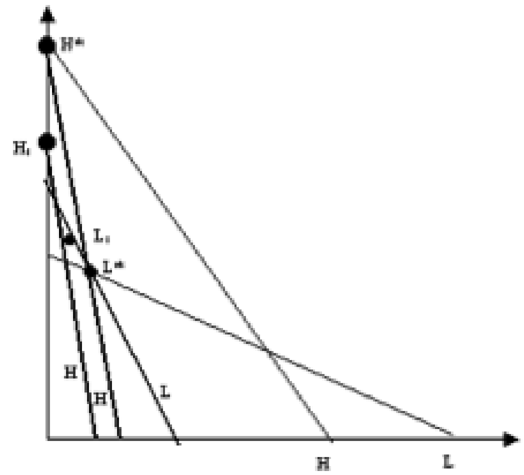
Separating equilibria:



If the proportion of low risk is high enough, the pooled profit line will be very close the L zero profit line and, therefore, we could have a pooled contract attracting both types and yielding positive profits, breaking the separating equilibria.



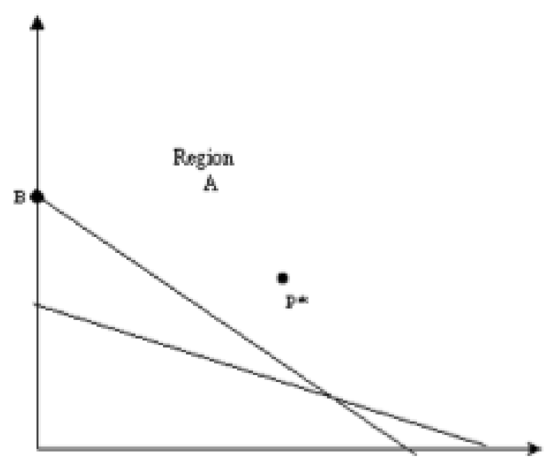
It is also possible that another pair of separating contract breaks the separating equilibria. Contract (H_1, L_1) may be a profitable deviation because we are losing money from H but gaining from L.



c) If a separating equilibrium exists, is it constrained Pareto efficient?

If such an equilibrium exists, it is constrained Pareto optimal (unless there is another pair of contracts which would give higher utility to both types and which would yield exactly zero profits). It is not, of course a Pareto optimum, since the first best is zero collateral for both.

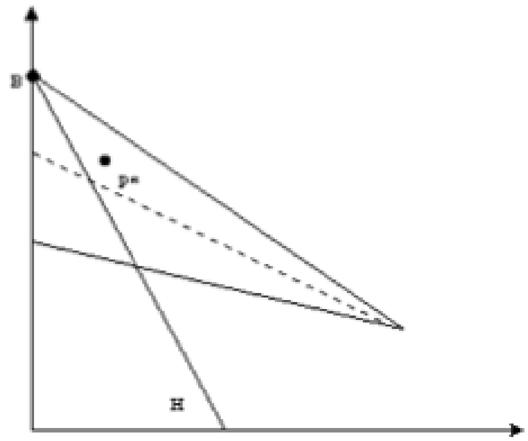
d) We now have a signaling model where the bank moves last and either accepts or rejects the offer of the entrepreneur.



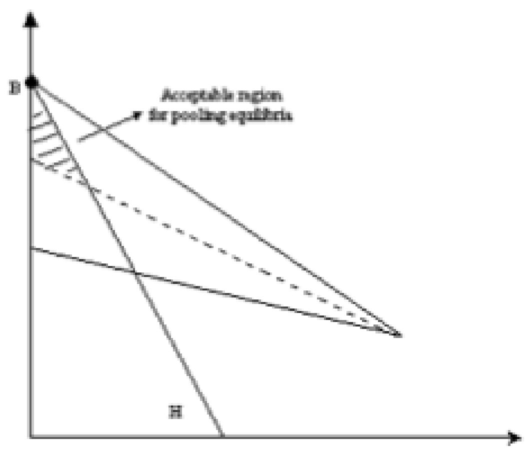
The bank will accept any offer in A. But then, for any point p^*, the H agent would rather propose B (and the bank would accept it, yielding a higher utility). So no point in A other than B is a candidate for a pooling equilibrium.

But if a pooling equilibria exists, it must be above the pooled zero profit line for the bank to accept.

Then, the only possible pooling equilibria are between these two lines:



But p^* cannot be a pooling equilibrium, since the H agents would rather propose B (the bank would accept) and get higher utility. So, the only possible pooling equlibria must be in the other area.

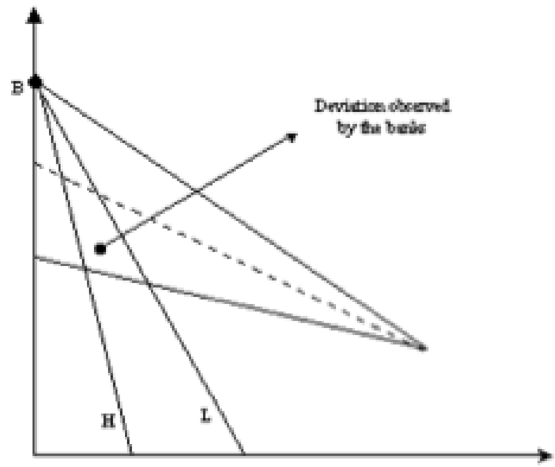


Both types play p^*

Banks' beliefs: $\begin{cases} P(\theta = H/\ p^*) = \lambda \\ \text{out of equilibrium - } P(\theta = H/\ otherwise) = 1 \end{cases}$

Banks' actions: $\begin{cases} \text{If } p^*\text{, gives credit (accept offer)} \\ \text{If } (r, C) \text{ - accepts above high break-even and rejects otherv} \end{cases}$

Intuitive criterion: banks must have reasonable beliefs.



If banks deviate, they give one of the types a worse outcome than in equilibrium:

- If high type deviates and bank rejects - gets 0;

bank accepts - also worse off.

Hence, it´s not high type (prior beliefs do not satisfy the intuitive criterion) who deviates.

So, $P(\theta = H) = 0$ in this region: and low type will deviate.