Microeconomics II
Spring 2023
Midterm Exam

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You have a total of 120 minutes (2 hours) to solve the exam.
Identify each sheet with your Student Number and Name.
Good luck!

I (3.5 points)

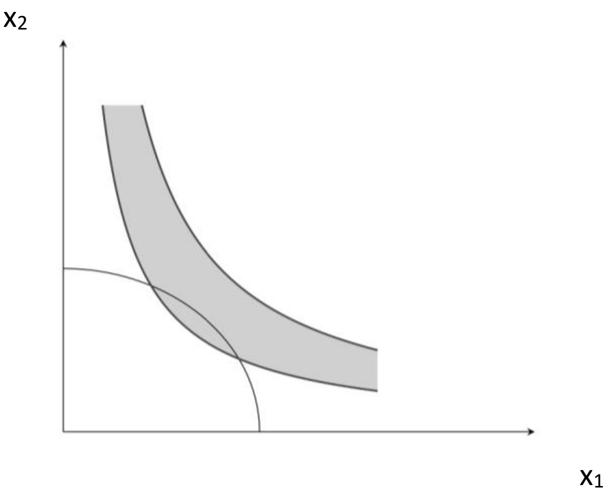
In the context of an economy with a constant returns to scale technology, state and prove the First Welfare Theorem.

Grading: 1 point for correct statement, 2.5 for correct proof.

II (2 points)

Exhibit a one-firm, one-consumer economy (with two goods) in which the production set is convex, the preference relation is continuous and convex, and there is nevertheless a Pareto optimal allocation that can be supported neither as a price equilibrium with transfers nor as a price quasi-equilibrium with transfers. Which condition of the Second Fundamental Theorem of Welfare Economics fails?

Without local nonsatiation, efficiency is not longer ensured. In the graph, there is a thick indifference curve and interior points of the production possibility set may now be Pareto optimal – but never achievable as walrasian equilibria (profit maximization, in particular, would fail).



Grading: 1 for LNS, 1 for example

III (6 points)

Consider a pure exchange economy with two goods, x and y, and three agents: 1, 2, and 3. The respective endowments are: $\omega_1 = (0, \bar{y})$, $\omega_2 = (0,0)$ and $\omega_3 = (0,0)$.

Agents 2 and 3 also own identical firms with a constant returns to scale technology, where $x = -2y$.
The preferences of agent 1 are represented by the utility function $U_1(x_1, y_1) = 2\sqrt{x_1} + y_1$.
The preferences of both agents 2 and 3 are represented by the utility function $U_i(x_i, y_i) = y_i$

(a) Find the walrasian equilibrium for this economy.

As we saw in the example from class, the CRS technology together with profit maximization means that equilibrium can only hold if p=1/2.
Since 2 and 3 have 0 income, both consume zero of all goods. Agent 1 will consume $x_1 = 4$ and $y_1 = \bar{y} - 2$. Feasibility then implies $x_P = 4$ and $y_P = -2$

Grading: 1 for argument and equilibrium price, 1 for consumption vectors, 0.5 for production plan, 0.5 for conclusion.

(b) Show that the equilibrium allocation coincides with the core for the economy.

There are three individual coalitions, three 2-agent coalitions and one 3-agent coalition.
From the individual coalitions, we get that $U_1 \geq \bar{y}$ and $U_2 \geq 0$ and $U_3 \geq 0$

2-agent coalitions

- 1 and *i, i=2 or i=3*

  $\max 2\sqrt{x_1} + y_1$

  s.t. $y_i = \overline{u_i}$

  $x_P = -2y_P$

  $x_1 = x_P$

  $y_1 + y_i = \bar{y} + y_P$

  Yielding $x_1 = 4$, $y_1 + y_i = \bar{y} - 2$ (and $x_P = 4$ and $y_P = -2$)
- 2 and 3 can never get more than 0 utility.

3-agent coalition

At any Pareto optimal allocation, $x_2 = x_3 = 0$ (otherwise 1 could improve with more of this good without hurting 2 and 3).
Pareto optimal allocations will be the solution to:

$\max 2\sqrt{x_1} + y_1$

s.t. $y_2 = \overline{u_2}$

$y_3 = \overline{u_3}$

$x_P = -2y_P$

$x_1 = x_P$

$y_1 + y_2 + y_3 = \bar{y} + y_P$

Yielding $x_1 = 4$, $y_1 + y_2 + y_3 = \bar{y} - 2$ (and $x_P = 4$ and $y_P = -2$)

But from the 2-agent coalitions of 1 and *i*, we get $y_1 + y_2 \geq \bar{y} - 2$ and $y_1 + y_3 \geq \bar{y} - 2$. Together with $y_1 + y_2 + y_3 = \bar{y} - 2$, it must then be that $y_3 = y_2 = 0$ (they cannot be negative due to the one-person coalitions). And therefore $y_1 = \bar{y} - 2$ and this coincides with the equilibrium allocation.

Grading: 0.5 for notion of core, 1 for Pareto efficient, 1 for 3-agent coalitions, 0.5 for 1-agent coalitions

IV (4.5 points)

a) Specify preferences and endowments for a pure-exchange economy with 2 agents, 2 goods and 2 states at date 1 (and assume there is no utility derived from consumption at date 0, utility is achieved only from the consumption of goods at date 1). Find the Arrow-Debreu equilibrium for this economy, with trade occurring at date 0.
b) What would be an equivalent rational expectations equilibrium with Arrow securities?

Grading: 3 for correct A-D equilibrium, 1.5 for equivalent Radner equilibrium.

V (4 points)

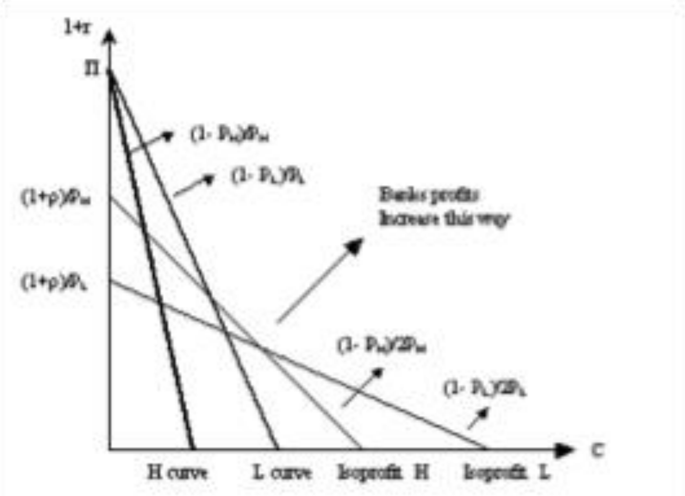
There are two types of risk-neutral entrepreneurs, Low risk ($\theta = L$) and High risk ($\theta = H$). The proportion of type H agents is λ. Each entrepreneur needs to borrow one dollar to undertake a risky project.

The outcome (*success* or *failure*) of a project is publicly observable *ex post*, but types are private information: only the entrepreneur herself knows her own type θ. The probability of success for type θ is p_θ where $0 < p_H < p_L < 1$. If the project succeeds, the profit is $\Pi > 0$; if it fails, the project yields zero.

There are two risk-neutral profit-maximizing banks. A bank's cost of funds is $1+\rho > 1$ (the supply of deposits is perfectly elastic at deposit interest rate $\rho > 0$). Assume $p_H\Pi > 1+\rho$.

A *credit contract* between a bank and an entrepreneur specifies a loan interest rate r and an amount of collateral $C \geq 0$. If the project succeeds, the entrepreneur must pay the bank $1+r$ but gets back the collateral; if the project fails, the entrepreneur pays nothing but the bank keeps the collateral. Liquidating the collateral is costly for the bank, so the value of the collateral to the bank is only $C/2$. This implies that if the bank signs a credit contract (r,C) with an entrepreneur of type θ, the bank's expected profit from this transaction is $p_\theta(1+r) + (1-p_\theta)C/2 - (1+\rho)$ and the entrepreneur expects to get $p_\theta[\Pi - (1+r)] - (1-p_\theta)C$.
Banks compete by simultaneously proposing credit contracts (as many as they want). Then, the entrepreneurs decide which contract (at most one!) to accept.

a) Show that no pooling equilibrium exists (Hint: use a diagram in the (C, 1+r) space and first make sure you know how to map indifference curves and profit lines).

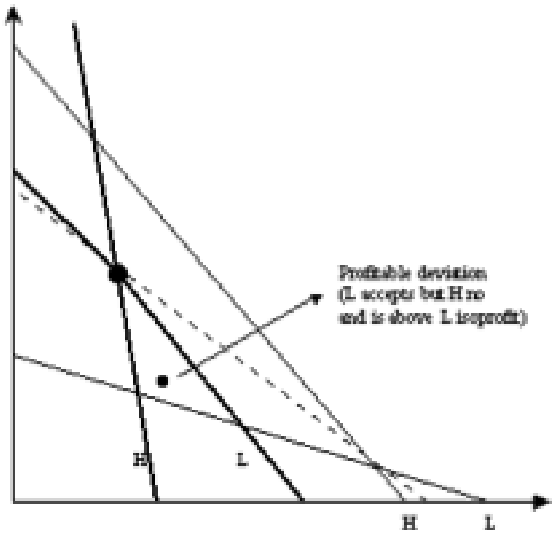


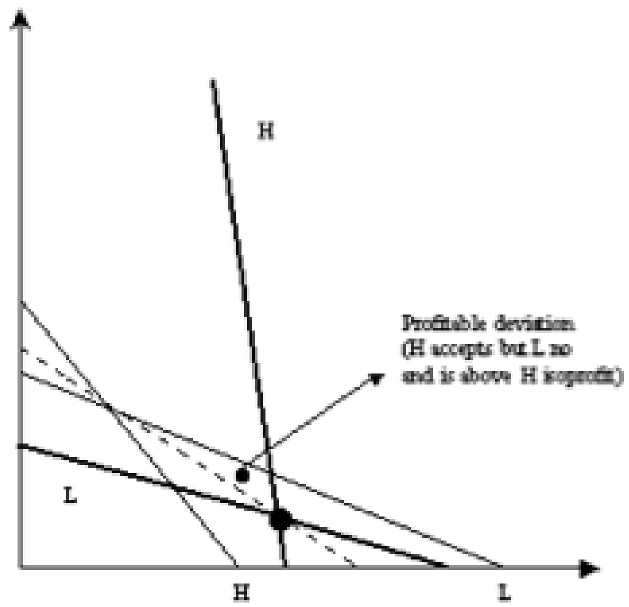
Lemma: In any equilibrium, banks make zero profits.

Proof: Suppose not: (r_L, C_L) and (r_H, C_H) are the equilibrium contracts with aggregate profits Π. The other bank could deviate and get $(r_L - \varepsilon, C_L)$ and $(r_H - \varepsilon, C_H)$. If (r_L, C_L) and (r_H, C_H) are incentive compatible, so are $(r_L - \varepsilon, C_L)$ and $(r_H - \varepsilon, C_H)$. Deviating bank makes profits close to Π.

Lemma: No pooling equilibria can exist.

Proof: Suppose not.





$\lambda\left[P_H(1+r)+(1-P_H)\frac{C}{2}\right]+(1-\lambda)\left[P_L(1+r)+(1-P_L)\frac{C}{2}\right]=1+\rho.$

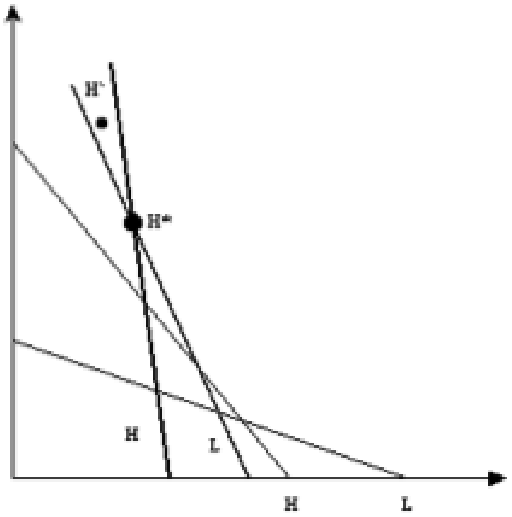
With $\overline{P}=\lambda P_H+(1-\lambda)P_L$ comes $\overline{P}(1+r)+(1-\overline{P})\frac{C}{2}=1+\rho$ (pooling break-even line).

a) Show what a separating equilibrium looks like, if it exists. Explain why it is possible that no separating equilibrium exists.

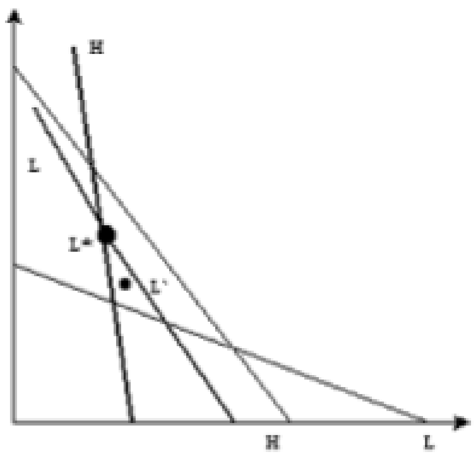
Lemma: In a separating equilibrium banks make zero profits from each type.

Proof: Assume not:

- Making positive profits from high type: if the initial contract for H is above the zero profit line, we can always draw a contract H' such that the H type would rather take it, but the L type will choose her initial contract. Then, we can capture all the H-types and with H' close enough to H, profits will be positive.

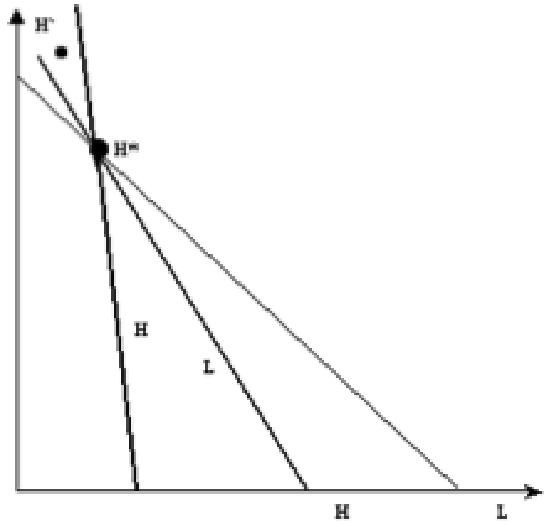


- Making profits from low type: symmetric reasoning applies. A bank can offer L' and capture all L and make positive profits.



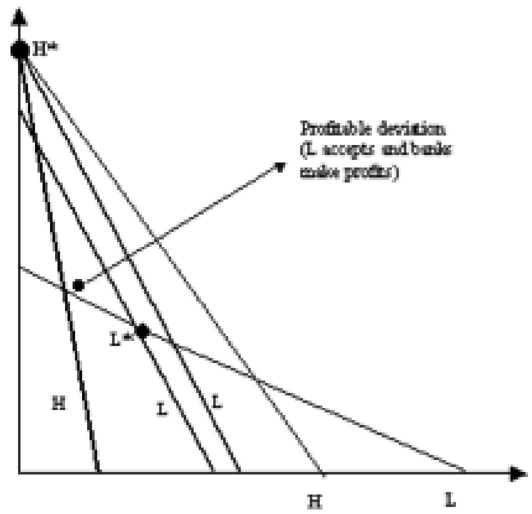
Lemma: High type allocation is $\left(\frac{1+\rho}{P_H}, 0\right)$ - distortion at the top.

Proof: Suppose not. We know that H contract must be on zero profit line. Suppose $C_H > 0$. Then, if H is the contract with $C_H > 0$, there is a profitable deviation to H' (clearly the L contract must be below U_L and therefore the L type will keep choosing her contract).

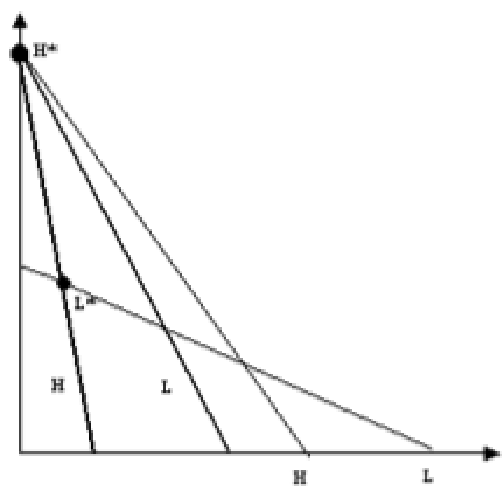


Lemma: H type will be indifferent.

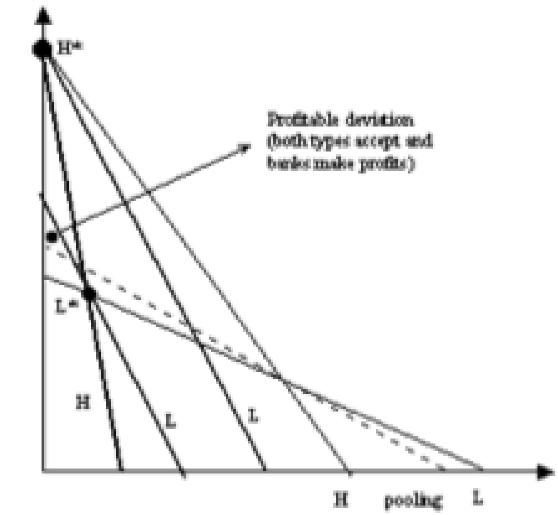
Proof: Suppose not. L^* must be on the zero profit line such that H doesn´t prefer L^* and L doesn´t prefer H^*. Then, if L were such that type H were not indifferent between H^* and L^*, we could find a profitable deviation:



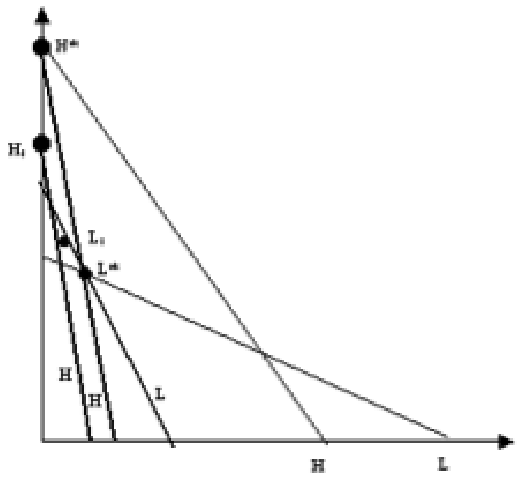
Separating equilibria:



If the proportion of low risk is high enough, the pooled profit line will be very close the L zero profit line and, therefore, we could have a pooled contract attracting both types and yielding positive profits, breaking the separating equilibria.



It is also possible that another pair of separating contract breaks the separating equilibria. Contract (H_1, L_1) may be a profitable deviation because we are losing money from H but gaining from L.



Grading: 1.5 for plotting of indifference curves and isoprofit lines, 1 for pooling, 1.5 for separating