**Rules (Please read carefully!)**

We kindly ask you to direct any questions you may have first to the Moodle forum for this assignment. We will regularly check and answer in the Moodle forum, so that everyone has the exact same information set.

* Upload in Moodle .pdf file before Sunday May 11 midnight.
* Written document for each group of four or five students may not exceed five pages (12pt font; 1.5 line spacing). This is excluding title page and any introduction. The tables and graphs are to be collected in an appendix. The tables should be numbered and have a short heading, for instance:

|  |
| --- |
| **Table 1: This table gives an example of what a table should look like. It reports the price and YTM of two bonds, Bond 1 and Bond 2.** |
|  | *Bond 1* | *Bond 2* |
| *Price* | 1 | 2 |
| *YTM* | 2 | 3 |

* These rules are strict. The grader will only take into account the first five pages of answers to the questions asked below and will consider only correctly referenced and formatted tables.
* Upload an Excel file or clearly commented code from another program with your calculations in Moodle.
* This assignment counts towards 15% of your final grade!

**Assignment and questions:**

You are thinking of setting up a fixed income fund. To construct your portfolio, you need to perform a detailed analysis of the available investment opportunities in both treasury and corporate bond markets.

1. Let us start with treasuries. Use the data reported in the sheet Yield curve. These are yields on zero coupon Treasury bonds (STRIPS) with maturities ranging from 1 month to 30 years at the end of December 2023 and at the end of December 2024. Using interpolation, graph the Treasury yield curve on both dates as well as the 12-month changes in these yields. Describe how the shape of the yield curve changed and briefly discuss the underlying reasons why.
2. Consider the five annual maturities $m\_{1}:m\_{5}$ assigned to your group as provided in the Excel file. For each of these maturities, calculate the one-year ahead forward rate from the point of view of the end of December 2023. To be specific, the forward rate for maturity of k years in December 2023 is the rate on a forward contract that invests 1$ in December 2024 and pays (1+f\_1,k+1)^k $ k years after December 2024. Compare these forward rates to the realized spot rates for these maturities on December 2024. Did the forward rates in December 2023 predict spot rates in December 2024 well? If not, what drives the difference?

Now suppose it is the end of December 2024 and you believe that high yield corporate bonds are severely underpriced. For this reason, the general idea of your fund is to go long in an index of high yield bonds and to hedge out the interest rate risk by taking offsetting positions in Treasury STRIPS. The index reinvests all proceeds from bonds that mature in new (but otherwise identical) bonds and pays out all coupons as dividends. You will invest a total of 1 billion $ in the high yield index.

1. Suppose that the expected dividend derived from the coupons of the bonds in the index is 0.06$ annually per 1$ of par value. Assuming that the discount rate for high yield bonds is equal to X, where X is $6\%+0.3\%\*median(m\_{1},m\_{2},m\_{3},m\_{4},m\_{5})$ for all maturities, what is the price per 1$ of par value? Using the interpolated treasury rates, calculate also the price per 1$ of par value if the dividends were considered risk-free and compare.
2. Calculate the duration and convexity of the high yield index. Discuss what they mean. To answer this question, be careful to account for the fact that your investment of 1 billion $ translates to more than 1 billion $ in par value.
3. To hedge the interest rate risk of your 1 billion $ investment in the index, you decide to use the two STRIPS with the shortest and longest maturity (among the five maturities assigned to your group). Discuss how much you need to buy or sell of these STRIPS to be insulated from interest rate changes up to a second order approximation. For your discussion, create also a plot that shows (i) the realized change in the value of the high yield index and (ii) the realized change in the value of the hedged position (that is, combining the change in value of the 1$ billion invested in the high yield index with the change in value of the offsetting position in the two STRIPS) when discount rates for all maturities and all bonds change by -2%, -1.5%, -1%,-0.5%, 0%, 0.5%, 1%, 1.5%, 2%.
4. Instead of hedging the interest rate risk by taking offsetting positions in STRIPS, you could also buy puts on the index. Suppose a broker offers you a European put contract on the 1 billion $ high yield index investment. The put option has maturity equal to 1 year and strike price K equal to 1$ billion \* (1-X%) where X is the median maturity of the five STRIPS assigned to your group. Use the Black-Scholes model to estimate the price of the put assuming the volatility of the index is 13% annualized.
5. Suppose the market price of the put option is higher than what you calculated in 6., is there an arbitrage opportunity? Regardless of your answer to this question, suppose the put is overpriced in the market, what positions must an arbitrage strategy take to take advantage of this fact?
6. Discuss the key differences between the hedging strategies from questions 5. (STRIPS) and 6. (put).
7. To reduce the cost of the put from 6., you ask the broker for a quote for a put with a knock-out feature. To price the put option with this feature, use a binomial tree approximation with up and down movements in the underlying stock price equal to $u=e^{σ\sqrt{t}} and d=e^{-σ\sqrt{t}}$, respectively, over each quarterly subperiod (i.e., t=1/4). Moreover, assume that one-fourth of the dividend is paid out at the end of every quarter. More specifically, the knock-out feature entails that if the price of the bond index investment falls three consecutive times, the option payoffs are zero. Hence, the price of the put option with this feature is zero in the down-down-down-state. What is the price of the put option with and without this feature and under what circumstances would you be interested in having the knock-out feature?