

**Exercise Set 9**

1. An index is currently trading at 25$. At-the-money call and put options of the index, with maturity of 1-year, are available at 3$ and 1$, respectively. You have an account with 10,000$. Draw the profit graph of investing the entire account:
   1. solemnly buying the index itself;
   2. solemnly buying call options;
2. What is the value of the account, from question 1, in one year if:

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| --- | --- | --- | --- | --- | --- |
| Index Price  Strategy | 15$ | 20$ | 25$ | 30$ | 35$ |
| a |  |  |  |  |  |
| b |  |  |  |  |  |
| c |  |  |  |  |  |

* 1. What is the return you got in that year?

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| --- | --- | --- | --- | --- | --- |
| Index Price  Strategy | 15$ | 20$ | 25$ | 30$ | 35$ |
| a |  |  |  |  |  |
| b |  |  |  |  |  |
| c |  |  |  |  |  |

1. Consider an at-the-money European put option on a stock with current price of $50 and 3 months maturity. The evolution of the stock price over the course of the next 3 months follows a binomial distribution, each month the stock either goes up by 20% or down by 20%. The continuously-compounded risk-free rate is 3% per annum. In the replicating portfolio of the put, what is the number of shares? (two decimal places, negative sign for short positions)
2. Consider the same setup as in the previous question. What is the value of the at-the-money call option? (two decimal places)
3. Consider an American call option with a strike of 4 on a stock with current price of 4.20 and 4 months to maturity. The stock pays a 30c dividend in 4 months and has annualized volatility of 30%. The continuously-compounded risk-free rate is 2.5% per annum. Using a two-step binomial model, what is the value of the option? (two decimal places).

1. A stock currently trading at 10$ has a daily volatility of 1%. The stock will pay a 1$ dividend in 80 trading days (assume that the stock will go ex-dividend at the close of trading on the 80th day, such that the American option holder can exercise just before and receive the dividend). The risk-free interest rate is 3%. Assume the year has 240 trading days. Using a three-step binomial model:
   1. Calculate the price of a European call option with strike of 10$ and maturity of one year;
   2. Calculate point 6.a. using the Black-Scholes formula;
   3. Calculate the price of an American call option with strike of 10$ and maturity of one year.
2. Price a European call-option on a stock index with the following characteristics: the index is currently trading at 39$; the exercise date is in 3 months; annual volatility is expected to be 18%; a 3-months T-bill is currently trading at 993.77$; and all the assumptions for the correct application of Black-Scholes model hold.
   1. Price a European call option on the index, with a strike of 40$, knowing that it pays no dividends;
   2. Price a European put option on the index, with a strike of 40$, assuming that the index has a continuously compounded dividend yield of 2% a year.
3. A stock, currently valued at 15$, will either rise in price by 3$ or fall by 3$, within the next year. The current risk-free rate is 2% and the market risk premium is 4%. The probability of the stock price going up is 70%, and falling is 30%. Using a one-step binomial model, price a European put option currently trading at-the-money?
4. Use the following information to answer the next 6 problems. Nova Inc. stock is currently trading at $60. The continuously compounded risk-free rate is 4.5%. The binomial factor *u* for a period of 1-month, which represents a movement up for the stock price over the next month, is equal to 1.021. Nova Inc. is paying a dividend of $2.5 in exactly one month.
   1. In the context of the Binomial Option Pricing Model, what is the Nova Inc. stock volatility implied by the factor *u*? Find the corresponding binomial factor *d*.

There is a European call option on Nova Inc. stock trading on the market. The strike price of this option is $55. The expiration of the option is in two months.

* 1. What is the lower bound for the value *c* of this call option?
  2. Find the exact value *c* for this option using the Binomial Option Pricing Model.
  3. There is also an American option written on Nova Inc. stock, with the same strike price. What is the price *C* of this option? Is it different than the price *c* of its European counterpart? If yes, explain why you observe this difference.

Suppose that you can also find European put options on Nova Inc. stock, with the same strike price and time to maturity of the call option you priced in point 3. Furthermore, suppose that you possess private information that make your expectations about Nova Inc. stock volatility exceed the market volatility estimates implied by option prices.

* 1. If you wish to take advantage of your private information, should you enter a short or a long *straddle*? What would be the initial cost of your strategy using European options?
  2. Draw the graph showing the *payoff* of your strategy. For what levels of the Nova Inc. stock price in 2 months your *profit* is positive?

1. Consider an American put option with maturity of 1 year on a non-dividend paying stock that is currently trading at $60. The put has a strike of $70, the risk-free interest rate is 5%, and the volatility of the stock is 30%. Compute the value of the put according to a two-period binomial model.