Investments

Masters in Finance

NOVA SCHOOL OF BUSINESS & ECONOMICS

Spring 2025, Martijn Boons

Book Chapter 10

Beta is dead!

- The CAPM is rejected in the data:
  - Huge body of work shows that the CAPM in its basic form does not explain well variation across assets in average returns:

    Corr(average returns, market betas) << 1

- Let us consider some of the most powerful evidence to this conclusion from Fama and French (1992)

- Sort stocks (at the end of month t) separately on two characteristics that are known to generate a spread in average returns (in month t+1)

1. Size (Market Cap): Small firms historically outperformed big firms. Why?
   - Perhaps small firms are less diversified?
   - But, investors diversify by investing in many stocks…

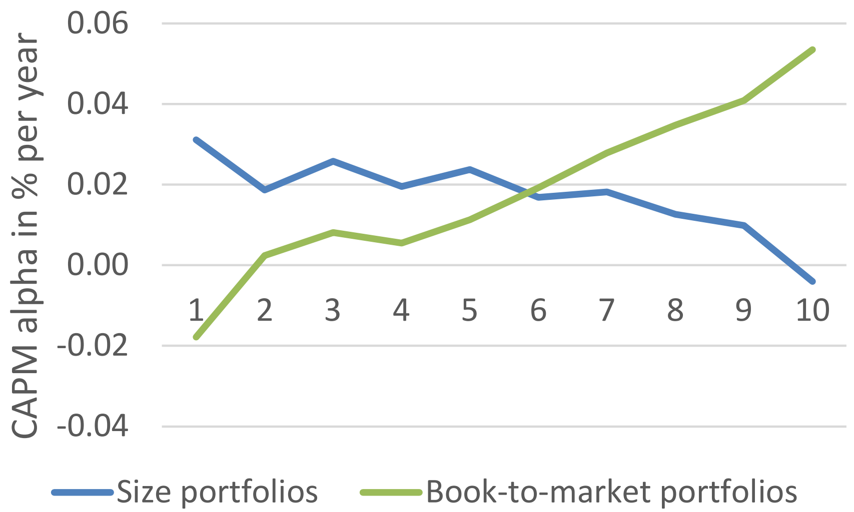
2. BM (book-to-market): High book-to-market firms (Value) historically outperformed low book-to-market firms (Growth). Why?
   - When market value is low relative to book value (of firm's projects), a large discount rate must have been applied.
   - Is this larger discount rate appropriate for the firm's risk or mispricing?

Fama and French (1992) Reject CAPM

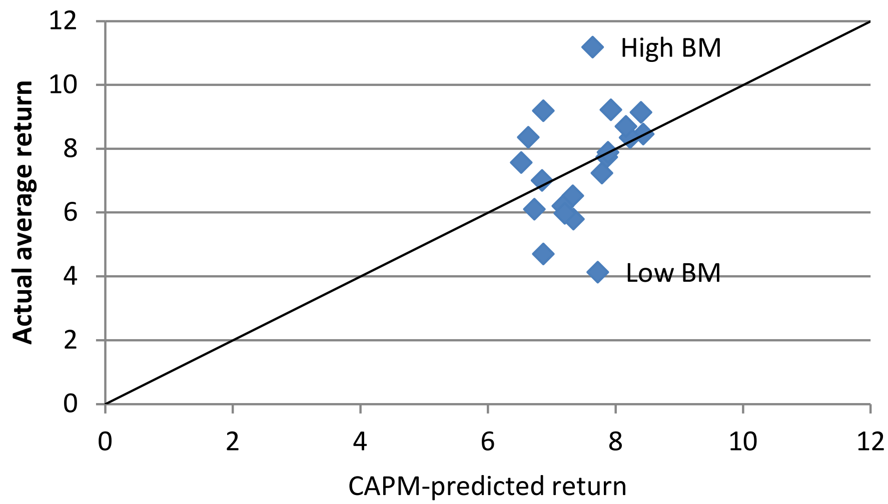
- Excel: excess returns on 2x10 value-weighted portfolios from 1961 to 2010. (Market portfolio=value-weighted portfolio of all US stocks and r_F=1 month t-bill return.)

1. Time series: CAPM alpha almost monotonically decreasing (increasing) in Size (BM), with large alpha of 3.5% (7.1%) for Small-Big (High-Low).
2. Cross-section: $\lambda_M = 5.8\%$, which is close to average excess market return, but R^2 still only 13% → CAPM beta explains very little variation in average returns across the portfolios.

TS: CAPM alphas



CS: CAPM-predicted vs average returns



The Fama-French three-factor model

- This evidence is commonly interpreted as meaning that there must be additional factors that investors care about:
  - Small-minus-Big size factor or SMB
  - High-minus-Low book-to-market factor or HML

- Formally, the Fama-French three factor model (FF3M) is written as:

$$E(r_i^e) = \beta_{i,M} E(r_M^e) + \beta_{i,SMB} E(r_{SMB}^e) + \beta_{i,HML} E(r_{HML}^e)$$

  - Compare to CAPM: $E(r_i^e) = \beta_{i,M} E(r_M^e)$

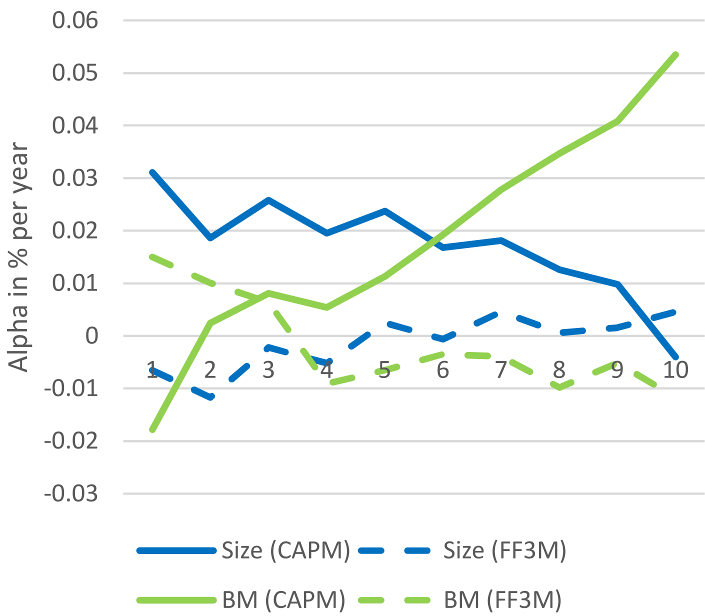
- FF3M often fares much better empirically explaining cross-sectional variation in average returns.

  - Consider, for instance, the size and book-to-market sorted portfolios:

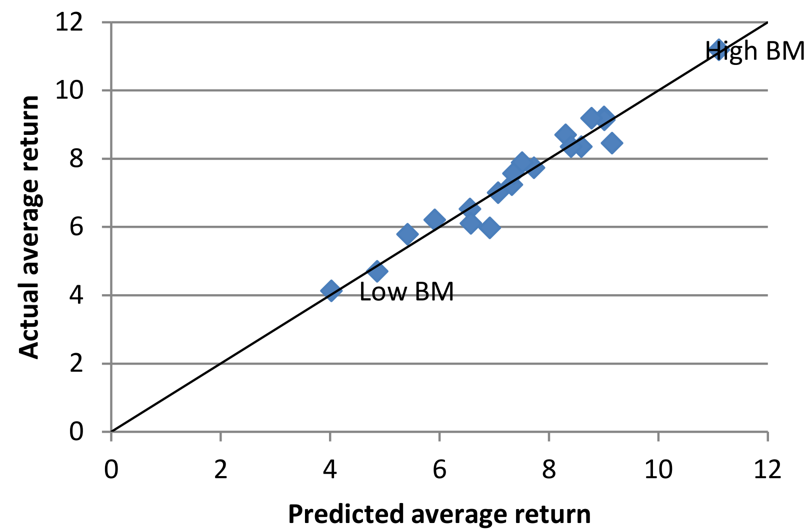
FF3M vs CAPM

1. Time series: small alphas for portfolios sorted on size, book-to-market
   – Same conclusion applies to portfolios sorted on many other characteristics (Fama and French (1996))
2. Cross-section: $\widehat{r^e_{i,t+1}} = \lambda_0 + \lambda_M \widehat{\beta_{i,M}} + \lambda_{SMB} \widehat{\beta_{i,SMB}} + \lambda_{HML} \widehat{\beta_{i,HML}} + a_i$
   – Exposure to SMB and HML factors improves R^2 dramatically (95% vs 13% before), i.e., SMB and HML betas explain lots of cross-sectional variation in average returns!

TS: CAPM vs FF3M alphas



CS: FF3M-predicted vs average returns



Size and book-to-market factors

- We have a better factor model: great!
- But, what risks do SMB and HML capture? How do these factors follow from investors' portfolio choices (like CAPM follows from observation that market=tangency)?
- Fama-French allude to interpretation that the new factors provide investors exposure to business cycle risk and therefore capture a risk premium (over and on top of market beta)
  - Small and High BM stocks have episodes with large losses that broadly coincide with bad times/recessions
  - Business cycles do not perfectly align with poor market returns

→ Thus, these stocks are systematically risky: Low P & High E(R)

Carhart's four factor model (FFCM)

- FFCM adds to the FF3M a momentum factor:

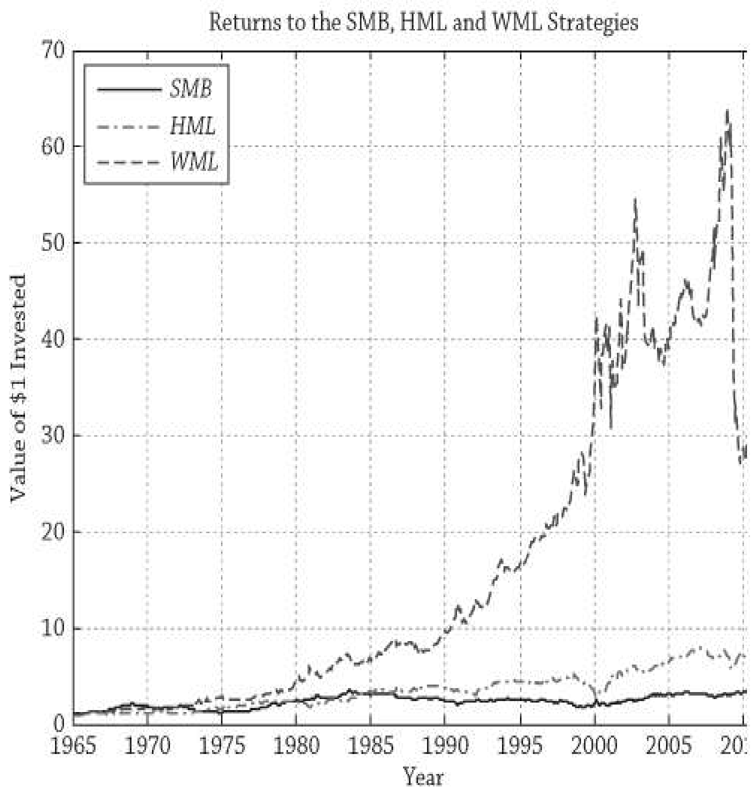
$$E(r_i^e) = \beta_{i,M}E(r_M^e) + \beta_{i,SMB}E(r_{SMB}^e) + \beta_{i,HML}E(r_{HML}^e)$$

$$+\beta_{i,WML}E(r_{WML}^e),$$

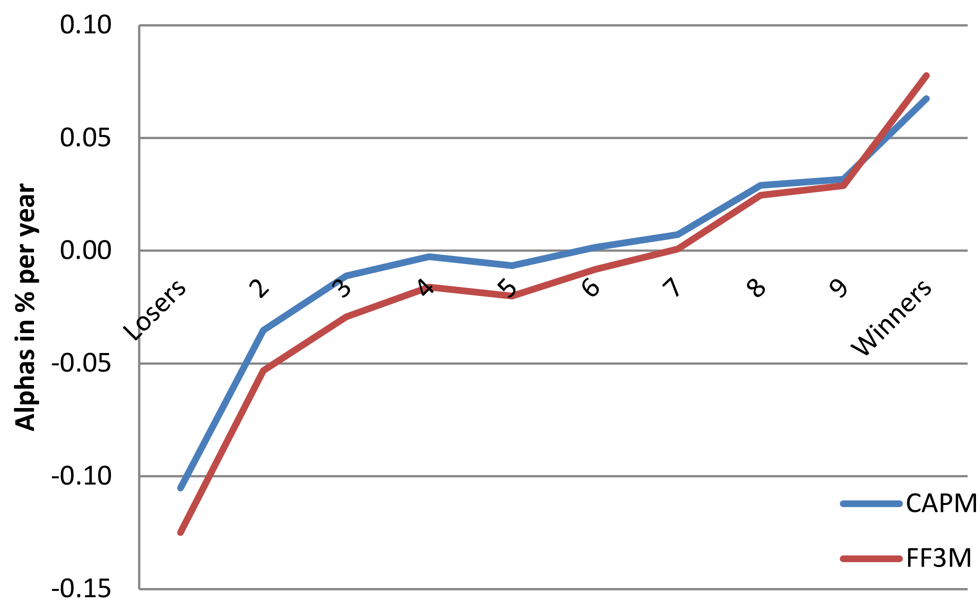
  - where WML is the difference in returns between winners (good performance over the last 12 months or so) and losers (bad recent performance)

- Where does this momentum factor come from?

Momentum returns are huge…



CAPM and FF3M alphas for Momentum portfolios sorted on past returns



Momentum returns are risky

- Momentum crashes: Of the eleven largest momentum crashes, seven occurred during the Great Depression in the 1930s, one occurred in 2001, and the other three occurred during the financial crisis
    - WML strategy lost > 50% (!) from March to April 2009
- Since momentum combines large average returns with business cycle risk, WML is a natural risk factor.
- Momentum strategies provide large on average, but risky returns in almost any asset class
    - "Value and Momentum Everywhere" (Asness, Moskowitz, Pedersen (2011))

On the origins of multifactor models

- Arbitrage Pricing Theory was developed by Steve Ross
- APT: If firm-specific risks are uncorrelated, they can be diversified away in large portfolios → Idiosyncratic risk should not be priced
- Factor risk cannot be avoided
  - Factors capture risks that determine bad times for a sufficiently large set of investors
    - Candidates: interest rate shocks, inflation shocks, oil price shocks, illiquidity shocks, monetary policy shocks, and so on.
    - Factor affect large set of assets through covariance
  - If these shocks do not perfectly align with market returns, factor portfolios must receive a risk premium
    - Factor portfolios are long stocks with high exposure to one or more of these risks and short stocks with low exposure
    - Long-short factor portfolios capture a non-zero CAPM alpha
      - Sign of the alpha depends on whether increase in the risk factor is good or bad news: inflation versus GDP news

APT return generating process

The APT posits that K factors explain all common or systematic variation in asset returns:

$$r_{i,t}^{e} = \underbrace{\beta_{i,1} f_{1,t} + \beta_{i,2}\, f_{2,t} + \cdots + \beta_{i,K}\, f_{K,t}}_{\text{Factor risk: cannot be diversified away}} + \underbrace{\varepsilon_{i,t}}_{\text{Firm-specific risk}}$$

Consequently, exposure to each of the K factors is priced in equilibrium:

$$E(r_i^e) = \beta_{i,1}\lambda_1 + \beta_{i,2}\,\lambda_2 + \cdots + \beta_{i,K}\,\lambda_K$$

where λ_k for $k = 1{:}K$ are risk premia. When the factors are traded, each $\lambda_k = E(f_{k,t})$, i.e., the expected return of the factor portfolio. Thus, we get:

$$E(r_i^e) = \beta_{i,1} E(f_{1,t}) + \beta_{i,2}\, E(f_{2,t}) + \cdots + \beta_{i,k}\, E(f_{k,t}).$$

The non-traded factor case is not so relevant practically, because you can always convert a non-traded factor in a traded factor by projecting the non-traded factor on the asset space.

APT vs CAPM

- A security's expected return depends on how risky its payments are
  - The security is attractive if it provides high returns in bad times
  - You are prepared to accept low expected return on these securities
  - Like paying a premium for insurance!

→ CAPM: bad times are when the market return is low

→ APT: there is more to bad times than just low market returns

- The CAPM is a special case of the APT, where the first and only factor $f_{1,t}$ is the market portfolio
- With more factors, the APT is an extension of the CAPM
  - APT has same uses as CAPM: valuation of projects and assets, portfolio choice, etc.
- In applications of APT, first factor is always assumed to be a market portfolio.
  - CAPM requires knowledge of the inherently unobservable true market portfolio of all assets.
  - APT doesn't require that. Rather, APT requires a market portfolio that explains lots of common variation across assets.
    - First principal component of stock returns in the data is highly correlated to the S&P500
    - Principal component: a statistical portfolio of all stocks that explains as much of the common variation in returns as possible

Overview

- Non-diversifiable factor risk is priced, idiosyncratic risk is not, e.g., a two-factor APT:

$$r_{i,t} = \beta_{i,M} r_{M,t} + \beta_{i,2} f_{2,t} + \varepsilon_{i,t}$$

- $f_{2,t}$ captures risk that is not perfect aligned with the market
  - E.g., business cycle risk: bad times for many investors (with houses, small businesses, human capital etc) are recessions, while market returns can be low in expansions too

- Think of $f_{2,t}$ as the return of a portfolio long stocks that comove strongly with the business cycle ($\beta_{i,2}$ high) and short stocks that comove weakly ($\beta_{i,2}$ low)
  - This long-short portfolio is risky, and thus captures a premium $\lambda_2 = E(f_{2,t}) > 0$

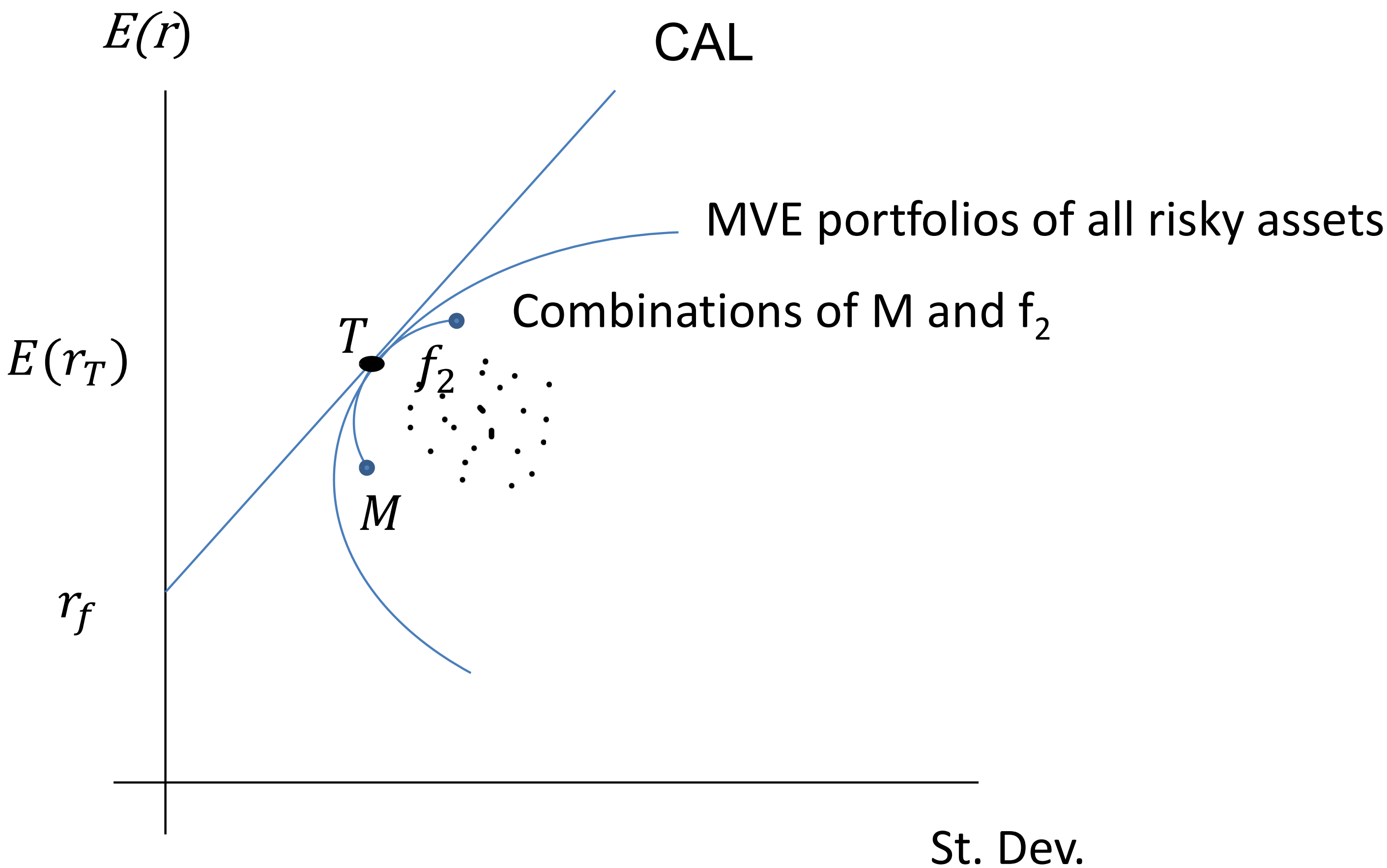
- Utility-based interpretation:
  - CAPM follows from max U=f(E(r_p), Var(r_p))
  - APT follows from max U=f(E(r_p), Var(r_p), Cov(r_p, r_{F2}))
- Relation to T:

  \+ − −

  - CAPM: $r_{T,t} = r_{M,t}$
  - APT: $r_{T,t}$ is a combination of $r_{M,t}$ and $f_{2,t}$ (aggregate demand for stocks differs from Tangency portfolio, because investors want to hedge against covariance with $f_{2,t}$)

Graphical illustration of a two-factor APT

$$r_{i,t} = \beta_{i,M} r_{M,t} + \beta_{i,2} f_{2,t} + \varepsilon_{i,t}$$



- Market portfolio is not efficient anymore (i.e., M≠T)
- Rather, M and f_2 can be combined to get T
  → This means that M and f_2 together price assets

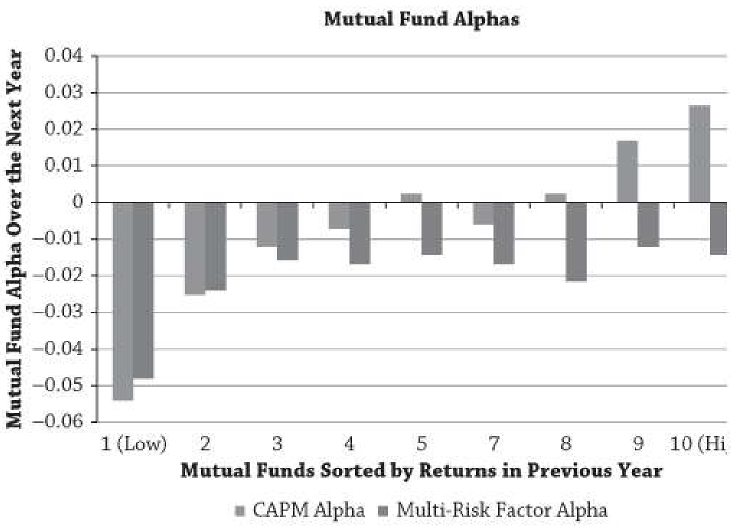
APT implementations

- Following Fama and French's preferred interpretation of the factors in their FF3M, empirical factors are routinely considered APT risk factors
  - Empirical factor returns comove with the risks that determine bad times for investors
    - SMB and HML correlate strongly with second and third principal component of stock returns in the data, so they do capture considerable comovement.
    - WML does not seem to capture as much comovement and Fama and French explicitly do not include it in their model...
      - They consider WML an anomaly (next topic)
  - Factors are long-short portfolios (excess returns): aggregated over all investors, they must still own the market portfolio
  - More recently, Fama and French have been advocating for a 5 factor model: FF3+profitability and investment factors

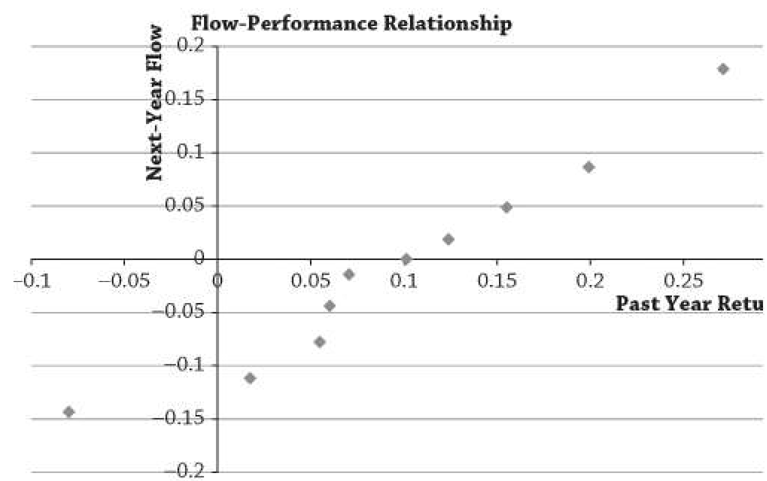
Application 1

- Perhaps unsurprisingly, multifactor models typically outperform the CAPM by a large margin in asset pricing tests, i.e., explaining (i) cross-sectional variation in historical average returns of portfolios and (ii) returns of mutual funds and hedge funds
  - Did the manager do something special or did he just load on something we knew about already?

- From Andrew Ang's Asset Management book:



1. Negative average alpha (CAPM and FFCM)
2. No persistence in high Carhart 4-factor alphas
3. Yet, investors chase returns:



The reality of active mutual fund performance

- Mutual funds advertise high past performance (in general)
  - Marketing: average return, Sharpe ratio, CAPM alpha, FF3 alpha,…?
  - Survivorship bias: Average return of live (covered by databases) minus dead (not covered) funds is about 4% per year!

- The reality: three stylized facts
  1. The typical active mutual fund delivers a negative alpha after costs, and, at best, a slightly positive alpha before costs
     – Large funds perform worst: alpha opportunities are hard to scale, as concentrated in small, illiquid, distressed stocks
  2. In addition, positive alpha is not persistent (in contrast to negative alpha)
  3. Yet, investors chase returns!

- What can explain these stylized facts?
  - low: Why are we paying fund managers so much through fees?
  - Interview with Jonathan Berk (https://www.youtube.com/watch?v=a41LhZE5Iec)

Intuition (Berk & Green, 2005; Berk & v. Binsbergen, 2015)

- A rational equilibrium model that fits these stylized facts:
  - Managers have differential talent
  - Decreasing returns to scale

- Investors chase returns: 1st-best historical return is most likely achieved by the most-skilled manager (with highest E(return))
  - This fund receives all new money flows from investors
  - Fund becomes bigger, and its E(return) will decrease until its equal to the 2nd-best E(return)
    - Now, new money will flow to the 1st- and 2nd-best fund and their E(return) decreases to 3rd-best fund … and so on …

- In equilibrium
  - E(return) on all funds are equal and investors will be indifferent between active and passive investing
  - Highly skilled managers do not obtain a positive alpha after fees, but will manage larger funds and earn higher fees
  - Skill is not measured by alpha after fees, but by

[alpha before fees * size of the fund]!

Application 2

- Are these multifactor models useful for valuation as well?

  – Let us apply the CAPM, FF3M and FFCM to calculate the forward-looking expected returns ($E(r_i)$, i.e., the cost of capital) of the 17 industries

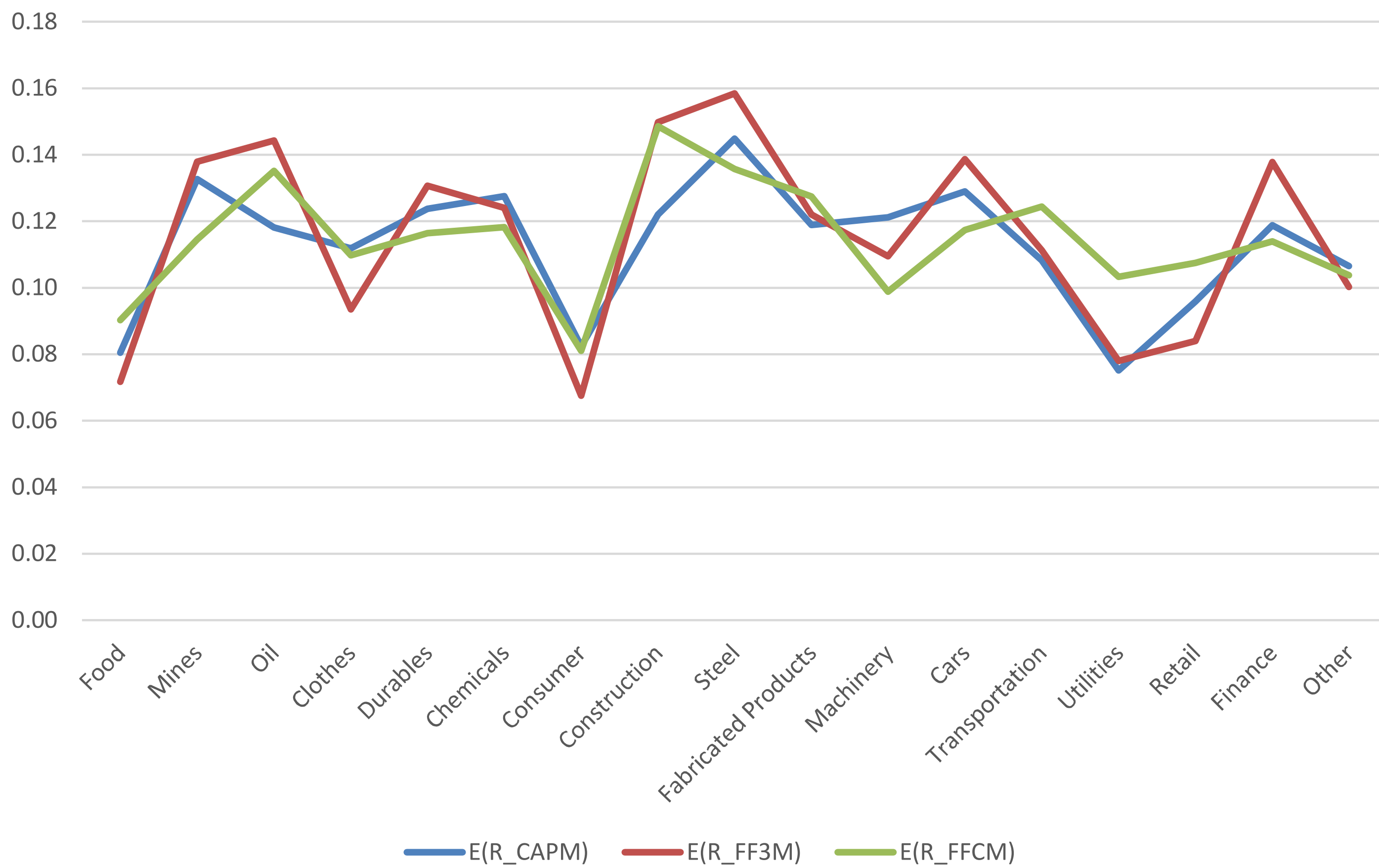
$$E(r_i^e) = \beta_{i,M} E(r_M^e) + \beta_{i,SMB} E(r_{SMB}^e) + \beta_{i,HML} E(r_{HML}^e) + \beta_{i,WML} E(r_{WML}^e)$$

  – β_{ik} : Estimate factor betas using a time series regression of returns on the factors using last 5 years of monthly data

  – E(f_k): Estimated using historical average returns of the factors

    - Assumption: factor portfolios have stable exposure to risk, such that risk premium is best estimated using as much historical data as possible.

Forward-looking expected returns of 17 industries



- Expected returns similar across the three models, suggesting that the CAPM may not be such a bad model to do valuation after all.
- Many analysts are working under this assumption!

Conclusions

- Our chosen application with well-diversified industry portfolios is quite friendly to the multi-factor models
  - Exposures
    - When the models are applied to individual stocks, additional noise will be introduced by having to estimate multiple betas over a recent window
  - Risk premia
    - We have used average excess factor returns over a long history as our estimates of factor risk premia. If you want to improve your forward-looking estimates of the factor risk premia, you will need to estimate them, which introduces additional noise.
  - As a result, forward-looking expected returns from multifactor models may be extreme and farther off from reality than a simpler model like CAPM.

Factor models (1/2)

- Factor models are extremely useful to estimate the variance-covariance matrix for many assets
  - This is a key input in portfolio optimization
  - Regardless of whether you believe factors are truly APT risk factors or just capturing mispricing (e.g., due to correlated trading and investor sentiment)

- Returns of assets have two components
  - *Systematic risk*
    - Small number of factors
    - Proxy for economic events (changes in interest rates, inflation, GDP growth)
    - Affect large numbers of assets
  - *Non-systematic risk*
    - Unique to each asset (new product innovations, changes in management, lawsuits, labor strikes, etc)
    - Uncorrelated across assets

- Equations that break down an asset's return into these two components are called *factor models*

Factor models (2/2)

$$r_i - r_f = \alpha_i + \beta_{i1} f_1 + \beta_{i2} f_2 + \ldots + \beta_{iK} f_K + e_i$$

where

- α: expected excess return when factors are zero
- *f*: common factors (1…K)
- β: sensitivities of assets to factors
- *e*: idiosyncratic (firm-specific) risk, uncorrelated with the factors and across firms, with E(*e*) = 0 → *shocks* or *surprises* about the firm

Portfolio risk

- Portfolio returns also follow the factor model

$$r_p - r_f = \alpha_p + \beta_{p1} f_1 + \beta_{p2} f_2 + \ldots + \beta_{pK} f_K + e_p$$

$$\alpha_p = \sum_{i=1}^{N} w_i \alpha_i \quad , \quad \beta_{pk} = \sum_{i=1}^{N} w_i \beta_{ik}$$

$$e_p = \sum_{i=1}^{N} w_i e_i \quad , \quad \sigma^2(e_p) = \sum_{i=1}^{N} w_i^2 \sigma^2(e_i)$$

$$\sigma_p^2 = \sum_{k=1}^{K} \sum_{l=1}^{K} \beta_{pl} \beta_{pk} \sigma\left(f_k, f_l\right) + \sigma^2(e_p)$$

- In matrix form: $\sigma_p^2 = \beta_p' \Sigma_f \beta_p + \sigma_{e_p}^2$ with

$$\beta_p = \left(\beta_{p1}, \ldots, \beta_{pK}\right)' \text{ and } \Sigma_f = \begin{pmatrix} \sigma_{f_1}^2 & \cdots & \sigma_{f_1, f_K} \\ \vdots & \ddots & \vdots \\ \sigma_{f_1, f_K} & \cdots & \sigma_{f_K}^2 \end{pmatrix}$$

- What is the idiosyncratic risk of the portfolio if it is well-diversified? 0
- What is the covariance between two portfolios *p* and *p**?

$$\beta_p{}' \Sigma_f \beta_{p*}$$

Factor models and MV analysis

- Even in the US, there are about 7,000 listed stocks

- MV analysis requires 24.5 million numbers
  - 7,000 variances
  - 24,496,500 covariances

- When $N>T$, there is a lot of estimation error and the covariance matrix is *ill conditioned* – it will blow up when inverted for portfolio optimization

- With K factors, you need fewer numbers $(7{,}000+K/2)(K+1))$
  - $7{,}000K$ betas
  - 7,000 residual variances
  - $K(K+1)/2$ factor variances and covariances

- More robust estimate of covariance matrix
  - All real-world risk management tools use a factor approach
  - E.g., Bloomberg factor model

Factor models and MV analysis

- Factor models are a good way to estimate the covariance matrix of stocks

$$\Omega = B\Sigma B^\top + U$$

B is an *N*×*K* matrix with the estimated betas of all stocks
Σ is the covariance matrix of the factors
U is a diagonal matrix with the variances of the epsilons

- The variance of a portfolio with a vector of weights *w* is then

$$\sigma_p^2 = w^\top \Omega w$$

- This approach to estimating the covariance matrix of stocks has much less estimation error than the sample covariance matrix

- This is the basic building block of any risk management system

Example: Setup

- Construct the tangency portfolio of the 17 industry portfolios:

$$w_T = \frac{\Omega^{-1}\mu^e}{i'\Omega^{-1}\mu^e}$$

- We will use:

  - Realized variance-covariance matrix

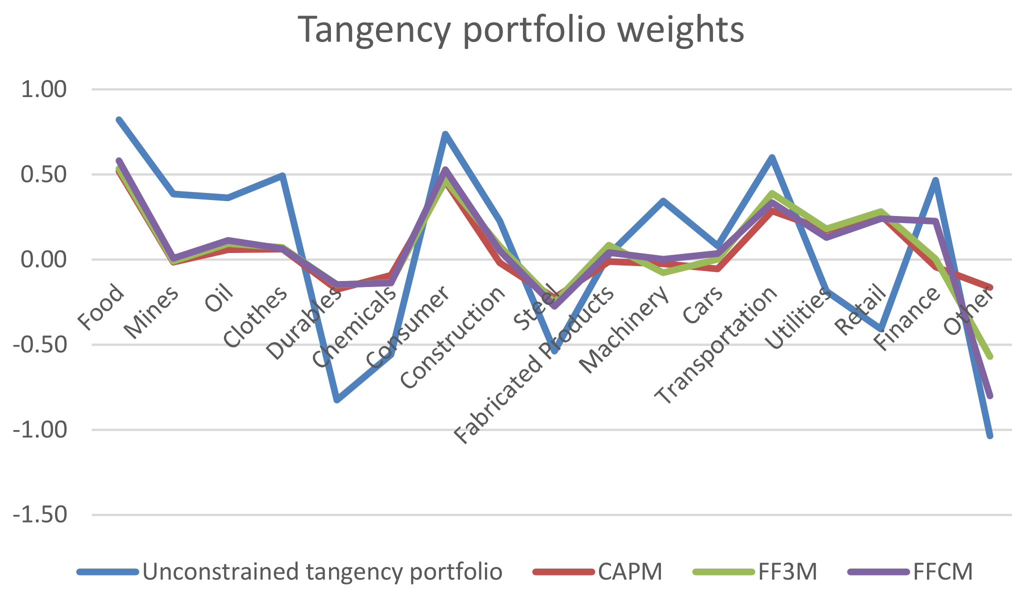
vs

  - Variance-covariance matrix estimated using a factor model
    - In Excel, the factor model approach is applied step by step for the FFCM

- For expected returns, we use the full sample historical average in the hope of dealing with noise. For (co-) variances, we use the last 60 months of returns to make sure they are timely.

- See Excel calculations!

Example: Results



- Results: Optimal weights are much more extreme for unconstrained approach than for factor-model approach
  - Among factor models, smaller models have less extreme weights.
  - Less extreme weights are cheaper to invest and tend to perform better in practice when subsequent portfolio performance is considered in an out-of-sample test.
    - This is what assignment analyzes with holdout sample!

Summing up

- Multifactor models useful to explain returns of (real-world) portfolios and popular for estimating portfolio risk:
  - given assets' betas wrt to the factors and a forward-looking estimate of the factors' variance-covariance matrix, you can predict portfolio risk.

- CAPM more popular for valuation and capital budgeting, because it relies on fewer inputs
  - Risk premia: What is the forward-looking expected return of factors like SMB, HML, and WML?
    - If risk: past average return is a good indicator.
    - If mispricing: the past is not a good indicator, because mispricing will be corrected.
  - Exposures: multifactor betas are harder to estimate and much less persistent (or predictable) than market betas
    - a firm that has a high-book-to-market ratio now will not likely have a high book-to-market ratio 5 years from now.
    - As a result, one should NOT discount all future cash flows at the higher rate applied to high book-to-market firms today.
  - The appropriate discount rate for firms' long-term cash flows is probably close to the market risk premium (see)
    - This is the assumption maintained by the vast majority of equity analysts.

Exercise

	Risk premium				
MKT	0.060				
SMB	0.030				
HML	0.050				
(Co-)variance matrix Σ_f	MKT	SMB	HML		Covariance of Stock A with each factor:
MKT	0.026	0.005	-0.004		0.031
SMB	0.005	0.011	0.000		0.006
HML	-0.004	0.000	0.011		-0.009

Consider the information above and assume the FF3M holds.

Q1: What is the CAPM alpha of stock A?
Hint: in a FF3M regression, the three betas can be calculated as: $\Sigma_f^{-1}\Sigma_{fA}$, where Σ_{fA} is the 3x1-vector [Cov(Stock A, MKT), Cov(Stock A, SMB), Cov(Stock A, HML)]′.

Q2: If stock A has idiosyncratic volatility equal to 40%, what is the fraction of stock A's variance coming from its exposure to the market? What are the remaining determinants of stock A's variance?

Q3: If stock B has FF3M betas equal to 1 (MKT), 0.8 (SMB), and 0.7 (HML) and the same variance as stock A; what is the correlation between stock A and B?