Investments

Masters in Finance

NOVA SCHOOL OF
BUSINESS & ECONOMICS

Spring 2025, Martijn Boons

Book chapters 8-9

Motivation

- Question: if everyone in the economy holds an efficient portfolio, how should securities be priced so that demand equals supply?

- If, for given expected returns, variances, and covariances, no investor wants to hold IBM, something is wrong
  - Price—and thus expected return—of IBM needs to adjust

- Equilibrium
  - Every investor is happy with her portfolio
  - Supply of assets equals demand for assets

CAPM equilibrium

- Investors are single-period mean-variance optimizers and have identical information on means, volatilities, and correlations.

- All investor’s portfolios lie on the capital allocation line: the Tangency portfolio of risky assets combined with the risk-free asset (two-fund separation)

- If borrowing and lending cancel out, the sum of all investors’ risky portfolios will be the Tangency portfolio.
  → Ultimately, a banks’ primary function is to receive savings and lend these out.

- In equilibrium, the sum of all investors’ desired portfolios must equal the supply of assets

- Aggregate supply of assets is the market portfolio

- Market portfolio = Tangency portfolio

From the tangency portfolio to an asset pricing model (1/2)

- Recall that we were solving for the case with N risky assets and a risk-free asset:

$$\min_{w_p} w_p{}' \Omega w_p, \text{ subject to } w_p{}' \mu^e = \mu_p^{e,desired}$$

  - Solution: $w_p = c_p \bar{w}, \text{where } c_p = \frac{\mu_p^{e,desired}}{\mu^{e\prime} \Omega^{-1} \mu^e}$ and $\bar{w} = \Omega^{-1}\mu^e$.
  - All optimal portfolio have the same relative weight in the risky assets determined by $\bar{w}$.

- The tangency portfolio that is invested 100% in risky assets and maximizes Sharpe ratio can be written as:

$$w_T = \frac{\Omega^{-1}\mu^e}{i'\Omega^{-1}\mu^e} \text{ with}$$

$$\mu_T^e = \frac{\mu^{e\prime}\Omega^{-1}\mu^e}{i'\Omega^{-1}\mu^e},\ \sigma_T^2 = \frac{\mu^{e\prime}\Omega^{-1}\mu^e}{(i'\Omega^{-1}\mu^e)^2}, \text{ and } \frac{\mu_T}{\sigma_T^2} = i'\Omega^{-1}\mu^e$$

- These are the ingredients we need to arrive at an asset pricing model!

From the tangency portfolio to an asset pricing model (2/2)

- For all *N* risky assets, with excess returns collected in the vector r^e, we can write covariance with the tangency portfolio as:

$$Cov(r^e, r_T^e) = Cov(r^e, r^{e\prime} w_T) = \Omega w_T = \frac{\mu^e}{i'\Omega^{-1}\mu^e} = \frac{\mu^e}{\mu_T^e / \sigma_T^2}$$

- The whole point is to understand equilibrium expected returns in μ^e, which are thus defined as:

$$\mu^e = \frac{Cov(r^e, r_T^e)}{\sigma_T^2} \mu_T^e$$

- Or, in more familiar notation for a single asset i:

$$E(r_i - r_f) = \beta_{i,T} E(r_T - r_f), \text{ where } \beta_{i,T} = \frac{Cov(r_i^e, r_T^e)}{Var(r_T^e)}$$

- Expected excess returns are linear in covariance with the tangency portfolio.

- Any difference in expected returns between two risky assets *i* and *j* follows from difference in $\beta_{i,T}$ vs $\beta_{j,T}$.

The Capital Asset Pricing Model (CAPM)

- The single-factor asset pricing model with r_T^e is just mathematics.
  - In any sample of returns, all average returns are explained perfectly by beta with the tangency portfolio constructed in that same sample.
  - However, r_T^e is not observable in practice and extreme weights in any sample make it a weird portfolio to interpret and work with.

- The CAPM guides us from mathematics to economics.

  - If in equilibrium $T = M$, we must also have:

$$E(r_i^e) = \beta_{i,M} E(r_M^e), \text{ where } \beta_{i,M} = \frac{Cov(r_i^e, r_M^e)}{Var(r_M^e)}$$

  - The only risk that is priced is systematic risk as measured by covariance with the market

The (ir)relevance of assumptions

- The assumption of fully informed rational mean-variance optimizers is probably violated in the real world
  - Asymmetric information
  - Behavioral biases
  - Skewness and kurtosis preferences
- Moreover, the CAPM assumes away frictions, such as trading costs, investment mandates, taxes etc

- Does this mean the CAPM is wrong?

"*Basic premises or assumptions of a model are not absolutely relevant. The gauge of a sucessful model is its ability to make correct predictions, not the empirical validity of the assumptions*"

Milton Friedman (1976 Nobel prize)

- CAPM captures risk in an intuitive manner that resonates with investors.

- Consequently, CAPM is ubiquitous in practice.

Risk in the CAPM

- High beta stocks are risky, and must therefore offer a higher return on average to compensate for the risk

- Why are high-beta stocks risky?

  - Because they pay up just when you need the money the least: when the overall market is doing well

  - And they lose money when you really need it: when the overall market is doing poorly

  - If anyone is to make this security a part of their overall portfolio, it must compensate them through a high expected return

- High-beta stocks have high <u>systematic risk</u>

  - Adding these stocks to a diversified portfolio increases portfolio variance a lot

Risk as covariance

- Why is risk measured by covariance (through beta)?
  - A small increment to the weight of an asset changes the variance of a portfolio by an amount proportional to covariance of the asset with the portfolio.

- Suppose you invest in the market and consider investing a little bit more in asset i (by borrowing at the risk-free rate). What will happen to portfolio variance:

$$\text{Var}(r_M) = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \, Cov(r_i, r_j)?$$

$$\frac{\partial \text{Var}(r_M)}{\partial w_i} = 2 w_i Cov(r_i, r_i) + 2 \sum_{j \neq i} w_j Cov(r_i, r_j) =$$

$$2 \sum_{j=1}^{N} Cov(r_i, w_j r_j) = 2 Cov\left(r_i, \sum_{j=1}^{N} w_j \, r_j\right) = 2 Cov(r_i, r_M)$$

Marginal variance = 2 x covariance

- Marginal variance determines the expected return and therefore the price of the asset (just like in economics it is the marginal cost that determines equilibrium price)

- Example: Suppose you invest one-third of your wealth in assets D, E, and C from last week with variance-covariance matrix:

	D	E	C
D	0.0144	0.0072	0.0000
E	0.0072	0.0400	0.0100
C	0.0000	0.0100	0.0225

Approximate how much portfolio variance changes if you increase the weight in D by 50 basis points?

The Capital Asset Pricing Model (CAPM)

$$E(r_i) = r_f + \beta_{iM} E\left(r_M - r_f\right)$$

- Why is the CAPM equation so important in practice?

  1. Systematic or priced risk is summarized through one single variable: β_{iM}.
  2. Given β_{iM} (and an estimate of the market risk premium), we can calculate present values of cash flows using appropriate discount rate.
  3. A passive strategy is mean-variance efficient; investors should hold ETFs
  4. All differences in portfolio return across investors or funds due to β_{iM}.

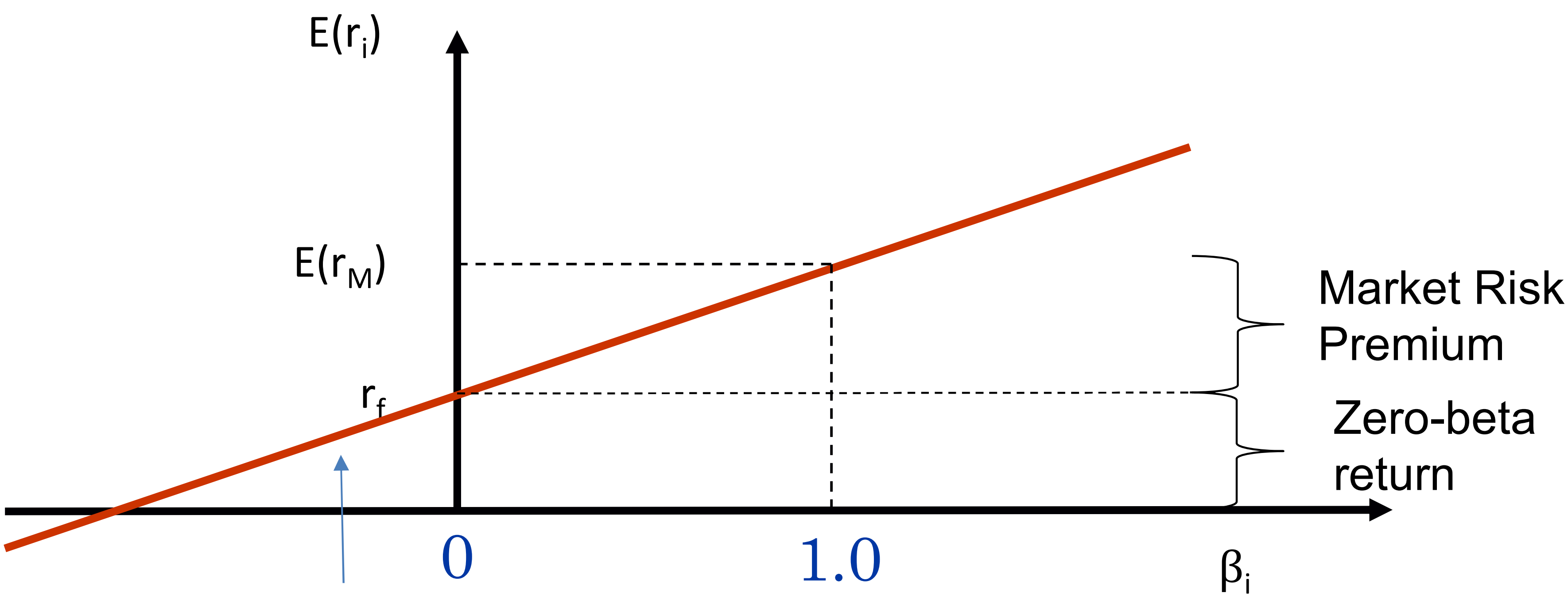
- Given a good proxy for the market portfolio, we can start applying the CAPM

  - Roll's (1977) critique: True CAPM market portfolio is unobservable, as it contains ALL assets traded in the world (also bonds, real estate, precious metals, etc).
  - Common proxy: portfolio of all stocks with weights according to each firm's market capitalization: $w_i = \frac{MarketCap_i}{\sum_i MarketCap_i}$

The Security Market Line

The SML presents the CAPM relation between expected return and beta

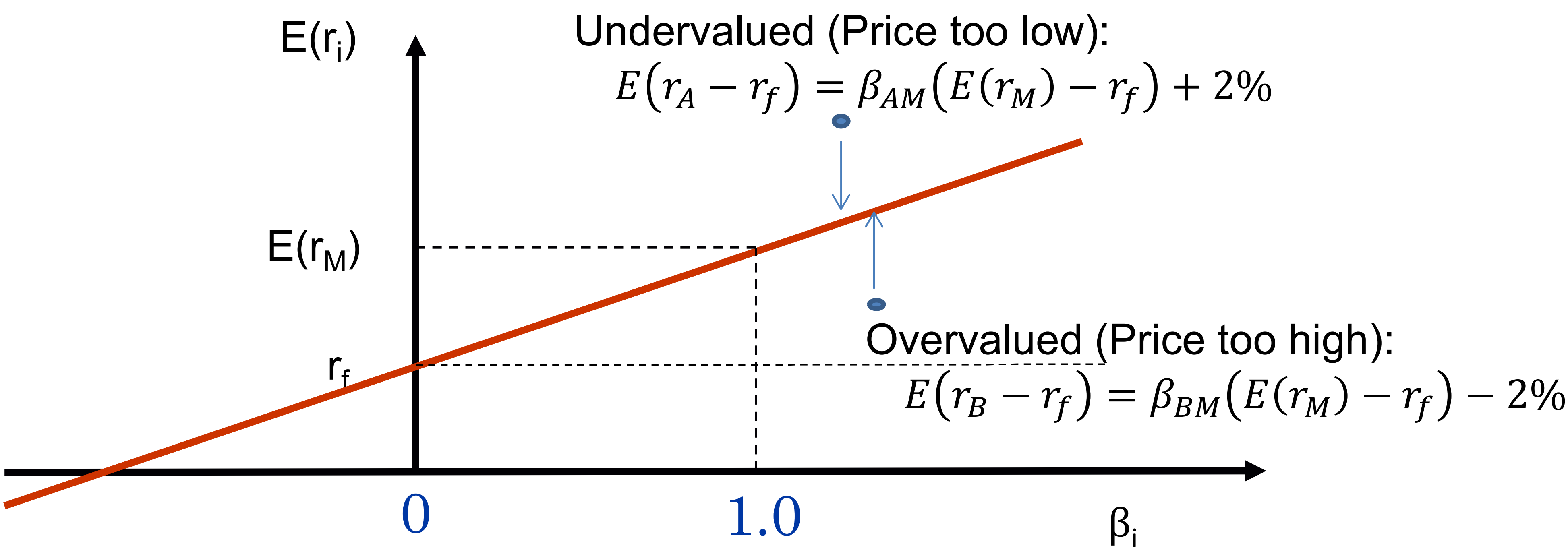
$$E(r_i) = r_f + \beta_{iM}(E(r_M) - r_f)$$



Betas and thus expected returns can be negative (insurance!)

CAPM equilibrium

- All assets are correctly priced and line up on the SML
- If not, investors correct mispricing instantaneously



Capital Market Line (CML)

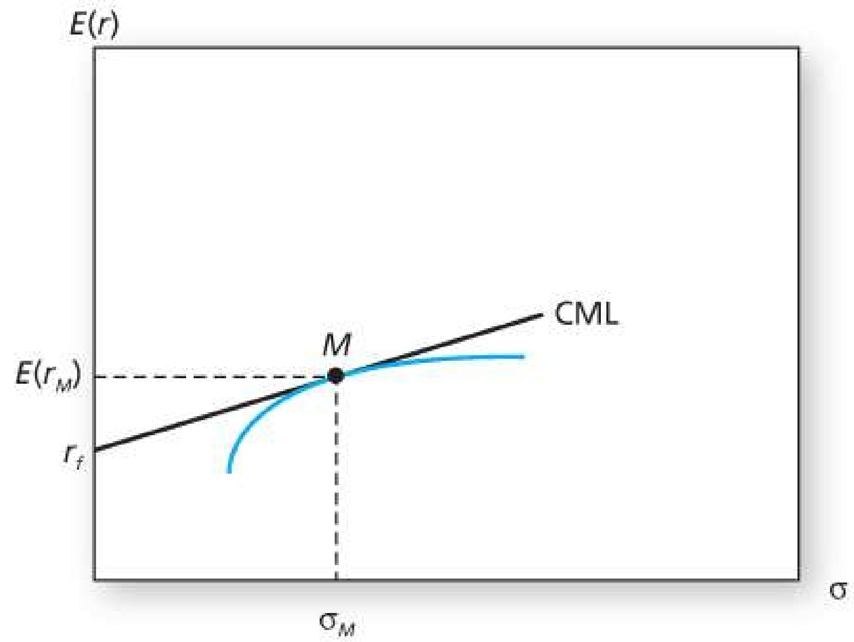


Figure 9.1 The efficient frontier and the capital market line

- CML plots relation between expected return and standard deviation
- It is the CAL when the risky portfolio is the market

The Market Risk Premium

- In the CAPM, investors optimally split wealth between the risk-free asset and the market portfolio

- Recall: Optimal capital allocation for any investor solves

$$\max_{y} U = r_f + y\left[E[r_M] - r_f\right] - \frac{A}{2} y^2 \sigma_M^2$$
$$\rightarrow y = \frac{E[r_M] - r_f}{A\sigma_M^2}$$

- Aggregating over all investors: $y = 1$, such that we can write the following expression for the market risk premium

$$E[r_M] - r_f = \bar{A}\sigma_M^2$$

where $\bar{A}$ is the “average” risk aversion across investors (=price of risk) and σ_M^2 is the variance of the market portfolio (=amount of risk)

- E.g., with σ_M =15% and $\bar{A}$=4, market risk premium equals 9%

Risk premia on all N assets

- Many extensions of the standard mean-variance framework are considered in practice.

- One famous example is the **Black-Littermann** approach: the first, but critical, step in this approach is analogous to the previous slide.

  - Optimal weights in N risky assets when there is a risk-free asset:

$$w_p = A^{-1}\,\Omega^{-1}\mu^e \text{ (compare to: } y = \frac{E[r_M]-r_f}{A\sigma_M^2}\text{)}$$

  - Consider the investor with risk aversion $\bar{A}$ such that for him/her it is optimal to invest 100% in the tangency portfolio ($w_p = w_T$)

  - If the tangency portfolio is the market portfolio ($w_T = w_M$), we then have that:

$$\mu^e = \bar{A}\Omega w_M \text{ (compare to: } E[r_M] - r_f = \bar{A}\sigma_M^2\text{)}$$

Beta in practice

- Beta is usually estimated through a time-series regression:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_{iM}\left(r_{m,t} - r_{f,t}\right) + \varepsilon_{i,t}$$

  - Example: Berkshire Hathaway

- Most important determinants of beta: industry/product-type (utilities vs luxury watches), financial leverage (debt vs equity), operational leverage (fixed vs variable costs)

b=1 systematic risk equals that of the market
b>1 return varies more than the market
0<b<1 return varies less than the market

Procyclical

b=0 market-neutral
b<0 "market hedge" (Countercyclical)

- Beta of a portfolio is weighted average of individual assets betas:

$$\beta_p = \sum_i w_i \, \beta_i$$

  - Industry beta is the value-weighted average of firms in that industry; company beta is the value-weighted average of its projects

Systematic vs idiosyncratic risk

- The CAPM regression splits the variance of the return of the asset into systematic risk + Idiosyncractic risk

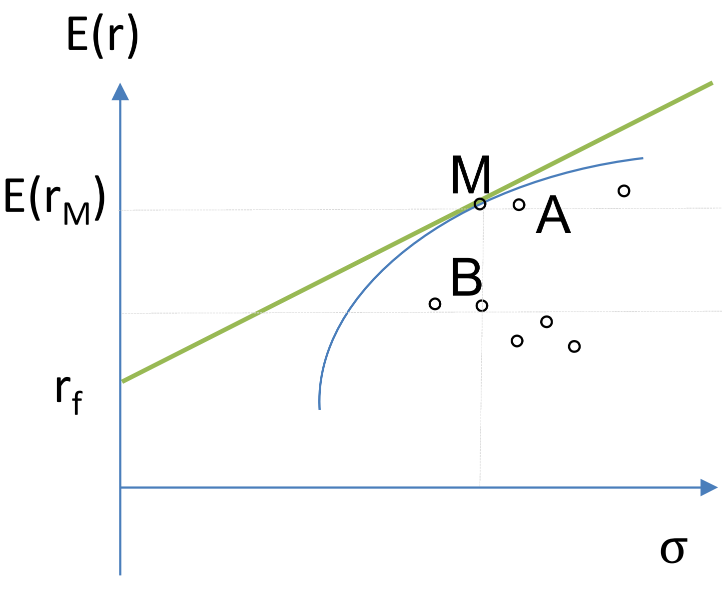
$$Var(r_{i,t} - r_{f,t}) = \underbrace{\beta_{iM}{}^2\, Var\left(r_{M,t} - r_{f,t}\right)}_{\text{systematic risk}} + \underbrace{Var\left(\varepsilon_{i,t}\right)}_{\text{Idiosyncractic risk}}$$

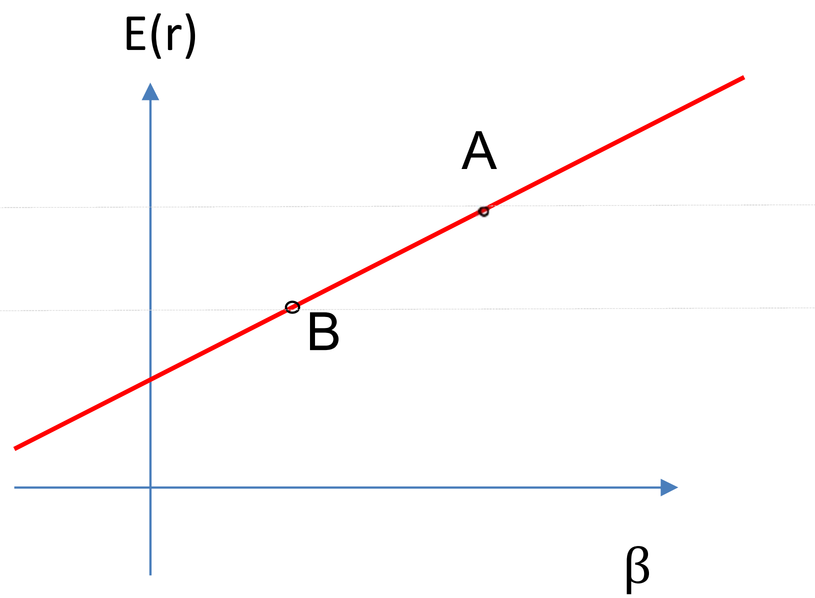
  - It is only systematic risk that investors require compensation for!
  - Idiosyncratic risk is diversified away by investing in a large portfolio of assets ($Var\left(\sum_{i=1}^{N} \frac{1}{N} \varepsilon_{i,t}\right) \to 0$ for large N).

- The R^2 of the regression thus estimates the proportion of the risk (variance) of a firm that can be attributed to systematic risk:
  - $R^2 = \beta_{iM}{}^2\, Var\left(r_{M,t} - r_{f,t}\right) \;/\; Var(r_{i,t} - r_{f,t})$
  - The rest ($1 - R^2$) is idiosyncratic risk.
  - Typical R-sq. is around 25% for individual firms, much larger for diversified portfolios.

CML vs SML

- All portfolios, whether efficient or not, must lie on the SML, but only efficient portfolios are on the CML
- Consider:
  - A has the same expected return as market, but higher volatility.
  - B has the same volatility as market, but lower expected return.





Why? Idiosyncratic risk:

→ $Var(r_{A,t} - r_{f,t}) = \beta_{AM}{}^2\, Var(r_{M,t} - r_{f,t}) \quad + \quad Var(\varepsilon_{A,t})$ with $\beta_{AM} = 1$ it must be that $Var(\varepsilon_{A,t}) > 0$

→ $Var(r_{B,t} - r_{f,t}) = \beta_{BM}{}^2\, Var(r_{M,t} - r_{f,t}) \quad + \quad Var(\varepsilon_{B,t}) = Var(r_{M,t} - r_{f,t})$, with $\beta_{BM} < 1$ it must be that $Var(\varepsilon_{B,t}) > 0$

Alpha or abnormal return

- The intercept α_i measures the average excess return over that predicted by the CAPM:

$$\alpha_i = \widehat{r_{i,t}^e} - \beta_i \widehat{r_{M,t}^e}$$

  - CAPM implies that the intercept α_i is equal to 0 for all assets!
    - $\alpha_i > 0$: asset provided average return above what CAPM predicted, i.e., larger than what was justified by its systematic risk, and is therefore called “underpriced”
    - $\alpha_i < 0$: asset called “overpriced”

- In the industry, known as Jensen’s alpha and used to measure abnormal portfolio performance.

- Be careful: (i) historical $\alpha_i > 0$ may not repeat in the future and (ii) CAPM may also be wrong (next week)

- What is the alpha of Berkshire Hathaway?

Exercise

- Consider the following data on two stocks and the market and risk-free asset

	Exp. Ret.	St. Dev.	$Corr(r_i^e, r_M^e)$
Nova	12%	25%	0.7
Catolica	7%	20%	0.5
Market	10%	15%	
Risk-free	4%		

1. Are Nova and Catolica correctly priced by the CAPM?
   - No, CAPM does not hold. β_{Nova}=25%*0.7/15%=7/6, so that CAPM-E(Rnova)=4%+7/6*(10%-4%)=11% and CAPM-E(Rcatolica)=8%. Thus, Nova is underpriced (α_{Nova} = 1%, β_{Nova} = 7/6) and Catolica is overpriced ($\alpha_{Catolica}$ = -1%, $\beta_{Catolica}$ = 2/3).
2. For which stock does the market explain the largest fraction of return variation?
   - Nova, since R^2=correlation^2.
3. Construct a portfolio of Nova, Catolica and the risk-free asset (with a total of 1$ invested) that is market-neutral, but which has positive alpha.
   - Long Nova and short Catolica to benefit from difference in alphas, but weight them such that the beta of the portfolio is zero:

$w_{Nova} = 1;\ w_{Catolica} = -(7/6)/(2/3) = -1.75,\ w_{rf} = 1 - (1 - 1.75)$ =1.75

→ $\beta_P = \frac{7}{6} - \left(1.75 * \frac{2}{3}\right) + 1.75 * 0 = 0; E(r_P^e) = 1 * 12\% - 1.75 * 7\% + 1.75 * 4\% - 4\% = 2.75\%$ ($= \alpha_{Nova} * 1$+ $\alpha_{Catolica}$ $*$-1.75)

The inputs (1/2)

- The market: a broad stock index, because stocks from many different industries have exposures to many different risk factors.

- What is the expected return on this market portfolio?
  - Forward-looking estimates reported by professional analysts and consultants or derived through other methods or from option prices, but all these different estimates are not as similar as one would hope...

- What risk-free rate?
  - In theory: short-term (1 month) treasury-bill rate
  - In practice, most investors pay a substantially higher rate to borrow
    - Surveys suggest most practitioners use 10 to 30 year treasury bond yield as input to CAPM
    - Paradox: the long-term bond is not risk-free, so combinations of the market and this bond will not lie on a straight line in mean-standard deviation space (i.e., there is no CML).

The inputs (2/2)

- If you are interested in a forward-looking estimate of the systematic risk of a stock and thus its CAPM expected return, make sure to use relatively recent data.
    - Betas may change over time
    - Five years of monthly data is typical for individual firms
    - Daily data is too noisy and may be sensitive to microstructure issues.
    - Beta estimated through a linear regression, but fine-tuning is common in the industry:

        - Beta shrinkage (e.g., Bloomberg adjustment):

        Adjusted beta = 0.66 x Unadjusted beta + 0.34 * 1. (**Why 1?**)

Formal tests of the CAPM

CAPM tested in 2 ways for individual assets *i*, or preferably, portfolios (less noise in averages as estimates of expected returns)

1. Time-series regressions (Black, Jensen, Scholes (1972))
   - $r^e_{i,t+1} = \alpha_i + \beta_{i,M} r^e_{M,t+1} + \varepsilon_{r,t+1}$
   - $H_0^{TS}: \alpha_1 = \cdots = \alpha_N = 0$ (Individual and joint tests – Gibbons, Ross, Shanken (1989))

2. Cross-sectional regressions (Fama and MacBeth (1973))
   - CAPM predicts that all cross-sectional variation in expected returns is explained by variation in beta. Thus, we should run

$$E(r^e_{i,t+1}) = \lambda_0 + \lambda_M \beta_{i,M} + a_i,$$

   to test H_0^{CS}: $\lambda_0 = 0$, $\lambda_M = E(r^e_{M,t+1})$, and $R^2=1$

   - No arbitrage: if idiosyncratic risk can be diversified away, any asset with $\beta_{i,M}=1$ must have $E(r^e_{i,t+1}|\beta_{i,M}=1)=E(r^e_{M,t+1})$!

- Two-step procedure, because both sides unobservable
  1. Estimate $\widehat{\beta_{i,M}}$ from $r^e_{i,t+1} = \alpha_i + \beta_{i,M} r^e_{M,t+1} + \varepsilon_{i,t+1}$
  2. Estimate $\widehat{\lambda_0}$ and $\widehat{\lambda_M}$ from $\widehat{r^e_{i,t+1}} = \lambda_0 + \lambda_M \widehat{\beta_{i,M}} + a_i$

Let's test the CAPM (i)

Using data from Kenneth French's data library (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html)

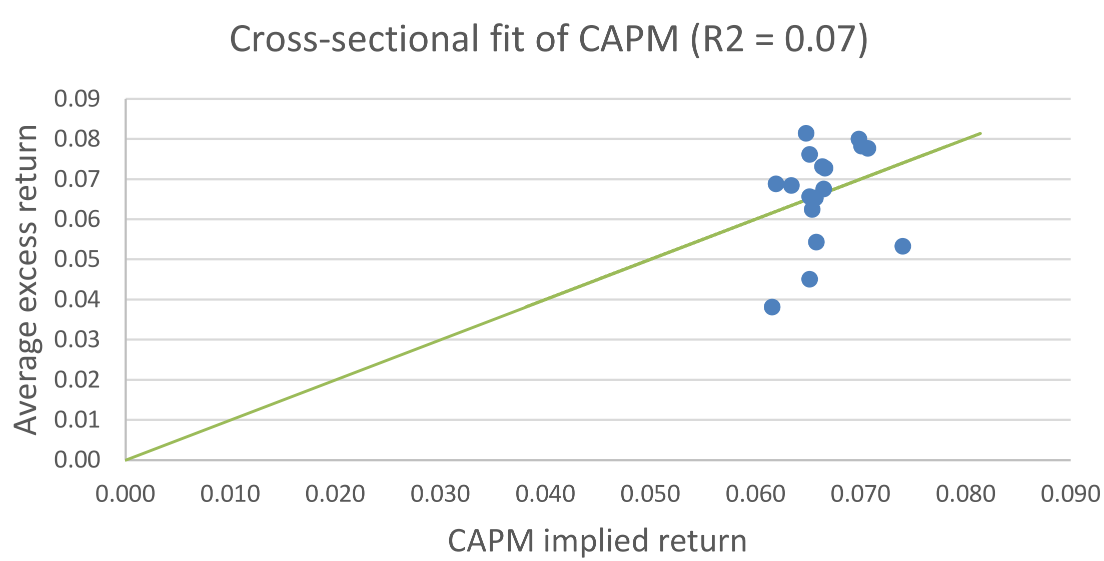
- Market portfolio: value-weighted portfolio of all stocks traded in the United States
- Risk-free rate: one month t-bill return
- Test assets: 17 industry portfolios
- Monthly data from 1962-01 to 2014-12

- Answer the following questions

1. Time-series regressions:
   - What is the largest, smallest and average beta over the 17 industry portfolios?
     Betas range from 0.54 (Utilities) to 1.29 (Steel) and average to ≈ 1 (by construction, because market value-weighted-average of industry portfolios = market portfolio).
   - What is the largest, smallest and average alpha?
     Alphas range from -3.8% (Steel) to 3.4% (Food) and average to ≈ 0 (by construction) Lots of noise in estimating abnormal returns: Steel and Food alphas are only marginally significant using >50 years of data!
     - Steel: if you overestimate beta, you will underestimate alpha → estimation error!
   - What is the largest, smallest and average R^2?
     R^2's range from 0.36 for Utilities to 0.92 for Other. R^2's are considerably higher for portfolios of stocks than for individual stocks, perhaps unsurprisingly.

Let's test the CAPM (ii)

2. Cross-sectional regressions:
   - <u>What is the estimated market risk premium, λ_M? Is it close to the average excess market return?</u>
     $\widehat{\lambda_M} = -1.7\%$, which is far from $E(r^e_{M,t+1})$=5.9%.
   - <u>Does market beta explain all variation in average excess return of the 17 industry portfolios as implied by the CAPM?</u>
     Market beta explains almost no variation in the average excess return across the 17 industry portfolios: whereas average excess returns range between 3.8% and 8.1%, CAPM predicted returns are all close to 6.5%. Graphically:



- From this test, we must conclude that the CAPM does not do a great job explaining cross-sectional variation in average industry portfolio returns.

Testing the CAPM – Results

- Although the outcomes of a test of the CAPM depends on the chosen set of test assets, this conclusion is not so different from the typical finding in the literature:

- Black, Jensen, Scholes (1972) get mixed results
  - ➢ They do find that higher beta stocks earn higher returns on average, but their estimates of $\widehat{\lambda_M}$ are small, insignificant, and vary wildly across subsample periods.

- Later work by Fama and French (1992) finds even weaker results

  - Early 1990s it was clear that there are firm characteristics, like size and book-to-market, that explain cross-sectional variation in average returns.
  - No relation between beta and returns after controling for such characteristics.
  - Leads Fama, French, and many others in their following, to pronounce “beta is dead!”

Standard interpretation of the CAPM's shortcomings

1. Perhaps we are estimating the CAPM incorrectly.
    - More advanced methods to estimate beta fare much better:
        - using high-frequency data (+techniques)
        - betas that are conditional functions of time or firm-characteristics.
        - Downside betas: Investors care most about the comovement of a stock's returns with negative market returns.
    - Returns over longer horizons than a single month are much better explained by the CAPM.

2. Missing factors or elements of risk
    - ➢ Multi-factor models (next week): perhaps there are additional risk factors that investors care about (aside from poor market returns) and which therefore capture a risk premium (e.g., recessions)

3. Behavioral biases
    - ➢ Perhaps prices are not correct, but large mispricing due to behavioral biases. E.g., try to rationalize the return of Bitcoin using any asset pricing model…

Wrapping up

- Assumptions of CAPM are quite restrictive

- Research shows it is sometimes not very accurate

- Still: widely used in corporate finance and investments.

- Why? The risk-return trade-off in the CAPM is elegant and makes perfect sense:
    - As a result of diversification, risk is a property not of an asset in isolation, but how assets co-move with investor's diversified portfolios.
    - Investors desire insurance for bad times, which in CAPM world means low returns on the market portfolio.
    - High beta assets do not provide insurance → low P, high E(R)

- Next week: Maybe there are other risks that investors care about?