Investments

Masters in Finance

NOVA SCHOOL OF
BUSINESS & ECONOMICS

Spring 2025, Martijn Boons

Chapter 14-15-16

Bonds defined

- Bonds promise to pay coupons at pre-specified dates and a fixed principal amount (face value) at the maturity date
  - Fixed coupons (most common): % of face value (typically 100$ or 1000$)
    - 0% coupon = zero coupon bond
    - Variable coupon (rare)
  - Maturity ranges from overnight to > 30 years
  - Nominal vs real
  - Default risk-free vs default risk
    - US vs Greek government
    - Government vs corporations

- Bonds are traded at discount (P<face value; e.g., zero coupon bonds), par (P=face value) or premium (P>face value)

- Large variety of bonds issued by single issuer
  - E.g., US Treasury
    - T-Bills: 4,13, 26, and 52 weeks (no coupons)
    - T-Notes: 2,3,5,7 and 10 years (semi-annual coupons)
    - T-Bonds: 30 years (semi-annual coupons)
  - Big companies like BoA and GE have ≈ 50 different bond issues outstanding
    - Different maturities & coupon rates

Bonds defined

- The price of a bond is found by discounting its future cash flows using spot interest rates r_t, appropriate for the default risk and maturity of the bond's cash flows:

$$\sum_{t=1}^{T} \frac{Coupon}{(1+r_t)^t} + \frac{Face\,Value}{(1+r_T)^T}$$

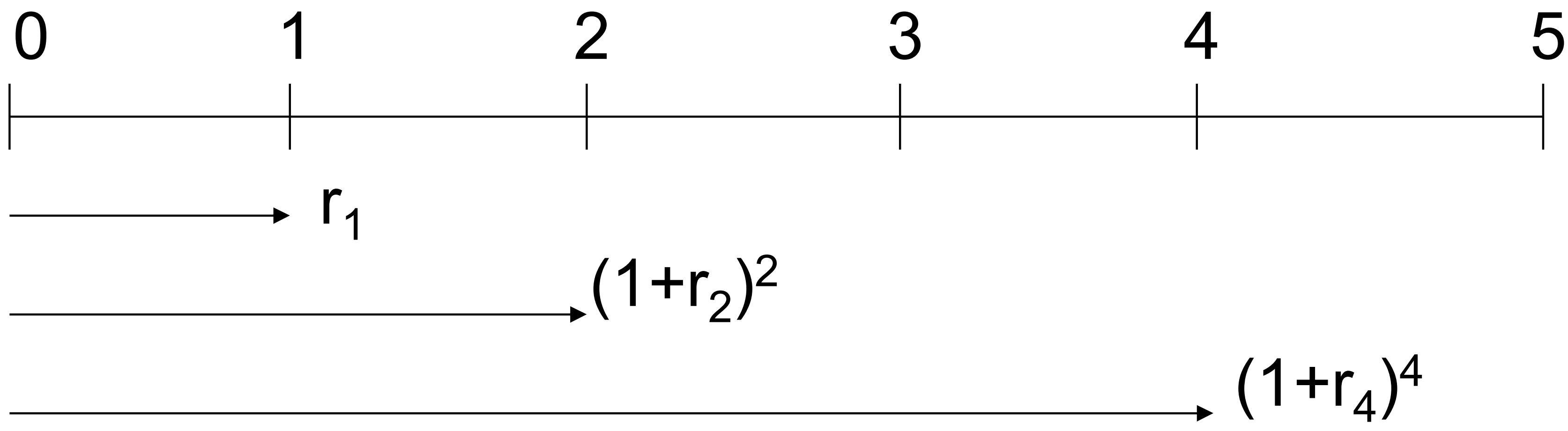
- Note, when $r_t = r$ for all t, we can apply the annuity formula:

$$\frac{\text{Coupon}}{r}\left(1 - \frac{1}{(1+r)^T}\right) + \frac{Face\,Value}{(1+r)^T}$$

- **<u>Yield-To-Maturity (YTM)</u>**: the single rate *y* that sets the present value of the bond's (promised!) future payments equal to the bond price:

$$P = \frac{\text{Coupon}}{y}\left(1 - \frac{1}{(1+y)^T}\right) + \frac{Face\,Value}{(1+y)^T}$$

Spot rates



- To discount a bond's future cash flows, we need spot interest rates r_t that determine the relevant discount rate from t to 0.

- We find these spot rates from the price of zero-coupon bonds ($P_{z,t}$):

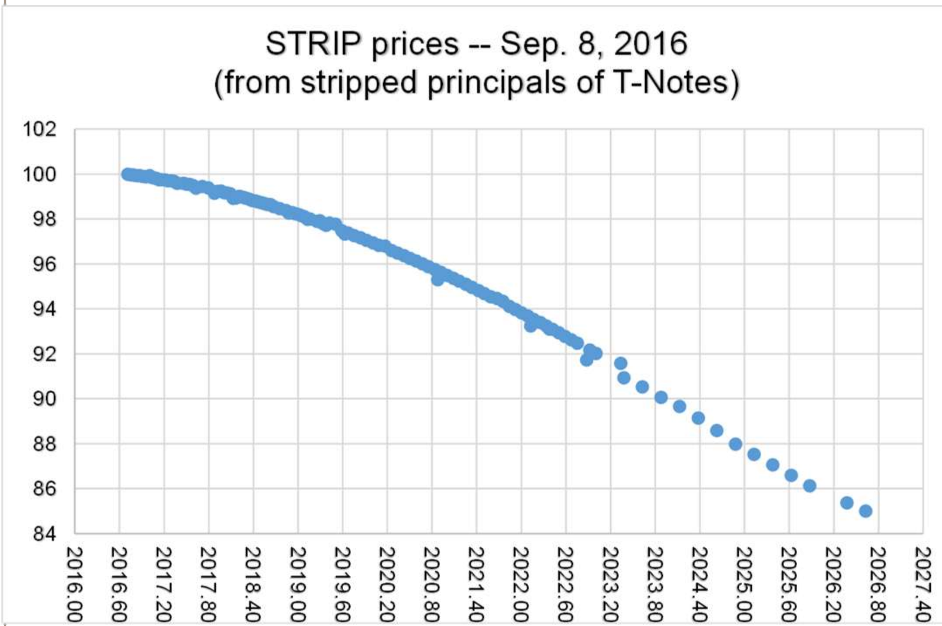
  Spot rate $r_t = (100/P_{z,t})^{1/t} - 1$ (annualized for comparison purposes)

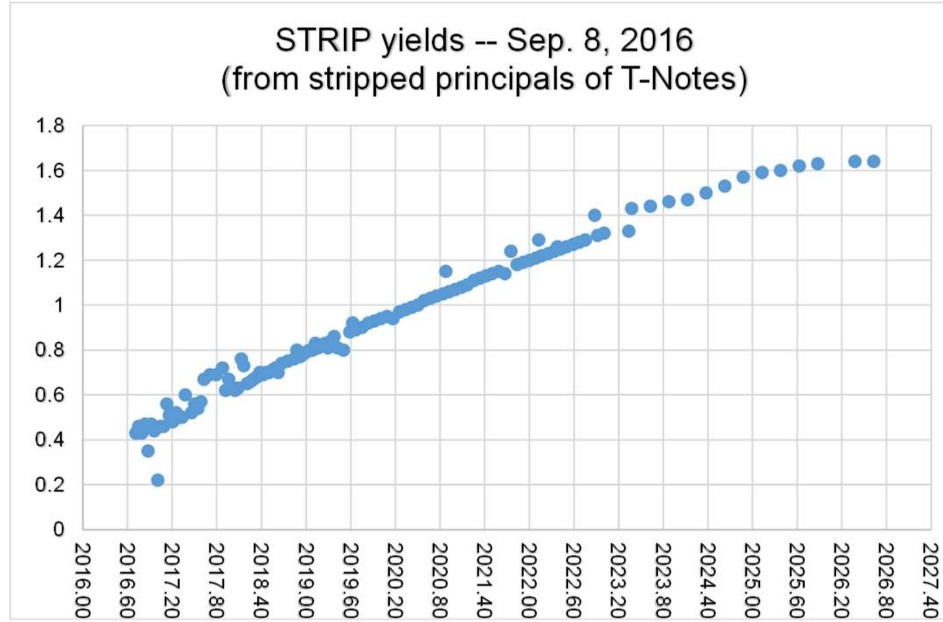
  – Price of a one-year zero = 98 → $r_1 = 100/98 - 1 = 2.04\%$
  – Price of a two-year zero = 95 → $r_2 = (100/95)^{1/2} - 1 = 2.60\%$
  – And so on …

STRIPS

- More precisely, spot rates are found using STRIPS
  - acronym for Separate Trading of Registered Interest and Principal of Securities
  - Started trading in January 1985
- A Treasury note with 10 years remaining to maturity consists of
  - A single principal payment at maturity
  - 20 coupon payments (semi-annually for 10 years)
- When this note is STRIP'd, the 20 coupons and the principal become separate securities, i.e., zero coupon bonds.
  - Spot rates are the yields of these <u>nominally risk-free</u> zero coupon bonds.
  - Yield curve

STRIP prices and the yield curve





Source: Wall Street Journal

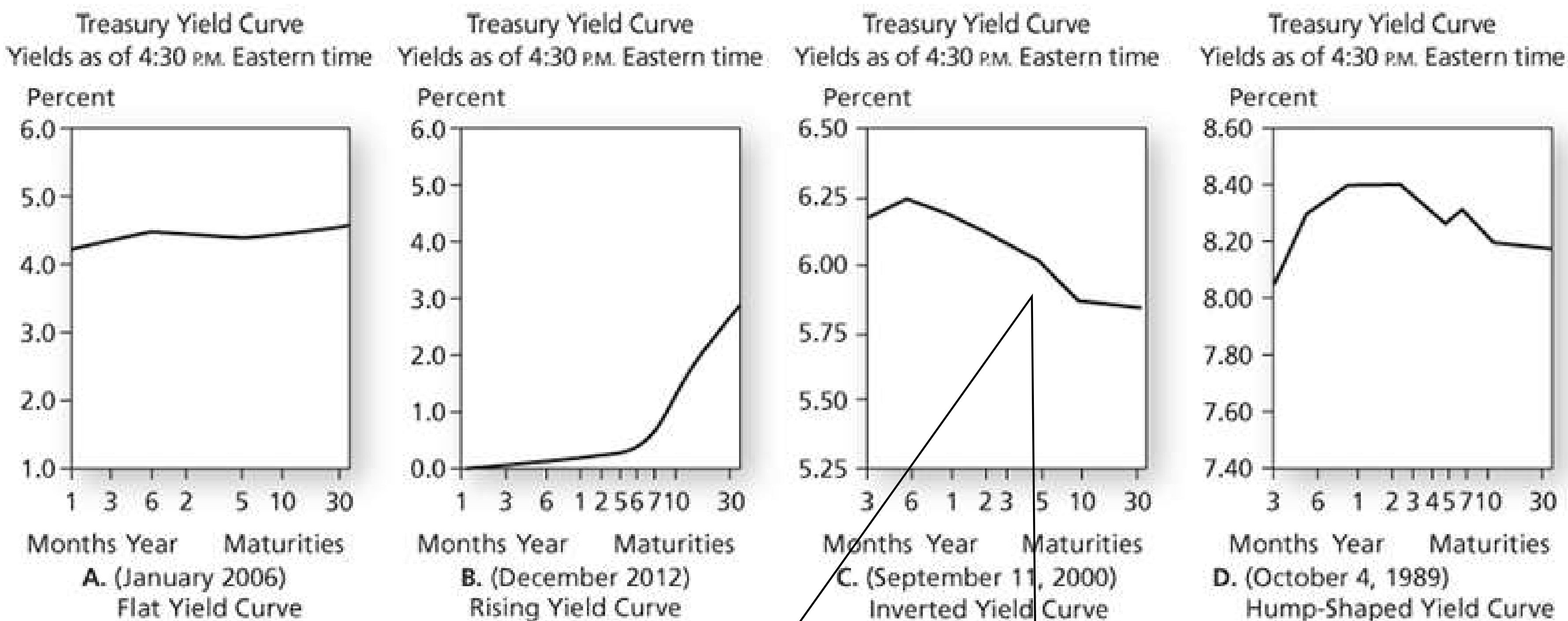
How did we get from the 3 and 10 year STRIP price to their yields?

r_3 = [redacted]

r_{10}=

Yield curve over time

- Usually upward sloping, because uncertainty about interest rates and inflation increase with maturity.
- But, there is considerable time-variation:
  - Important insight: interest rates vary over time and with maturity!



Inversions tend to occur when bad times are approaching: investors fear short-term economic uncertainty and avoid reinvestment risk from buying short-term bonds, such that these prices (yields) are relatively low (high).

Forward rates (1/3)

- We can decompose a T-period loan (which is what a bond is...) into spot and <u>forward</u> components

- Example:
  - Consider 2-year zero with a YTM of 5% (=r_2)
  - Suppose the 1-year spot rate (=r_1) is 4%
  - A 2-year 5% loan can be thought of as a 1-year spot loan at the rate of 4%, followed by a 1-year loan starting at time 1 at the forward rate $f_{1\to 2}$
  - The forward rate, $f_{1\to 2}$, simply solves

$$(1+4\%)(1+f_{1\to 2}) = (1+5\%)^2 \Leftrightarrow$$
$$f_{1\to 2} = \frac{1.05^2}{1.04} - 1 = 6.01\%$$

Forward rates (2/3)

- This is a no-arbitrage argument:
  - A forward rate $f_{T\rightarrow T+K}$ is an interest rate quoted today for a future period T to T+K
  - The forward contract invests 1$ at T and pays off $(1+f_{T\rightarrow T+K})^K$ at T+K
  - We can replicate the forward contracts payoffs and thus find that

$$f_{T\rightarrow T+K} = ((1+r_{T+K})^{T+K}/(1+ r_T)^T)^{1/K}-1$$

	The forward contract	Replication strategy		
		Borrow present value of 1$ at T and invest for T+K periods		Total
Today	0	$+1/(1+r_T)^T$ (short T year bond)	$-1/(1+r_T)^T$ (long in T+K year bond)	0
T	-1	-1	0	-1
T+K	$(1+f_{T\rightarrow T+K})^K$	0	$+(1+r_{T+K})^{T+K}/(1+ r_T)^T$	$(1+r_{T+K})^{T+K}/(1+ r_T)^T$

- Notion of <u>no-arbitrage</u> (or <u>law of one price</u>), key in economics and finance

Forward rates (3/3)

- Forward rates reflect two things
  - Expectations about future spot rates
    - If yield curve is upward (downward) sloping, then forward rates are higher (lower) than spot rates
  - Risk premium, since future spot rates are uncertain
    - By agreeing today on a forward rate, you are taking risk that interest rates move against you.
    - You want to be compensated for this risk.
- The ``expectations hypothesis of the term structure'' states that risk premia are zero (or, at least, constant), such that any variation in forward rates reflects variation in expected future spot rates
  - Confidently rejected in the data (Fama and Bliss, 1987; many others)

Yield to maturity (1/2)

- The yield-to-maturity (YTM) of a bond provides a measure for the return of investing in the bond
  - The YTM corresponds to the single rate at which discounting future cash flows generates the bond’s current price
    - It is like an internal rate of return: the discount rate that sets an NPV = 0
  - Annualized holding period return if you hold the bond until maturity (and there is no default)
  - However, YTM does not correspond to the realized return in each future period
    - The reason is that interest vary over time and with maturity
    - Thus, coupons are reinvested at uncertain rates

Yield to maturity (2/2)

- Suppose that a 5-year pure discount bond with a face value of $100 is selling for $95. What is the yield to maturity on this bond?

$$95 = \frac{100}{(1+r_5)^5}$$

$$(1+r_5)^5 = \frac{100}{95}$$

$$r_5 = \left(\frac{100}{95}\right)^{1/5} - 1 = 1.031\%$$

- If I hold until maturity, my realized return (nominal!) will be:

  $r_{t \to t+5} = \frac{100}{95} - 1 = 5.26\%$ or 1.031% per year

- If, instead, I sell in one year, my realized return will depend on what r_4 will be one year from now.

Coupon bonds (1/4) – example

- You can purchase a 5-year coupon bond with an annual coupon rate of 7% and a face value of $1,000

- You know the following information for zero coupon bonds from the market for Treasury Strips (recall: prices are quoted per $100 of face value)

	1	2	3	4	5
Strip Price	98	95	92	89	85
Spot rate	2.04%	2.60%	2.82%	2.96%	3.30%

- What would you pay for the coupon bond?

Coupon bonds (2/4) – example

$$\text{Price} = 70\times 0.98 + 70\times 0.95 + 70\times 0.92 + 70\times 0.89 + 70\times 0.85 + 1{,}000\times 0.85 = 1{,}171 > 1{,}000$$

- We have essentially constructed a replicating portfolio of zeros
  - 70$ worth of 1,2,3,4-year zero coupon bonds
  - 1070$ worth of 5-year zero coupon bond

- This is the no-arbitrage price of the coupon bond.
  - What if the market price is 1,150?
    - Arbitrage opportunity! Buy bond, short replicating portfolio.
      - Payoff today = 21 = 1171-1150
      - Payoff at all dates 1,…,5 in the future = 0

Coupon bonds (3/4) – YTMs

- Consider the previous 5-year coupon bond with annual coupon rate of 7%, face value of $1,000, and price of $1,171. What is the yield to maturity on this bond?

- It turns out that

$$1{,}171 = \frac{70}{(1+3.24\%)} + \frac{70}{(1+3.24\%)^2} + \frac{70}{(1+3.24\%)^3} + \frac{70}{(1+3.24\%)^4} + \frac{1070}{(1+3.24\%)^5}$$

- In Excel: Solver or IRR

Coupon bonds (4/4) – YTMs

- Yield to maturity on the coupon bond is 3.24%
  - In practice, firms usually aim to issue bonds at (close to) par: they offer a coupon rate that is approximately equal to the yield to maturity of the bonds to be sold.
  - As time passes, bond's risk may change and thus yield may differ from coupon rate: bond will trade at discount or premium

– The YTM is a weighted-average of the yield of zero-coupon bonds with equal and shorter maturities
  - The YTM (3.24%) is closest to the 5-year zero-yield (3.30%), because this yield is used to discount the bond's largest cash flow!

- As the coupon rate increases, weight of YTM in earlier strip yields increases
  - When the yield curve is upward sloping: higher coupons → lower YTM.

Decomposing bond returns

- Two components:

$$R_{t+1} = \frac{P_{t+1}+C_{t+1}}{P_t} - 1 = \underbrace{\frac{P_{t+1}-P_t}{P_t}}_{Capital\ gain} + \underbrace{\frac{C_{t+1}}{P_t}}_{Coupon\ yield\ or\ current\ yield}$$

- Suppose you buy the 5-year coupon bond with an annual coupon rate of 7% and a face value of $1,000 today at a price of 1171 and sell it one year from now (just after the first coupon is paid). What is your return assuming the bond's YTM remains at 3.24%?

$$R_{t+1} = \underbrace{\frac{1139-117}{1171}}_{Capital\ gain} + \underbrace{\frac{70}{1171}}_{Coupon\ yield} = -2.74\% + 5.98\% = 3.24\%$$

- If the YTM does not change, your return is the YTM!
  - Since coupon rate > YTM, capital gain is negative → the bond is traded at a premium and price decreases over the life of the bond towards the face value to be received at maturity.
- In reality, YTM will likely change though…

Yields and returns

- Yield-to-maturity ≠ expected return in each year over the life of a bond
  - Exception: holding a default-free zero-coupon bond until maturity.
  - Expected return depends on yield of bonds with shorter maturities, which for coupon bonds means the rate at which coupons can be reinvested.

- Example: what is the expected return if I buy the 5 year coupon bond for 1171 and sell it after 1 year? Assume the "expectations hypothesis of the term structure" is correct.
  - Forward rate = expected future spot rate (+ risk premium=0).

	1	**2**	**3**	**4**	**5**
Strip Price	98	95	92	89	85
Spot Rate	2.04%	2.60%	2.82%	2.96%	3.30%
Coupon Bond Price = Present Value of all future CF's at t=0					
Coupon Bond	70	70	70	70	1070
Present Value at t=0	68.6	66.5	64.4		
Bond Price at t=0	1171.3				
Expected Spot rates at t=1 equal Forward Rates at t=0					
Forward Rates		f(1-->2)	f(1-->3)	f(1-->4)	f(1-->5)
E.g., (1+2.6%)^2 / (1+2.04%) -1= 3.16%		3.16%	3.21%	3.26%	3.62%
Expected return from t=0 to t=1					
Expected Present Value at t=1	70.0	67.9	65.7	63.6	928.1
Expected Bond Price at t=1	1195.2				
Expected Return	2.04%				

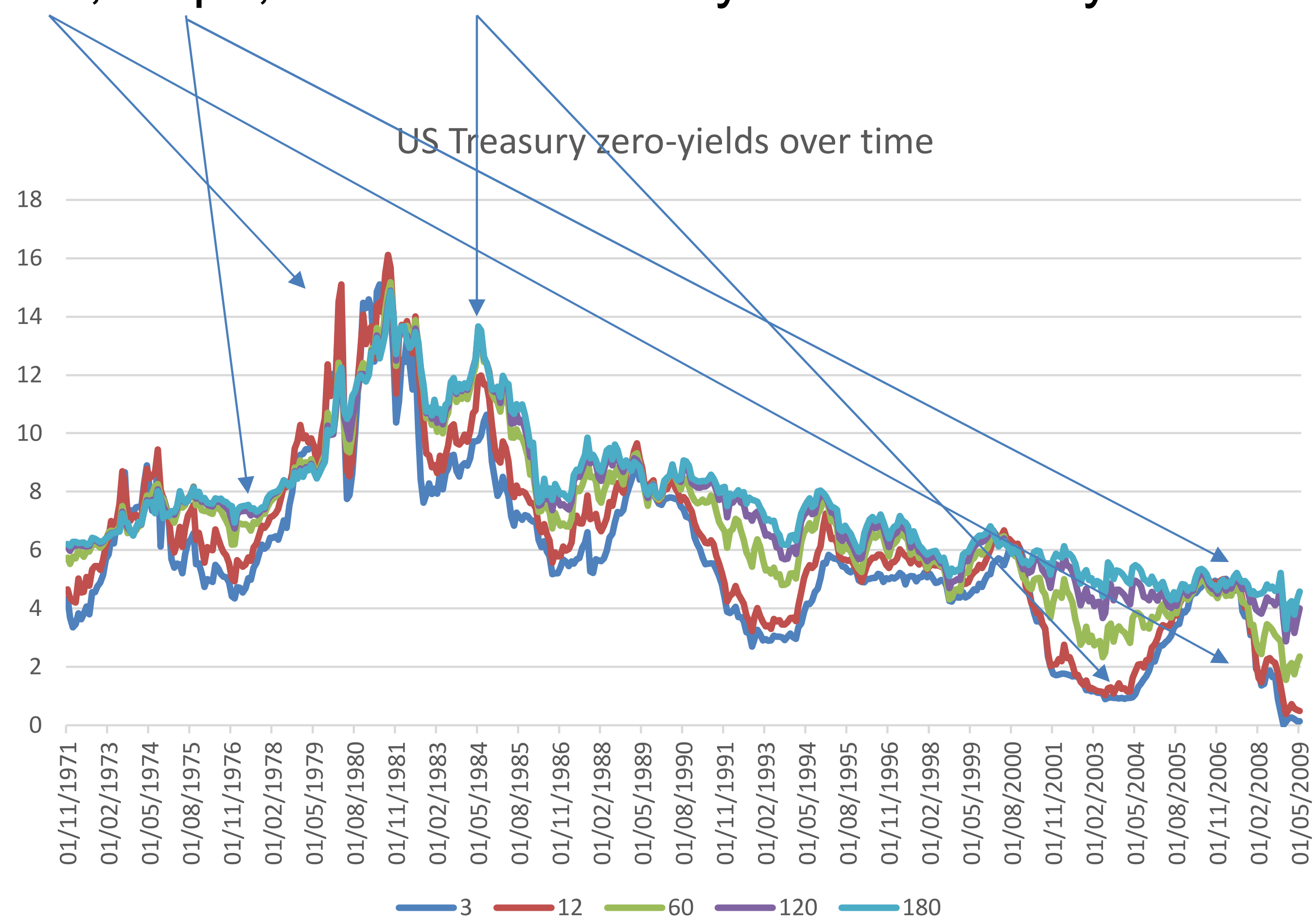
This is what we expect the 1yr spot rate to be at t=1!

≠YTM, which was 3.24%!

Interest-rate risk

Interest rate risk (1/2)

- Even if coupon and principal are guaranteed: Bond returns are risky!
  - There is an inverse relation between interest rates/yields and prices
  - Level, slope, and curvature of yield curve vary over time



Interest rate risk (2/2)

- Consider a 15-year bond with annual coupon of 8 and principal of 100
- Suppose that the yield curve is flat at 5%
  - The bond then sells at 131.14 (**why?**)

- When interest rates move, bond prices change

  - Suppose, immediately after you buy the bond, the yield curve falls by 100 basis points to 4%

    - Bond price goes from 131.14 to 144.47
    - Return = 10.2%

  - Instead, suppose the yield curve goes up by 100 basis points to 6%

    - Bond price goes to 119.42
    - Return = -8.9%

  - Convexity: price decrease for yield increase is smaller than price increase for equal decrease in yield (-8.9% vs 10.2%)

Some math

- Taylor expansion:

$$f(x) - f(x_0) = \frac{f'(x_0)}{1!}(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \frac{f'''(x_0)}{3!}(x - x_0)^3 + \ ...$$

- In finance, we routinely estimate the sensitivity of prices to discount rates using a second order expansion, because yields don't change suddenly by huge amounts.

- What happens to bond price $= \mathrm{P} = \sum_t \frac{C_t}{(1+y)^t}$ when yield changes by Δy?

- 1st derivative $f'(x_0) = \frac{\partial P}{\partial y} = -\frac{1}{(1+y)} \sum_t \frac{C_t}{(1+y)^t} \times t$

- 2nd derivative $f''(x_0) = \frac{\partial^2 P}{\partial y^2} = \frac{1}{(1+y)^2} \sum_t \frac{C_t}{(1+y)^t} \times (t^2 + t)$

- $\Delta P = -\left(\frac{1}{(1+y)} \sum_t \frac{C_t}{(1+y)^t} \times t\right) \Delta y + \left(\frac{1}{2(1+y)^2} \sum_t \frac{C_t}{(1+y)^t} \times (t^2 + t)\right) (\Delta y)^{\ 2}$

- Let's go over this result step by step.

What determines bond price changes?

- Let us study the first part and consider the percentage change in the bond price:

$$\frac{\Delta P}{P} = -\left(\frac{1}{(1+y)} \times \underbrace{\frac{1}{P} \times \sum_t \frac{C_t}{(1+y)^t} \times t}_{Duration\ D}\right) \Delta y$$

- Duration: Value-weighted average of how long you have to wait for the present value of each of the cash flows C_t of a bond with price P:

$$D = \frac{1}{P} \sum_t \frac{C_t}{(1+y)^t} \times t$$

- Thus, a first-order approximation to the percentage change in the bond price for a change in y can be written as:

$$\frac{\Delta P}{P} \approx - \underbrace{\frac{D}{(1+y)}}_{Modified\ Duration} \times \Delta y$$

Duration: examples

- For zero coupon bonds: duration = maturity

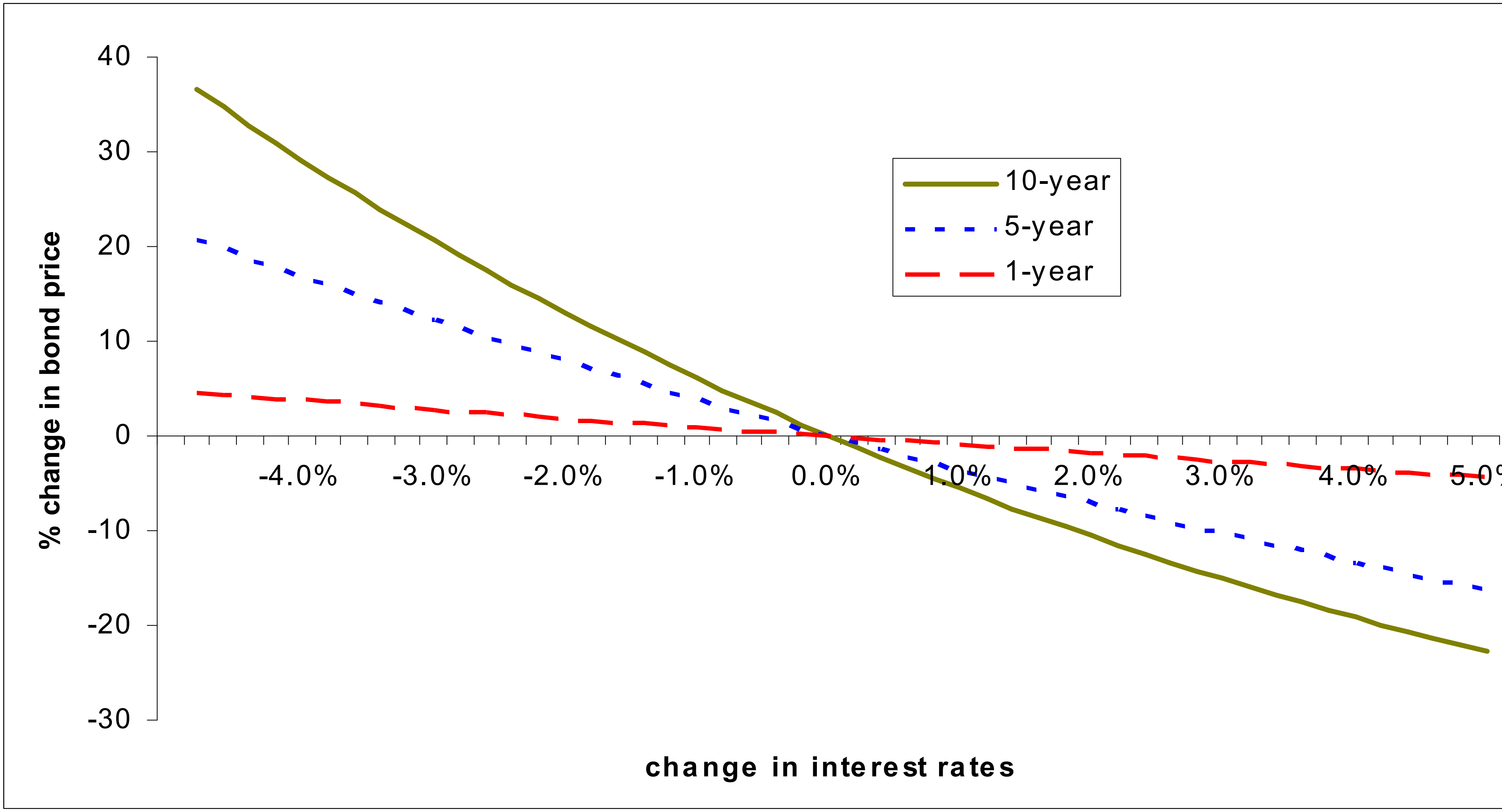
- Duration of a growing perpetuity: (1+y)/(y-g)
  - Often used to approximate the duration of the stock market: (1+10%)/(10-3%) ≈ 15 years.

- Approximate the change in price of the 15 year bond using duration and compare to the actual change.

  - Duration = 9.996 years

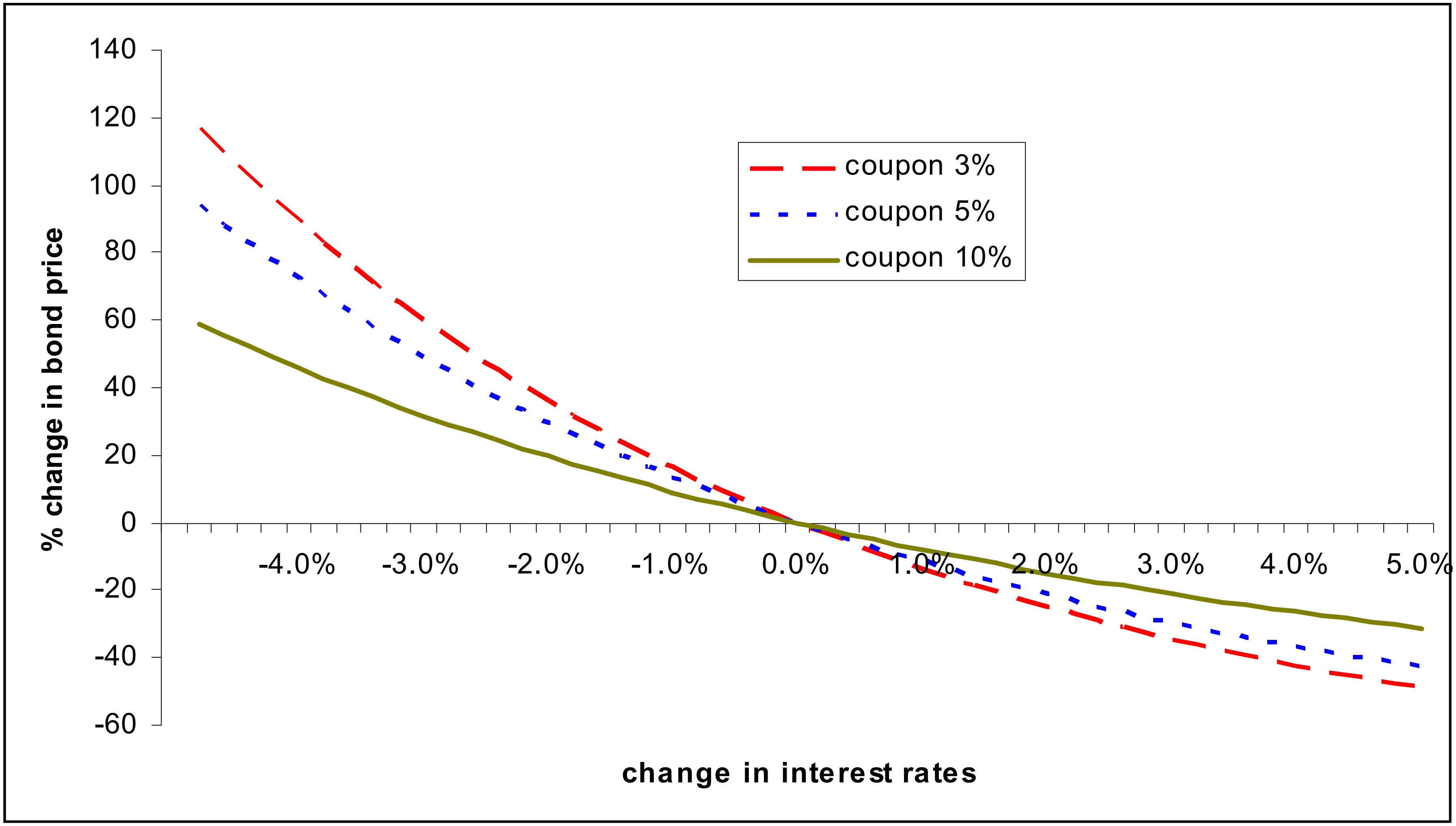
	Approximated % change	Approximated new price	Actual new price
to 4%	-9.996/(1+5%)*-1%=0.095	143.623	144.474
to 6%	-0.095	118.655	119.424

Price risk (1/2)



Long term bonds are more sensitive to interest rate changes than short-term bonds. Why?

Price risk (2/2)



High-coupon bonds are less sensitive than low-coupon bonds. Why?

Convexity (1/2)

- First-order, duration-based approximation works well for smallest changes in yields.

- For larger swings in yields, consider second-order approximation:

$$\frac{\Delta P}{P} = -\frac{D}{(1+y)} \times \Delta y + \frac{1}{2} Convexity \times (\Delta y)^{2}$$

where $Convexity = \frac{1}{P(1+YTM)^2} \sum_t \frac{C_t \times (t^2+t)}{(1+YTM)^t}$

- Intuition for convexity:
  - YTM ↑, duration ↓, interest rate sensitivity ↓
  - YTM ↓, duration ↑, interest rate sensitivity ↑

  → Higher (lower) sensitivity to decreases (increases) in yields

Back to our example

- Approximate the change in price of the 15 year bond using duration and convexity

  - Duration = 9.996 years
  - Convexity = 123.29

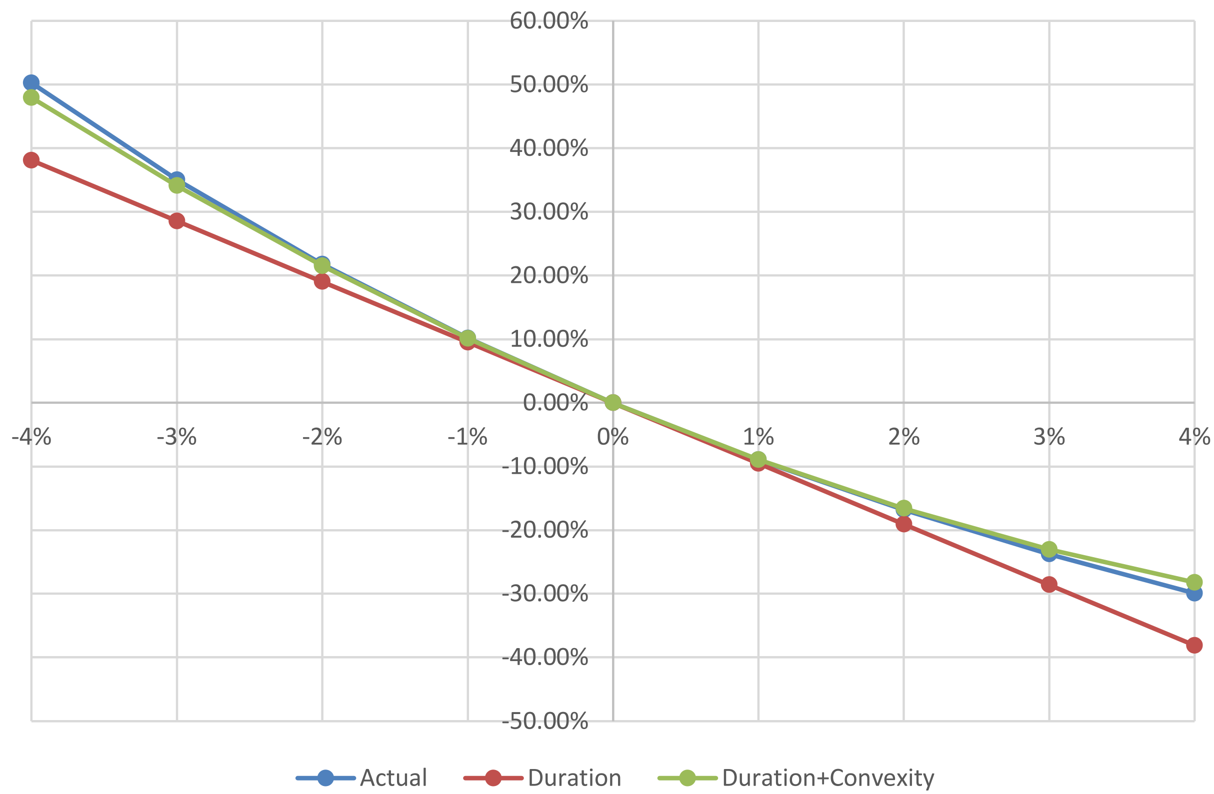
	Approximated % change	Approximated new price	Actual new price	Abs(Diff)
First order				
to 4%	0.095	143.623	144.474	0.85
to 6%	-0.095	118.655	119.424	0.77
Second order				
to 4%	0.101	144.432	144.474	0.04
to 6%	-0.089	119.463	119.424	0.04

  - We are getting closer due to non-linearity in 2^{nd} vs 1^{st} order approximation!

  - Still, even larger changes in yields require even higher order approximations:

Convexity

(Approximated) Returns for changes in the yield from 5% to 1%, 2%, …, 9%



Interest rate risk management

- In interest rate risk management (for bond funds, pension funds, insurance companies etc) duration-matching is of first-order importance, while convexity-matching is of second-order importance.

- Example: Pension fund asset-liability management (ALM)
  - Liabilities: long-term pension promises with duration ~20 years
  - Assets are chosen to match this duration

    Duration matching: D^*_{Assets} x Assets = $D^*_{\text{Liabilities}}$ x Liabilities
    (using D* is modified duration)

  - Unsurprisingly, pension funds invest large fraction of portfolio in long-term bonds and long-term assets, such as real estate.

  - Similarly, we can match convexity: C_{Assets} x Assets = $C_{\text{Liabilities}}$ x Liabilities

Interest rate risk management

- Suppose you have invested 1 million $ in the 15 year bond with
  - YTM = 5%, Price = 131.14$
  - Duration = 9.996 years
  - Convexity = 123.29

- You are worried about interest rate risk over the next few months and desire to hedge against it. Suppose there are two very liquid zero-coupon Treasury bonds that you decide to hedge with:
  - 5-year maturity with a 3% yield
  - 10 year maturity with a 4% yield

1. How much do you need to short and/or long of each bond to hedge interest rate risk (to a second-order approximation)?

2. Analyze how good your hedge is if all yields suddenly increase by 1%.

Solution

1. We have two equations, which we need to solve for two unknowns:
Use D* = modified duration = D/(1+y)

(a) D*bond = w5 x D*5 + w10 x D*10
(b) Cbond = w5 x C5 + w10 x C10

(a) - D*5/C5 x (b) = D*bond - D*5/C5 x Cbond =
w5 x (D*5 - D*5/C5 x C5) + w10 x (D*10 - D*5/C5 x C10)

→ w10 = D*bond - D*5/C5 x Cbond / (D*10 - D*5/C5 x C10)
→ w5 = (D*bond - w10 x D*10) / D*5

w10 1.485 → you need to short 1.485*1 million worth of 10 year zeros
w5 -0.980 → you need to long 0.980*1 million worth of 5 year zeros

2. By approximation, you have hedged perfectly: the hedged return = 0.
In reality, you are 0.4 basis points off, but this is less than 1/1000 of the bond's return!

Returns			
	Approximated		
	First order	Second order	Actual
ZC5	-0.0485	-0.0471	-0.0472
ZC10	-0.0962	-0.0911	-0.0913
Payoffs:			
Hedge portfolio	0.0952	0.0890	0.0893
Bond	-0.0952	-0.0890	-0.0893
Hedged	0.00000	0.00000	-0.00004

Issues with duration (+convexity) matching

- Requires active rebalancing
  - Duration and convexity change with yields and as time passes
- All our calculations are made using the YTM of bonds, which is a weighted average of yields of different maturities
  - For instance, two bonds with the same YTM may be differentially impacted when the slope and curvature of the term structure change, while the level of yields remains the same.
  - Take derivatives with respect to r_1, r_2, r_3 ... (partial or key duration and convexity) or with respect to the level, slope and curvature factors that drive 99.X% of the variation in these yields.

Some practical issues

Bond pricing conventions

- In practice, bond prices are expressed as % of face value (100, basically)
  - Number after the hyphen denotes 32nds (also called “ticks”)
  - A “+” sign means half a tick

- Examples
  - Price of 101-12 means 101.375% of face value
  - Price of 108-31+ means (108 + 31.5/32)% of face value

- Why?
  - Traditionally, trades and quotes were communicated by hand
  - In exercises, we will work with decimals.
    - Does this tick size and reporting method make sense given today’s technology?
    - There are more oddities in bond pricing that abstract from what is important:

Accrued interest (1/3)

- When purchasing a bond between coupon dates, the buyer must pay accrued interest to the seller in excess of the quoted price
  - The holder of the bond at coupon payment date receives the full amount of interest, whereas the seller actually held the bond over part of the current coupon period
  - Price that is paid including accrued interest: dirty price or full price
    - This is the present value of future cash flows (*and thus the theoretically correct price that we have considered so far*!)
  - Quoted price before accrued interest: clean price or flat price
    = Dirty price – accrued interest

Accrued interest (2/3)

- **Example**: what is the accrued interest for a 4% T-Note that matures on 31-12-2018, if the bond is transacted on 31-3-2018?

  1. Bond pays semi-annual coupons of 4/2 each December- and June-end until maturity;

  2. Number of days between December 31 2017 (last coupon date) and March 31 2018 (transaction date): 90;

  3. Number of days between coupon dates (December 31 2017 and June 30 2018): 181;

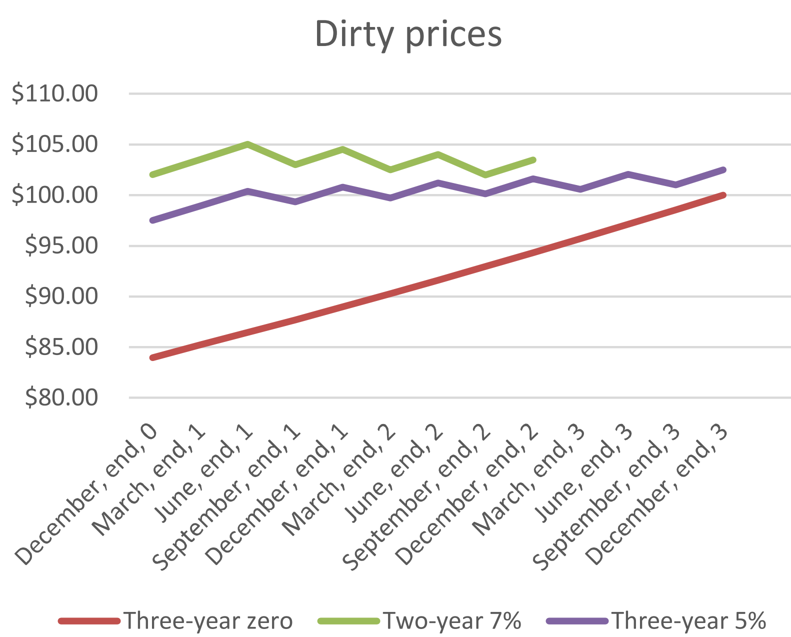
  4. <u>Accrued interest=4/2 x 90/181=0.99% of face value.</u>

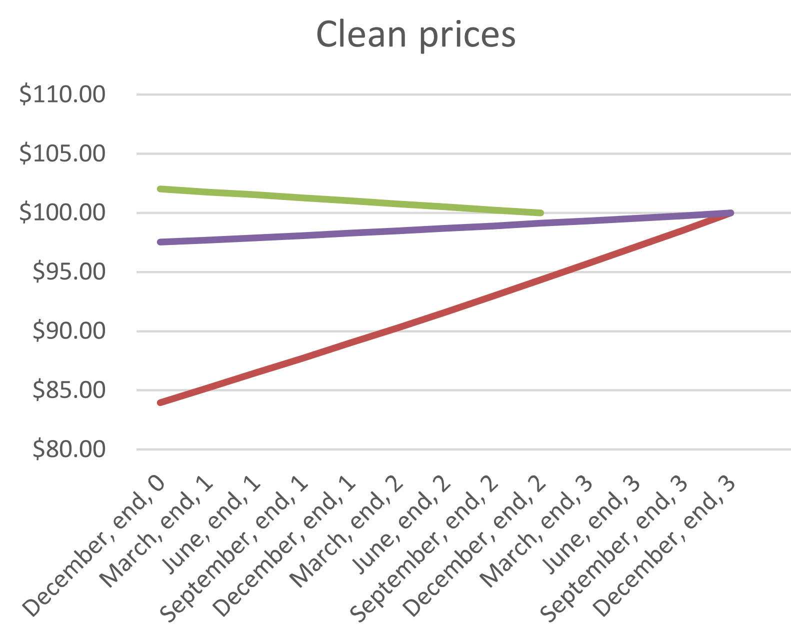
     - Buyer of the bond will pay dirty price to seller, which equals:

     quoted price (excl. June 30 2018 coupon) + accrued interest (claim to June 30 2018 coupon)

Accrued interest (3/3)

- The distinction between dirty and clean prices is important for quickly interpreting charts (e.g., in financial press)
  - Clean prices get rid of zig-zag pattern: closing in on next coupon P↑, but as coupon is paid P↓
  - Helps pinning down source of a change in bond price when interest rates vary over time and with maturity...





- Dirty = PV(of all future cash flows), Clean = Dirty - Accrued
- Accrued = 3.5 on (or just before) coupon date, 0.5*3.5 in between coupon dates

Wrapping up

- Exact calculation of bond transaction price depends on market conventions and fineprint
  - Coupons are usually paid semi-annually, i.e., 4% coupon means 4%/2 = 2% every 6 months.
  - In US yields are often quoted as APR's with semi-annual compounding, whereas in Europe yields are often quoted as EAR's.
  - Day count conventions: actual/360, actual/actual, …?
  - I don't find this super-interesting: as long as everyone agrees on the exact formula with which the price (or payoff, more generally) is calculated...

Corporate bonds

Overview of fixed-income markets

Category	Issuer	Outstanding ($Tr.)	% of Total
Treasury	Treasury	12.7	32%
Mortgage-related	Federal / Private	8.7	22%
Corporate Debt	Corporations, Financial Institutions	8.1	21%
Municipal	Local Governments	3.7	9%
Money markets	Corporations, Financial Institutions	2.9	7%
Federal Agency securities	Federal Agencies	2.0	5%
Asset-backed	Financial Institutions	1.4	4%

Source: SIFMA, statistics, Q2 2015

Default risk and expected returns

- When the issuer is not the Treasury, there is default risk
  - Under default, lender does not receive the bond's promised payments, but only some recovery value which depends on
    - The bond's provisions such as seniority and collateral
    - The result of a complex bargaining process in the bankruptcy courts
    - The liquidation value of the firm's assets

– For a bond without default risk, YTM is a good estimate of expected return
  – YTM is a weighted average of the return you expect to make in each year over the life of the bond

– For a bond with default risk, YTM overstates the expected return
  – Recall: the yield is the single discount rate that sets the PV of the bond's promised (≠expected) payments equal to its current price
  – Risk-return trade-off: ceteris paribus, bonds with higher default risk have higher yields, but this higher yield translates less than 1-for-1 to higher expected return.

The expected return of risky debt

- Broadly two methods to adjust yield for default risk:
  1. Discount (i) probability of x (ii) loss in case of **default**
  2. Use the CAPM

- Each method relies on information from ratings

Moody' s	Standard & Poor's	Safety
Aaa	AAA	The strongest rating; ability to repay interest and principal is very strong.
Aa	AA	Very strong likelihood that interest and principal will be repaid
A	A	Strong ability to repay, but some vulnerability to changes in circumstances
Baa	BBB	Adequate capacity to repay; more vulnerability to changes in economic circumstances
Ba	BB	Considerable uncertainty about ability to repay.
B	B	Likelihood of interest and principal payments over sustained periods is questionable.
Caa	CCC	Bonds in the Caa/CCC and Ca/CC classes may already be
Ca	CC	in default or in danger of imminent default
C	C	C-rated bonds offer little prospect for interest or principal on the debt ever to be repaid.

Transition matrix (Moody's)

Historically, year t AAA bonds never defaulted in year t+1!

Average One-Year Letter Rating Migration Rates, 1920-2010

From/To:	Aaa	Aa	A	Baa	Ba	B	Caa	Ca_C	WR	Default
Aaa	86.556%	8.214%	0.827%	0.162%	0.032%	0.001%	0.001%	0.000%	4.206%	0.000%
Aa	1.195%	84.152%	7.238%	0.740%	0.167%	0.037%	0.006%	0.005%	6.392%	0.068%
A	0.079%	2.917%	84.575%	5.549%	0.683%	0.121%	0.029%	0.009%	5.946%	0.092%
Baa	0.041%	0.286%	4.467%	81.252%	4.996%	0.789%	0.131%	0.015%	7.742%	0.280%
Ba	0.007%	0.083%	0.474%	5.923%	73.373%	6.838%	0.576%	0.068%	11.367%	1.292%
B	0.006%	0.050%	0.154%	0.592%	5.768%	71.304%	5.551%	0.534%	12.260%	3.781%
Caa	0.000%	0.021%	0.029%	0.189%	0.808%	8.067%	62.742%	3.841%	11.945%	12.358%
Ca-C	0.000%	0.026%	0.113%	0.061%	0.468%	3.265%	7.691%	51.801%	13.225%	23.350%

Yield spread

Reuters Corporate Spreads for Industrials
03/28/2014

Rating	1 yr	2 yr	3 yr	5 yr	7 yr	10 yr	30 yr
Aaa/AAA	5	8	12	18	28	42	65
Aa1/AA+	10	18	25	34	42	54	77
Aa2/AA	14	29	38	50	57	65	89
Aa3/AA-	19	34	43	54	61	69	92
A1/A+	23	39	47	58	65	72	95
A2/A	24	39	49	61	69	77	103
A3/A-	32	49	59	72	80	89	117
Baa1/BBB+	38	61	75	92	103	115	151
Baa2/BBB	47	75	89	107	119	132	170
Baa3/BBB-	83	108	122	140	152	165	204
Ba1/BB+	157	182	198	217	232	248	286
Ba2/BB	231	256	274	295	312	330	367
Ba3/BB-	305	330	350	372	392	413	449
B1/B+	378	404	426	450	472	495	530
B2/B	452	478	502	527	552	578	612
B3/B-	526	552	578	604	632	660	693
Caa/CCC+	600	626	653	682	712	743	775
US Treasury Yield	0.13	0.45	0.93	1.74	2.31	2.73	3.55

- Yield spread = yield to maturity risky bond – yield to maturity risk-free government bond
- 1 year CAA yield = 600 bps + 13 bps= 6.13% vs 0.18% for AAA

Method 1: Adjusting the yield to maturity

Consider a one-year bond with YTM of y. For each $1 invested in the bond today, the issuer promises to pay $(1 + y) in one year. Suppose the bond will default with probability p, and L is the expected loss per $1 of debt in the event of default.

So the expected return of the bond

$$r_d = (1 - p)(1+y) + p(1+y - L) - 1$$
$$= y - pL$$

= Yield to Maturity − Prob(default) × Expected Loss

The probability of and loss in case of default

- This adjustment of the yield can be large!
  - The average loss rate for corporate bonds in case of default is about 60%
  - Default probabilities are easily obtained per rating category:

Rating:	**AAA**	**AA**	**A**	**BBB**	**BB**	**B**	**CCC**	**CC–C**
Default Rate:								
Average	0.0%	0.1%	0.2%	0.5%	2.2%	5.5%	12.2%	14.1%
In Recessions	0.0%	1.0%	3.0%	3.0%	8.0%	16.0%	48.0%	79.0%

Independent of year t being a normal or recession year, AAA bonds never defaulted in year t+1!

- Combining, expected return of a B rated bond =
  - Normal times: Yield – [5.5%*60% = 3.3%]
  - Recession: Yield – [16%*60% = 9.6%]

Method 2: CAPM expected return on debt

- Calculating the CAPM beta for an individual corporate bond is more difficult than for a stock
  - Bonds are traded infrequently and returns not covered in standard databases.

- Fortunately, we can use ratings again:

By Rating	**A and above**	**BBB**	**BB**	**B**	**CCC**
Avg. Beta	<0.05	0.10	0.17	0.26	0.31
By Maturity	(BBB and above)	1-5 year	5-10 year	10-15 year	>15 year
Avg. Beta		0.01	0.06	0.07	0.14

- Note that the rating category ignores industry, so beta should probably be scaled up (down) in procyclical (countercyclical) industries

Example

Estimating the Debt Cost of Capital

Problem

In mid-2012, homebuilder KB Home had outstanding 6-year bonds with a yield to maturity of 6% and a B rating. If corresponding risk-free rates were 1%, and the market risk premium is 5%, estimate the expected return of KB Home's debt.

Given the low rating of debt, we know the yield to maturity of KB Home's debt is likely to significantly overstate its expected return. Using the information from the previous slides, we can use either of the two methods to estimate the expected return:

Method 1: E(Rd) = yield – p*L = 6% - 5.5%*60% = 2.7%

Method 2: E(Rd) = 1% + 0.26*5% = 2.3%

While both estimates are rough approximations, they confirm that the expected return of KB Home's debt is well below its promised yield of 6%!

We'll come back to bond prices when there is default risk later, because we can price risky bonds w. option-pricing techniques.