MACROECONOMETRICS
Master in Economics

Markov Regime Switching Models

Markov Regime Switching Models

- Generalisation of the simple dummy variables approach to model structural changes in the parameters of a model

- Allow regimes (called states) to occur several periods over time

- In each period t the state is denoted by s_t .

- There can be m possible states: $s_t = 1,\dots, m$

Markov Regime Switching Models

Example: Dating Business Cycles

Let y_t denote the GDP growth rate

- Simple model with m= 2 (only 2 regimes)

$$y_t = \mu_1 + e_t \quad \text{when} \quad s_t = 1$$
$$y_t = \mu_2 + e_t \quad \text{when} \quad s_t = 2$$
$$e_t \text{ i.i.d. } N(0,\sigma^2)$$

- Notation using dummy variables:

$$y_t = \mu_1 D_{1t} + \mu_2 (1\text{-}D_{1t}) + e_t$$

where $D_{1t} = 1$ when $s_t = 1$,

$= 0$ when $s_t = 2$

Markov Regime Switching Models

- How does s_t evolve over time?

  – Model: Markov switching :

  $$P[s_t \mid s_1, s_2, ..., s_{t-1}] = P[s_t \mid s_{t-1}]$$

  – Probability of moving from state i to state j:

  $$p_{ij} = P[s_t = j \mid s_{t-1} = i]$$

Markov Regime Switching Models

- Example with only 2 possible states: m=2
- Switching (or conditional) probabilities:

$$\text{Prob}[s_t = 1 \mid s_{t\text{-}1} = 1] = p_{11}$$

$$\text{Prob}[s_t = 2 \mid s_{t\text{-}1} = 1] = 1 - p_{11}$$

$$\text{Prob}[s_t = 2 \mid s_{t\text{-}1} = 2] = p_{22}$$

$$\text{Prob}[s_t = 1 \mid s_{t\text{-}1} = 2] = 1 - p_{22}$$

Markov Regime Switching Models

- Unconditional probabilities:

$$\text{Prob}[s_t = 1] = (1\text{-}p_{22})/(2 - p_{11} - p_{22})$$

$$\text{Prob}[s_t = 2] = 1 - \text{Prob}[s_t = 1]$$

Examples:

If p_{11} =0.9 and p_{22} =0.7 then Prob[s_t = 1] =0.75

If p_{11} =0.9 and p_{22} =0.9 then Prob[s_t = 1] =0.5

Markov Regime Switching Models

Estimation:

- Maximize Log-Likelihood
- Hamilton filter - Filtered probabilities:

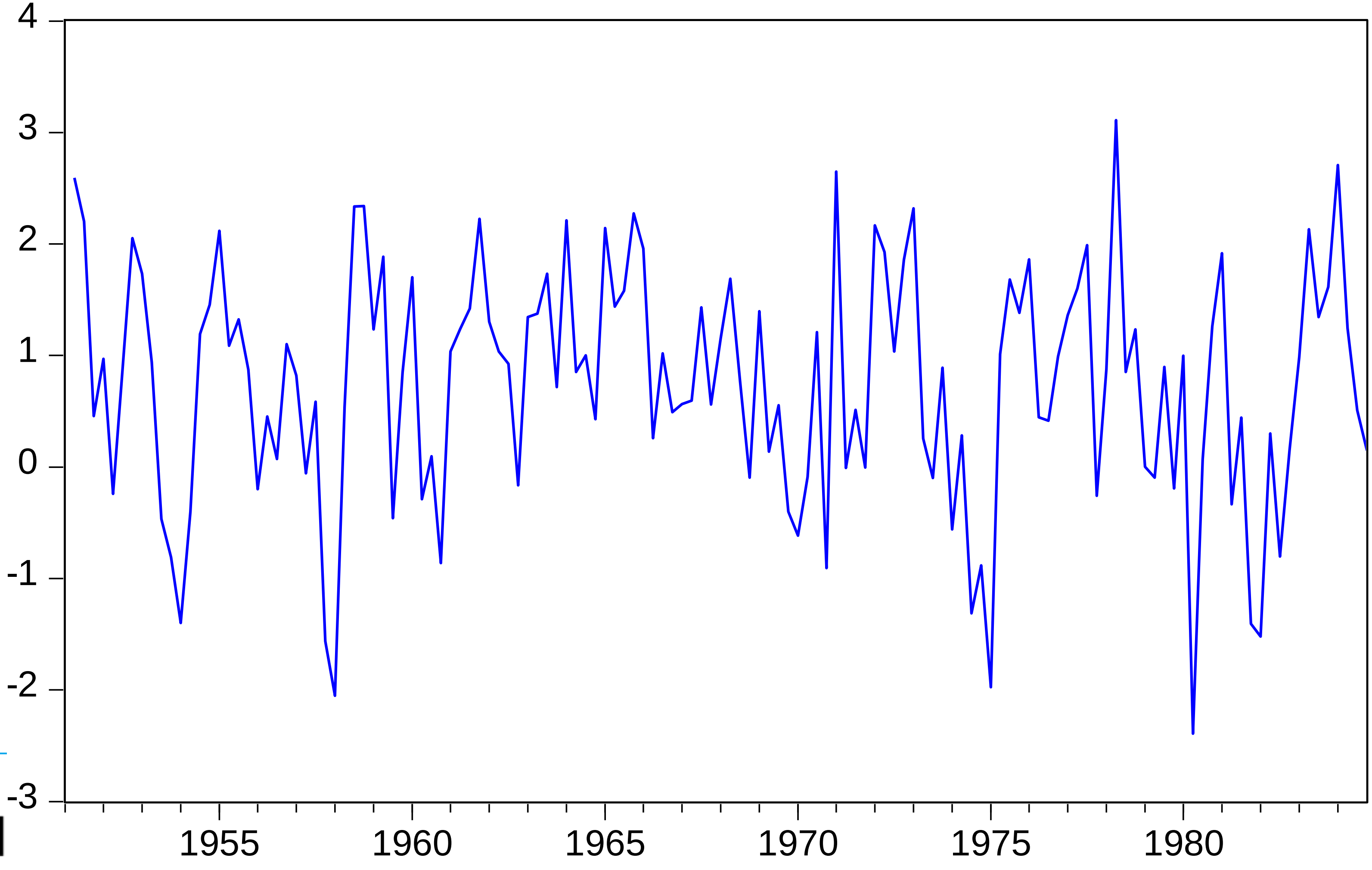
$$\pi_{1,t} = \text{Prob}[s_t = 1 \mid y_1, y_2, \ldots, y_t]$$

$$\pi_{2,t} = \text{Prob}[s_t = 2 \mid y_1, y_2, \ldots, y_t] = 1 - \pi_{1,t}$$

for t=1, …, T

Markov Regime Switching Models

Hamilton(1989)-GNP growth: gnp_hamilton.wf1



Markov Regime Switching Models

Variable	Coefficient	Std. Error	z-Statistic	Prob.
Regime 1				
C	1.104278	0.130904	8.435813	0.0000
Regime 2				
C	-0.486848	0.344455	-1.413388	0.1575
Common				
LOG(SIGMA)	-0.182101	0.075204	-2.421439	0.0155
Transition Matrix Parameters				
P11-C	2.314974	0.558760	4.143056	0.0000
P21-C	-0.785850	0.607785	-1.292973	0.1960

Markov Regime Switching Models

Estimated model with quarterly data,

y_t = 100*dlog(gnp):

$y_t = 1.1 + e_t$ when $s_t = 1$

$y_t = -0.5 + e_t$ when $s_t = 2$

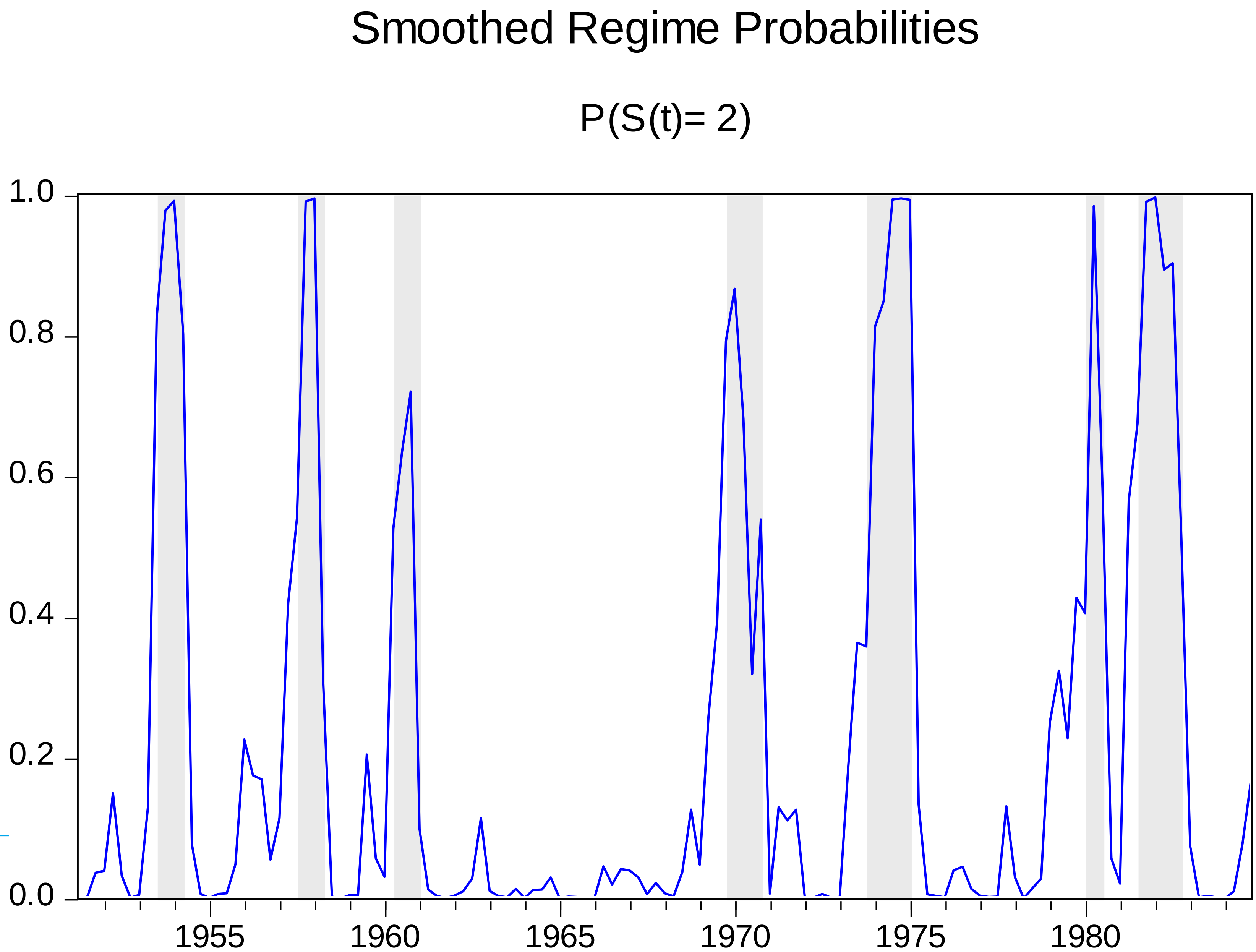
e_t i.i.d. $N(0,0.8^2)$

$p_{11} = 0.91$

$p_{22} = 0.69$

Expected durations of regimes 1 and 2: 11 and 3 quarters respectively

Markov Regime Switching Models



Markov Regime Switching Models

Time-Varying Transition Probabilities

- Example with only 2 possible states: $m=2$
- Switching (or conditional) probabilities:

$$\text{Prob}[s_t = 1 \mid s_{t\text{-}1} = 1] = p_{11} = \text{F}_{11}(\text{ a}_{11}+\text{b}_{11}\text{ xt})$$

$$\text{Prob}[s_t = 2 \mid s_{t\text{-}1} = 2] = p_{22} = \text{F}_{22}(\text{ a}_{22}+\text{b}_{22}\text{ xt})$$

- EViews uses multinomial logit for function F(.)

Markov Regime Switching Models

Time-Varying Transition Probabilities

- Example: Kim and Nelson (1999)

  kimnelson_tvp.wf1

Yt = log growth rate of industrial production (DLOGIP)

Xt = log growth rate of the Composite Index of Eleven Leading Indicators (DLOGIDX) → as a business-cycle predictor

Markov Regime Switching Models

Time-Varying Transition Probabilities

Variable	Coefficient	Std. Error	z-Statistic	Prob.
Regime 1				
C	0.517304	0.077907	6.640016	0.0000
Regime 2				
C	-0.865887	0.154375	-5.608983	0.0000
Common				
AR(1)	0.189474	0.050835	3.727263	0.0002
AR(2)	0.079344	0.051193	1.549889	0.1212
AR(3)	0.110945	0.052365	2.118690	0.0341
AR(4)	0.122252	0.050906	2.401551	0.0163
LOG(SIGMA)	-0.362469	0.038331	-9.456332	0.0000

Markov Regime Switching Models

Time-Varying Transition Probabilities

Time-varying transition probabilities:
P(i, k) = P(s(t) = k | s(t-1) = i)
(row = i / column = j)

		1	2
Mean	1	0.951975	0.048025
	2	0.200479	0.799521

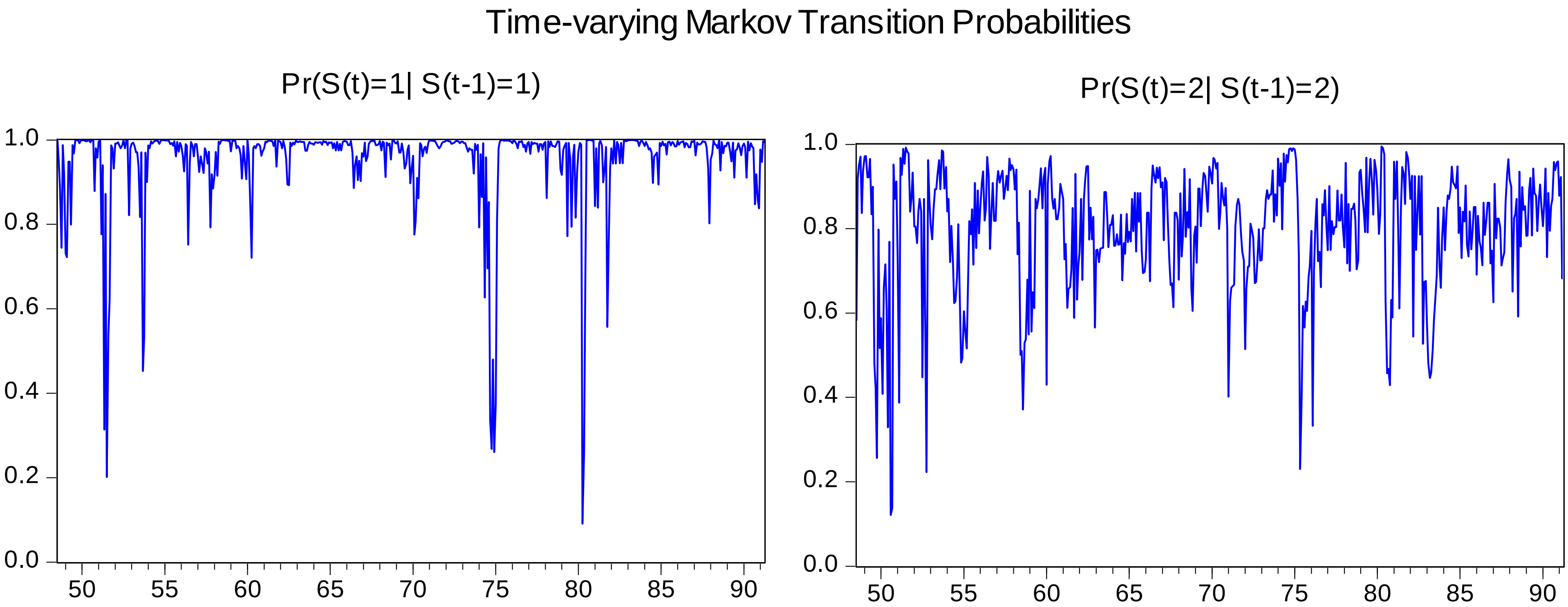
		1	2
Std. Dev.	1	0.112804	0.112804
	2	0.145240	0.145240

Time-varying expected durations:

	1	2
Mean	506.3750	10.47908
Std. Dev.	2959.387	16.94910

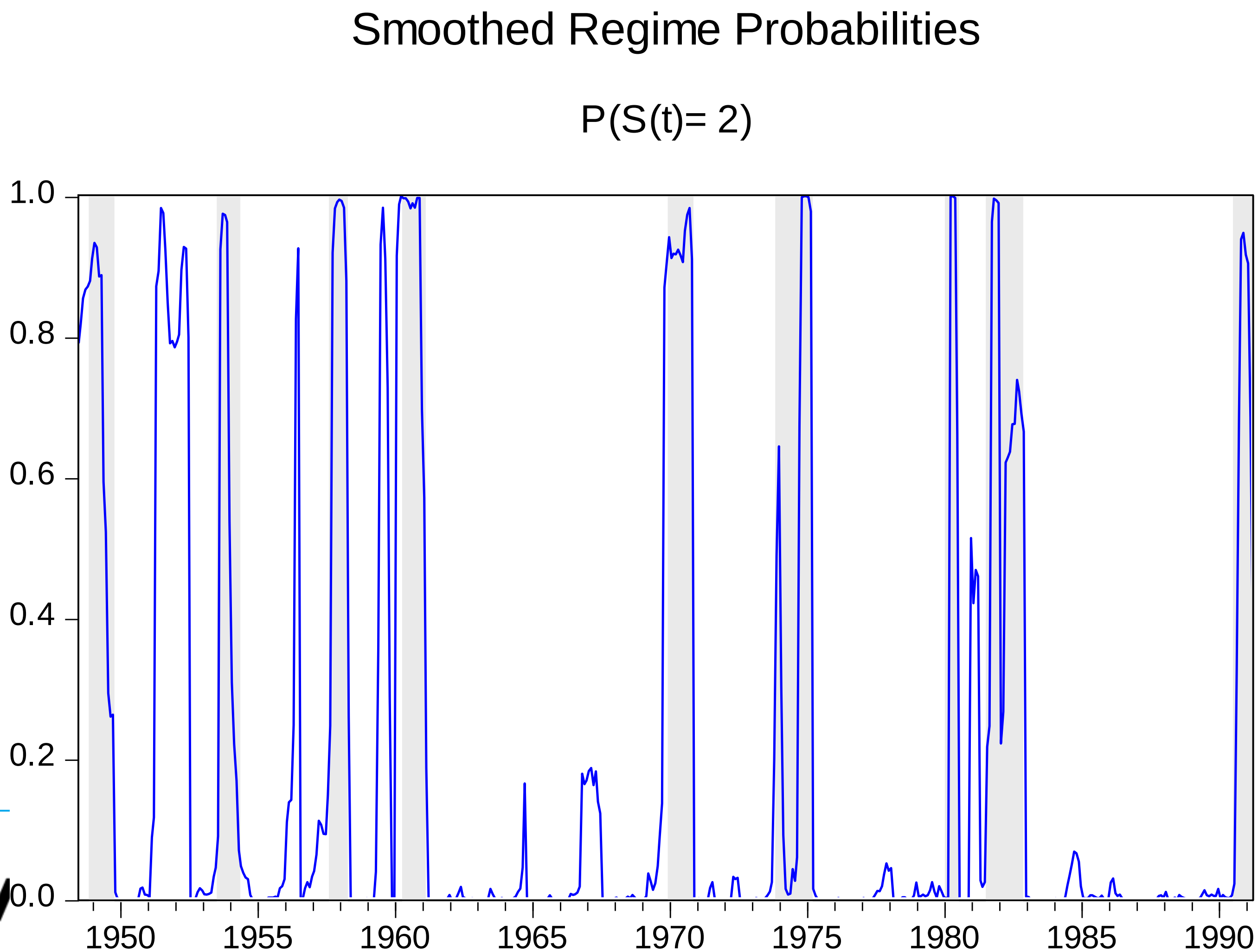
Markov Regime Switching Models

Time-Varying Transition Probabilities



Markov Regime Switching Models

Time-Varying Transition Probabilities



Markov Regime Switching Models

Regime Heteroskedasticity

Kim and Nelson (1999) - ew_excs.WF1

Yt = monthly CRSP equal-weighted excess returns

Assumptions:

- 3 regimes
- Mean excess returns = 0
- Variance excess returns vary across the 3 regimes