Regression with Time Series Variables

Regression with Time Series Variables

- With time series regression, Y might not only depend on X, but also lags of Y and lags of X
- *Autoregressive Distributed lag* (or ADL(p, q)) model has these features:

$$\begin{aligned} Y_t = \alpha + \delta t + \rho_1 Y_{t-1} + .. + \rho_p Y_{t-p} \\ +\beta_0 X_t + \beta_1 X_{t-1} + .. + \beta_q X_{t-q} + \varepsilon_t. \end{aligned}$$

- Here we mostly focus on one X, but same ideas hold for case with several.
- Estimation and interpretation depends on whether, X and Y, are stationary or not.
- Note: X and Y must have the same stationarity properties (either must both be stationary or both have a unit root).
- Before running any time series regression, you should do unit root tests for every variable in your analysis.

Time Series Regression when X and Y are Stationary

- When X and Y are stationary, standard OLS methods ADL(p, q) are fine
- E.g. hypothesis testing can be done using t-statistics or F-statistics. Sequential testing procedures can be used to select p and q, etc.
- Can rewrite ADL in more convenient form:

$$\Delta Y_t = \alpha + \delta t + \phi Y_{t-1} + \gamma_1 \Delta Y_{t-1} + .. + \gamma_{p-1} \Delta Y_{t-p+1}$$
$$+\theta X_t + \omega_1 \Delta X_{t-1} + .. + \omega_{q-1} \Delta X_{t-q+1} + \varepsilon_t$$

- Note: this form of ADL model less likely to run into multicollinearity problems.

- One thing researchers often calculate is the *long run* or *total multiplier*
- To motivate: suppose that X and Y are in an equilibrium or steady state. Then X rises (permanently) by one unit, affecting Y, which starts to change, settling down in the long run to a new equilibrium value.
- Difference between old and new equilibrium values for Y is long run effect of X on Y and called long run multiplier.
- This multiplier is often of great interest for policymakers who want to know the eventual effects of their policy changes.
- For ADL(p, q) model long run multiplier is:

$$-\frac{\theta}{\phi}$$

Time Series Regression When Y and X Have Unit Roots

- Now assume Y and X have unit roots.
- In practice, you would do Dickey-Fuller test to confirm this.

Spurious Regression

- Consider the regression:

$$Y_t = \alpha + \beta X_t + \varepsilon_t$$

- OLS estimation of this regression can yield results which are completely wrong.
- Even if the true value of β is 0, OLS can yield an estimate, $\widehat{\beta}$, which is very different from zero.
- Statistical tests (using the t-statistic or P-value) may indicate that β is not zero.
- Furthermore, if $\beta = 0$, then the R^2 should be zero. In fact, the R^2 will often be quite large.
- If Y and X have unit roots then *all* the usual regression results might be misleading and incorrect.
- This is called *spurious regression problem*.

- We will not prove, but stress the practical implication:
- With the one exception of cointegration (see below), *you should never run a regression of Y on X if the variables have unit roots. Same thing holds for ADL.*

Cointegration

- If Y and X are cointegrated, do not need to worry about spurious regression problem.
- Cointegration has nice economic intuition.
- Intuition for cointegration: errors in the above regression model are:

$$\varepsilon_t = Y_t - \alpha - \beta X_t$$

- Errors are just a linear combination of Y and X.
- Since X and Y both have unit roots you would expect the error to also have unit root.
- After all, if you add two things with a certain property together the result generally tends to have that property.

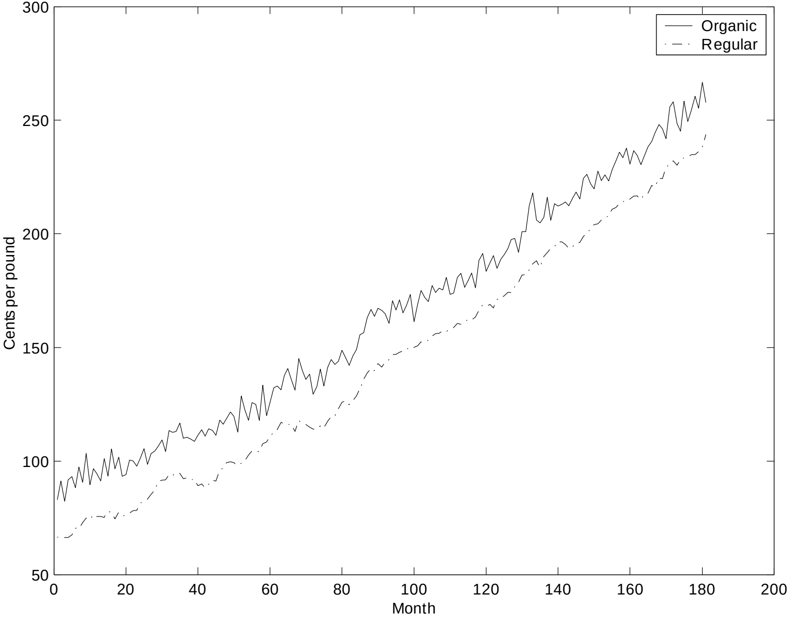
- Error does indeed usually have a unit root (this is what causes spurious regression problem).
- However, it is possible that the unit roots in Y and X "cancel each other out" and that the resulting error is stationary. This is *cointegration*,
- To summarize: if Y and X have unit roots, but some linear combination of them is stationary, then Y and X are cointegrated.

- Intuition for cointegration:
- X and Y have stochastic trends. However, if they are cointegrated, the error does not have such a trend. Y and X will not diverge from one another; Y and X will trend together.
- In economic model involving an equilibrium concept, ε is the equilibrium error. If Y and X are cointegrated then the equilibrium error stays small.
- If Y and X are cointegrated then there is an equilibrium relationship between them. If they are not, then no equilibrium relationship exists.
- If Y and X are cointegrated then their trends will cancel each other out.

Example: Cointegration Between the Prices of Two Goods

- Similar goods should be close substitutes for each other and therefore their prices should be cointegrated.
- Data for 181 months on the prices of regular oranges and organic oranges in a certain market.
- Although the prices of these two products will fluctuate due to the vagaries of supply and demand, market forces will always keep the price difference between the two goods roughly constant.
- Figure provides visual evidence for cointegration

Figure 7.1: The Prices of Regular and Organic Oranges



- Many examples of cointegration, especially in macroeconomics and finance.
- Short term and long term interest rates
- Purchasing power parity and the permanent income hypothesis, theories of money demand, etc. etc.

Estimation and Testing with Cointegrated Variables

- If Y and X are cointegrated, then the spurious regression problem does not apply and OLS methods are fine.
- Coefficient from this regression is the long run multiplier.
- Regression of Y on X is called *cointegrating regression*.
- But it is important to verify that Y and X are cointegrated.
- Many tests for cointegration exist and computer software packages like Gretl will do several tests
- We will cover two tests: *Engle-Granger* test and *Johansen Test*
- Basic idea of Engle Granger test: test for unit root in residuals of cointegrating regression
- Remember: cointegration occurs if errors do not have unit root (and residuals are estimates of errors)

- Engle-Granger test has following steps:

1. Run the regression of Y on an intercept and X and save the residuals.
2. Carry out a Dickey-Fuller test on the residuals (without including a deterministic trend).
3. If the unit root hypothesis is rejected then conclude that Y and X are cointegrated. However, if the unit root is accepted then conclude cointegration does not occur.

- Note: Critical values (see Table 7.2 in textbook) are slightly different from the critical values for the Dickey-Fuller test.
- Note: Usually you not include a deterministic trend when doing this test (i.e. if it were included it could mean the errors could be growing steadily over time. This would violate the idea of cointegration.)
- Regression in Step 2 above is usually:

$$\Delta\widehat{\varepsilon}_t = \phi\widehat{\varepsilon}_{t-1} + \gamma_1\Delta\widehat{\varepsilon}_{t-1} + .. + \gamma_{p-1}\Delta\widehat{\varepsilon}_{t-p+1} + u_t$$

- Remember: cointegration is found if we reject hypothesis of unit root in residuals (i.e. null hypothesis “no cointegration” and we conclude “cointegration is present” only if we reject unit root in errors hypothesis)

Table 7.2: Critical Values for the Engle-Granger Test				
	$T = 25$	$T = 50$	$T = 100$	$T = \infty$
1% Critical Value	-4.37	-4.12	-4.01	-3.90
5% Critical Value	-3.59	-3.46	-3.39	-3.33

Example: Cointegration Between the Prices of Two Goods (continued)

- Regression of Y = the price of organic oranges on X = the price of regular yields:
$$\widehat{Y}_i = 20.686 + 0.996X_i$$
- Engle-Granger test: carry out a unit root test on the residuals, $\widehat{\varepsilon}_t$, from this regression.
- Remember that first step in doing the unit root test is to correctly select the lag length.
- Using the sequential strategy, turns out that AR(1) specification for the residuals is appropriate.
- Dickey-Fuller strategy says we should regress $\Delta\widehat{\varepsilon}_t$ on $\widehat{\varepsilon}_{t-1}$.

- t-statistic on $\widehat{\varepsilon}_{t-1}$ in the resulting regression is -14.54.
- Since sample size is 180 and Table 7.2 says that the 5% critical value is between -3.39 and -3.33, we reject the unit root hypothesis and conclude that the residuals do not have a unit root.
- Thus, the two price series are indeed cointegrated.
- Since cointegrated, do not need to worry about the spurious regressions problem.
- Hence, estimate of the long run multiplier is 0.996.

More Practical Issues in Cointegration Testing

- Note: we have focussed on two variables, Y and X. In practice, you may have many more variables.
- Example: consider the three variables: income (Y), consumption (C) and investment (I).
- Some macroeconomists claim that the ratios $\frac{C}{Y}$ and $\frac{I}{Y}$ are roughly stable in the long-run.
- Common to take logs, so:

$$\ln(C) - \ln(Y) \approx \text{constant}$$

- and

$$\ln(I) - \ln(Y) \approx \text{constant}$$

- If $\ln(C), \ln(Y)$ and $\ln(I)$ all contain unit roots, reasoning above suggests that two cointegrating relationships might occur.

- Engle-Granger test (based on a cointegrating regression involving all three variables), would only find whether cointegration is/is not present (not tell you how many cointegrating relationships)
- What should you do in this case? One option is to use the Johansen test (to be discussed shortly)
- Or you could do multiple Engle-Granger tests using different combinations of your variables.
- E.g. do an Engle-Granger test with all three variables, $\ln(C), \ln(Y)$ and $\ln(I)$.
- If you find cointegration with this test, then at least one cointegrating relationship exists.
- Then you could do three more Engle-Granger tests: i) using $\ln(C)$ and $\ln(Y)$, i) using $\ln(I)$ and $\ln(Y)$ and iii) using $\ln(C)$ and $\ln(I)$. If two cointegrating relationships exist, then these latter tests will indicate it.

- Another issue: Often the researcher has a suspicion as to what the cointegrating relationship should be.
- E.g. if C/Y roughly constant the regression:

$$\ln(C) = \alpha + \beta \ln(Y) + \varepsilon$$

- should have coefficient $\beta = 1$.
- Step 1 of the Engle-Granger test uses OLS to estimate β.
- But you could set $\beta = 1$ if you wanted to test whether $\ln(C)$ and $\ln(Y)$ are cointegrated with a cointegrating coefficient of $\beta = 1$.
- You test this by constructing a new variable, Z, where

$$Z = \ln(C) - \ln(Y)$$

- and then test whether Z has a unit root using Dickey-Fuller test.
- If Z is found to be stationary, then you know $\ln(C) - \ln(Y)$ is stationary and this is a cointegrating relationship.

The Johansen Test for Cointegration

- This is a very popular cointegration test for cointegration
- Unfortunately, to explain this test in detail would require a discussion of concepts beyond the scope of this course.
- But Gretl does the Johansen test
- Accordingly,give intuitive description of this test and illustration of how to use it in practice.

- Suppose you have M time series variables
- It is possible to have up to $M - 1$ cointegrating relationships
- Johansen test tests for the number of cointegrating relationships
- "number of cointegrating relationships" is referred to as the "cointegrating rank" and you will see the word "Rank" on computer outputs for this test
- There is more than one variant of the Johansen test statistic, the main one is called the "Trace statistic"

- As with any hypothesis test, you compare test statistic to a critical value and, if test statistic is greater than critical value, you reject the hypothesis being tested.
- Or you can look at p-value. A p-value of less than 0.05 means you can reject the hypothesis at the 5% level of significance.
- Gretl provides a p-value for you critical value all these numbers for you.
- In your computer tutorial (computer session 4) you will see how this is done in detail. Here I will give an illustration.

Example: Consumption, Aggregate Wealth and Expected Stock Returns

- Based on paper in Journal of Finance in 2001, "Consumption, aggregate wealth and expected stock returns" by Lettau and Ludvigson
- cay variables are consumption (c), assets (a) and income (y).
- Theory suggests cay variables should be cointegrated and the cointegrating residual should be able to predict excess stock returns.
- Here we will focus only on testing for cointegration
- Unit root tests indicate that all of these variables have unit roots.
- Results for Johansen test using a lag length of one and intercepts in the model in Table 7.9.

Table 7.9: Johansen Test for Cointegration Using CAY Data		
Rank	Trace Statistic	5% Critical Value
0	37.27	29.68
1	6.93	15.41
2	0.95	3.76

- How should you interpret this table?
- Each row carries out one test.
- For row labelled Rank=0: hypothesis being tested is that 0 cointegrating relationships exist against alternative that more 0 exist
- Row labelled Rank=1: hypothesis being tested is that 1 cointegrating relationship exists against alternative that more 1 exist
- etc.
- Comparing Trace statistic to critical value we conclude:
- Reject Rank $= 0$ in favour of Rank > 0
- Fail to reject Rank $= 1$ in favour of Rank > 1
- Thus, we are finding one cointegrating relationship (same as Lettau and Ludvigson)

Time Series Regression when Y and X are Cointegrated: The Error Correction Model

- This section assumes that Y and X are cointegrated.
- Remember: you should always do Dickey-Fuller tests on your variables first. If your variables all have unit roots, then do cointegration test.
- Estimating the cointegrating regression will provide an estimate of the long run multiplier.
- But what if you are interested in short run properties? Want *error correction model* (ECM).
- *Granger Representation Theorem*, says that if Y and X are cointegrated, then the relationship between them can be expressed as an ECM.

- Begin with the simplest version of an ECM:

$$\Delta Y_t = \varphi + \lambda \varepsilon_{t-1} + \omega_0 \Delta X_t + e_t$$

- ε_{t-1} is the error obtained from the regression model involving Y and X (i.e. $\varepsilon_{t-1} = Y_{t-1} - \alpha - \beta X_{t-1}$)
- To avoid confusion, denote the regression error in ECM as e_t to distinguish from error in cointegration regression, ε_t
- The ECM has $\lambda < 0$ (for reasons shown below)
- Note: if we knew ε_{t-1}, then the ECM would be just a regression model (similar to an ADL)

- Interpretation: the ECM says that ΔY depends on ΔX – an intuitively sensible point (i.e. changes in X cause Y to change). But this is not new (same idea in ADL)
- New point: ΔY_t depends on ε_{t-1}. This is unique to the ECM and gives it its name.
- Remember that ε can be thought of as equilibrium error.

- Let us assume that $\Delta X_t = 0$ and $e_t = 0$ to show role that ε_{t-1} plays in the ECM.
- If $\varepsilon_{t-1} > 0$ then Y_{t-1} is too high to be in equilibrium.
- Since $\lambda < 0$ the term $\lambda\varepsilon_{t-1}$ will be negative and so ΔY_t will be negative.
- Thus: if Y_{t-1} is above its equilibrium level, then it will start falling in the next period and the equilibrium error will be "corrected"
- If $\varepsilon_{t-1} < 0$ opposite will hold.
- Note: this example shows why $\lambda < 0$ (if $\lambda > 0$ equilibrium errors will be magnified instead of corrected)

Estimation and Testing in the ECM

- Computer packages like Gretl will automatically estimate ECMs for you, but here we explain some details
- Do not have to worry about the spurious regression problem with.
- We have assumed that Y and X both have unit roots, thus ΔY and ΔX are stationary.
- We have assumed Y and X to be cointegrated, thus ε_{t-1} is stationary.
- Hence, dependent variable and all explanatory variables in the ECM are stationary.
- This means OLS estimation and testing (e.g. t-statistics) work in standard way.

- The only new issue: ε_{t-1} is an explanatory variable.
- Errors not directly observed, but we can replace them with residuals.
- A two step estimation proceeds as follows:

1. Run a regression of Y on X and save the residuals.
2. Run a regression of ΔY on an intercept, ΔX and the residuals from Step 1 lagged one period.

- General form for ECM is:

$$\Delta Y_t = \varphi + \delta t + \lambda \varepsilon_{t-1} + \gamma_1 \Delta Y_{t-1} + .. + \gamma_p \Delta Y_{t-p}$$
$$+ \omega_0 \Delta X_t + .. + \omega_q \Delta X_{t-q} + e_t$$

- This is just like an ADL (using differenced data) except for the error correction term.
- Same "correction of equilibrium error" interpretation, same two-step estimation procedure can be done.
- Can do sequential testing or use an IC to decide whether to include deterministic trend and select p and q

Example: Cointegration Between the Prices of Two Goods (continued)

- Above we found Y = price of organic oranges and X = the price of regular oranges, were cointegrated.
- This suggests that we can estimate an error correction model.
- Run a regression of Y on X and save the residuals.
- Then use residuals, $\widehat{\varepsilon}_t$, (in lagged form) in ECM:

$$\Delta Y_t = \varphi + \lambda \widehat{\varepsilon}_{t-1} + \omega_0 \Delta X_t + e_t$$

- Result is given in Table 8.3

Table 8.3: Two-step Estimation of the Simple Error Correction Model			
Variable	OLS Estimate	t-statistic	P-value
Intercept	-0.023	-0.068	0.946
$\widehat{\varepsilon}_{t-1}$	-1.085	-14.458	8.7×10^{-32}
ΔX_t	1.044	5.737	4.1×10^{-8}

What if Y and X Have Unit Roots but are NOT Cointegrated?

- Do not run regression of Y on X (spurious regression problem).
- Maybe you should rethink your basic model.
- E.g. instead of working with Y and X themselves, e.g., difference them. Remember that if Y and X have unit roots, then ΔY and ΔX should be stationary.
- Then you could estimate the original ADL model, but with changes in the variables:

$$\Delta Y_t = \alpha + \delta t + \gamma_1 \Delta Y_{t-1} + .. + \gamma_{p-1} \Delta Y_{t-p+1}$$
$$+ \omega_0 \Delta X_t + \omega_1 \Delta X_{t-1} + .. + \omega_{q-1} \Delta X_{t-q+1} + \varepsilon_t$$

- If Y and X have unit roots, then all the variables in the regression above will be stationary and OLS methods for estimation and testing can be used.
- Problem: sometime you end up with a regression where coefficients do not have interpretation you want. But sometimes, this is good solution.
- E.g. suppose $Y =$ log wages and $X =$ log prices and both have unit roots but are not cointegrated.
- If you work with ΔY and ΔX, then your variables have a nice interpretation as being wage inflation and price inflation

Summary and Further Directions

- So far we have shown how to build time series regression models for the three main cases:

i) when all variables are stationary,
ii) when all variables have unit roots and are cointegrated
iii) when all variables have unit roots but are not stationary.

- But what do you use these models for?
- One answer is the usual regression one (e.g. coefficients measure marginal effects)
- But there are lots of other things such as Granger causality, forecasting and issues which arise with Vector autoregressions (VARs).
- Textbook covers all of these topics
- We will only have time to discuss one: Granger causality

Granger Causality

- With correlation we warned "be careful since correlation does not necessarily imply causality"
- With regression one uses economic theory (or common sense) to try and choose X which causes Y
- But this may not be possible (leading to need for instrumental variable methods)
- With time series can stronger statements about causality simply by exploiting the fact that time does not run backward.
- If event A happens before event B, then it is possible that A is causing B.
- However, it is not possible that B is causing A.
- This is intuitive idea behind Granger causality

Granger Causality when X and Y are Stationary

- Since stationary, can use ADL model
- Begin with simple ADL model:

$$Y_t = \alpha + \rho Y_{t-1} + \beta X_{t-1} + \varepsilon_t$$

- Implies that last period's value of X has explanatory power for current value of Y.
- β is a measure of the influence of X_{t-1} on Y_t.
- If $\beta = 0$, then past values of X have no effect on Y
- Granger causality if $\beta \neq 0$
- If $\beta = 0$ then X does not Granger cause Y.
- In words: "if $\beta = 0$ then past values of X have no explanatory power for Y beyond that provided by past values for Y".

Granger Causality when X and Y are Stationary

- OLS methods can be used with ADL
- Thus, can use t-test of the hypothesis that $\beta = 0$
- If β is statistically significant then X Granger causes Y.
- With ADL(p, q) model:

$$Y_t = \alpha + \delta t + \rho_1 Y_{t-1} + .. + \rho_p Y_{t-p} + \beta_1 X_{t-1} + .. + \beta_q X_{t-q} + \varepsilon_t$$

- X Granger causes Y if any or all of $\beta_1, .., \beta_q$ are statistically significant.
- If X at any time in the past has explanatory power for the current value of Y, then we say that X Granger causes Y.
- Can use F-test of $H_0 : \beta_1 = 0, .., \beta_q = 0$ as a Granger causality test.
- Note: usually omit X_t from ADL, but can include if you want to test for contemporaneous causality

Example: Does Wage Inflation Granger Cause Price Inflation?

- Data from 1855 – 1987 on UK prices and wages.
- Dickey-Fuller tests indicate that both the logs of wages and prices have unit roots
- Engle-Granger test indicates they are not cointegrated.
- However, differences are stationary and can be interpreted as inflation rates (i.e. wage and price inflation).
- Table 7.4 contains OLS results from regression of $\Delta P =$ price inflation on four lags of itself, four lags of $\Delta W =$ wage inflation and a deterministic trend.

Table 7.4: ADL with Price Inflation as Dependent Variable			
Variable	OLS Estimate	t-statistic	P-value
Intercept	-0.751	-1.058	0.292
ΔP_{t-1}	0.822	4.850	0.000
ΔP_{t-2}	-0.041	-0.222	0.825
ΔP_{t-3}	0.142	0.762	0.448
ΔP_{t-4}	-0.181	-1.035	0.303
ΔW_{t-1}	-0.016	-0.114	0.909
ΔW_{t-2}	-0.118	-0.823	0.412
ΔW_{t-3}	-0.042	-0.292	0.771
ΔW_{t-4}	0.038	0.266	0.791
t	0.030	2.669	0.009

Example: Does Wage Inflation Granger Cause Price Inflation?

- P-values in table indicates that only deterministic trend and last period's price inflation have significant explanatory power for present inflation.
- All of the coefficients on the lags of wage inflation are insignificant.
- This suggests that wage inflation does not seem to Granger cause price inflation.
- Formally, we should do F-test of $H_0 : \beta_1 = 0, .., \beta_q = 0$ for Granger causality
- F-statistic is $F = 0.145$.
- The 5% critical value is approximately 2.37 (note: $F_{4,118}$ distribution)
- Since $0.145 < 2.37$ we cannot reject the hypothesis that $\beta_1 = 0, .., \beta_4 = 0$ at the 5% level of significance.
- Accept hypothesis that wage inflation does not Granger cause price inflation.

Causality in Both Directions

- Should past wage inflation cause price inflation or should the reverse hold?
- Causality may be in either direction, it is important that you check for it.
- It is possible to find that Y Granger causes X and that X Granger causes Y.
- This can be done by doing Granger causality test with Y as dependent variable
- Then repeat with X as dependent variable

Example: Does Price Inflation Granger Cause Wage Inflation?

- Previous example investigated whether wage inflation Granger caused price inflation (it did not)
- Does price inflation cause wage inflation?
- E.g. workers and unions look at inflation when deciding on their wage demands.
- Table 7.5 contains results from OLS estimation of regression of ΔW = wage inflation on four lags of itself, four lags of ΔP = price inflation and a deterministic trend.
- We do find evidence that price inflation Granger causes wage inflation.
- Coefficient on ΔP_{t-1} is highly significant (last year's price inflation rate has strong explanatory power for wage inflation).
- Confirmed by the F-test for Granger causality.
- We accept hypothesis that price inflation does Granger cause wage inflation.

Table 7.5: ADL with Wage Inflation as Dependent Variable			
Variable	OLS Estimate	t-statistic	P-value
Intercept	-0.609	-0.730	0.467
ΔW_{t-1}	0.053	0.312	0.755
ΔW_{t-2}	-0.040	-0.235	0.814
ΔW_{t-3}	-0.058	-0.348	0.728
ΔW_{t-4}	0.036	0.215	0.830
ΔP_{t-1}	0.854	4.280	0.000
ΔP_{t-2}	-0.217	-0.993	0.323
ΔP_{t-3}	0.234	1.067	0.288
ΔP_{t-4}	-0.272	-1.323	0.188
t	0.046	3.514	0.001

Granger Causality with Cointegrated Variables

- Testing for Granger causality among cointegrated variables is similar, except use ECM instead of ADL:

$$\Delta Y_t = \varphi + \delta t + \lambda \varepsilon_{t-1} + \gamma_1 \Delta Y_{t-1} + .. + \gamma_p \Delta Y_{t-p} + \omega_1 \Delta X_{t-1} + .. + \omega_q$$

- Remember this is ADL model (using differenced data) except for the term $\lambda \varepsilon_{t-1}$.
- Remember that $\varepsilon_{t-1} = Y_{t-1} - \alpha - \beta X_{t-1}$ so X_{t-1} enters
- X does not Granger cause Y if $\lambda = 0, \omega_1 = 0, .., \omega_q = 0$.
- Testing whether Y Granger causes X is achieved by reversing the roles that X and Y play in the ECM.
- One consequence of Granger Representation Theorem is:
- If X and Y are cointegrated then some form of Granger causality must occur.
- That is, either X must Granger cause Y or Y must Granger cause X (or both).

Chapter Summary

- If all variables are stationary, then an ADL(p, q) model can be estimated using OLS. Econometric techniques are all standard.
- A variant on the ADL model is often used to avoid potential multicollinearity problems. It provides a straightforward estimate of the long run multiplier.
- If all variables are nonstationary, great care must be taken in the analysis due to the spurious regression problem.
- If all variables are nonstationary but the regression error is stationary, then cointegration occurs.
- If cointegration is present, the spurious regression problem does not occur.
- Cointegration is an attractive concept for economists since it implies that an equilibrium relationship exists.

- Cointegration can be tested using the Engle-Granger test. This test is a Dickey-Fuller test on the residuals from the cointegrating regression.
- The Johansen test is another test for cointegration. Unlike the Engle-Granger test it allows you to find out how many cointegrating relationships there are.
- If cointegration is present you can either run a cointegrating regression or estimate an error correction model
- If the variables have unit roots but are not cointegrated, you should not work with them directly. Rather you should difference them and estimate an ADL model using the differenced variables.
- We also covered Granger causality testing for stationary and cointegrated cases