Financial Econometrics
Volatility Exercises

Q.1 Using US annual data 1873 - 2000 on log stock prices, ls_t an ARIMA(1,1,0) model was estimated

$$\Delta ls_t = \mu + \phi \Delta ls_{t-1} + \varepsilon_t$$

and a test for second order ARCH gave a p-value of 0.0434. The equation was re-estimated assuming GARCH(1,1) errors, i.e., assuming that $\varepsilon_t \sim N(0, h_t)$ with

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}.$$

The estimates (standard errors in parentheses) are $\mu = 0.043$ (0.016), $\rho = 0.071$ (0.109), $\omega = 0.007$ (0.007), $\alpha = 0.0149$ (0.092), $\beta = 0.625$ (0.0291).

a) Explain how a test for second order ARCH is conducted.

b) Explain how the GARCH(1,1) model is estimated.

c) What are the conditions required for the variances to be non-negative? Are they satisfied in this case.

d) Consider the following output regarding the estimation of a GARCH, an EGARCH and a GJR model for Japanese bonds:

GARCH

Dependent Variable: JAPAN
Method: ML - ARCH (Marquardt)
Date: 12/09/10 Time: 23:35
Sample(adjusted): 1/09/1990 4/09/1996
Included observations: 327 after adjusting endpoints
Convergence achieved after 13 iterations
Variance backcast: ON

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.162783	0.073572	2.212572	0.0269
@TREND	-0.000298	0.000162	-1.843565	0.0652
JAPAN(-1)	0.975177	0.010010	97.42092	0.0000
		Variance Equation		
C	0.001028	0.000405	2.539172	0.0111
ARCH(1)	0.082545	0.031575	2.614274	0.0089
GARCH(1)	0.826457	0.049896	16.56350	0.0000

R-squared	0.993228	Mean dependent var	4.968621
Adjusted R-squared	0.993122	S.D. dependent var	1.362088
S.E. of regression	0.112960	Akaike info criterion	-1.588421
Sum squared resid	4.095920	Schwarz criterion	-1.518880
Log likelihood	265.7068	F-statistic	9415.864
Durbin-Watson stat	1.848777	Prob(F-statistic)	0.000000

EGARCH

Dependent Variable: JAPAN
Method: ML - ARCH (Marquardt)
Date: 12/09/10 Time: 23:39
Sample(adjusted): 1/09/1990 4/09/1996
Included observations: 327 after adjusting endpoints
Convergence achieved after 7 iterations
Variance backcast: ON

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.178242	0.076203	2.339047	0.0193
@TREND	-0.000349	0.000157	-2.230184	0.0257
JAPAN(-1)	0.973420	0.010231	95.14083	0.0000
		Variance Equation		
C	-0.373594	0.130750	-2.857317	0.0043
|RES|/SQR[GARCH]...	0.157909	0.047557	3.320392	0.0009
RES/SQR[GARCH](1)	-0.046342	0.023248	-1.993377	0.0462
EGARCH(1)	0.943799	0.024775	38.09431	0.0000

R-squared	0.993251	Mean dependent var	4.968621
Adjusted R-squared	0.993125	S.D. dependent var	1.362088
S.E. of regression	0.112941	Akaike info criterion	-1.601392
Sum squared resid	4.081784	Schwarz criterion	-1.520261
Log likelihood	268.8275	F-statistic	7849.384
Durbin-Watson stat	1.851877	Prob(F-statistic)	0.000000

GJR

Dependent Variable: JAPAN
Method: ML - ARCH (Marquardt)
Date: 12/09/10 Time: 23:38
Sample(adjusted): 1/09/1990 4/09/1996
Included observations: 327 after adjusting endpoints
Convergence achieved after 5 iterations
Variance backcast: ON

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.183614	0.081797	2.244738	0.0248
@TREND	-0.000347	0.000170	-2.043156	0.0410
JAPAN(-1)	0.971913	0.010911	89.07935	0.0000
		Variance Equation		
C	0.000436	0.000163	2.674341	0.0075
ARCH(1)	0.020209	0.017138	1.179198	0.2383
(RESID<0)*ARCH(1)	0.099265	0.038378	2.586508	0.0097
GARCH(1)	0.902551	0.021743	41.50920	0.0000

R-squared	0.993229	Mean dependent var	4.968621
Adjusted R-squared	0.993102	S.D. dependent var	1.362088
S.E. of regression	0.113124	Akaike info criterion	-1.597947
Sum squared resid	4.095060	Schwarz criterion	-1.516817
Log likelihood	268.2644	F-statistic	7823.764
Durbin-Watson stat	1.843058	Prob(F-statistic)	0.000000

i) What characteristic of the data do the EGARCH and GJR model capture that GARCH does not?

ii) Based on the outputs, indicate which of the models you would chose and whether the characteristics indicated in i) is present in the data.

e) Explain how the GARCH-in-mean model works and why it may be interesting to consider.

f) Considering that your sample has 327 observations, of which the last three are 3.103, 3.099, 3.104, and that you saved 5 for forecast evaluation (3.119, 3.291, 3.158, 3.101, 3.26) compute a 5 step ahead forecast and the corresponding Root Mean Forecast Error, using the conditional expectation of the bond given in the GARCH output.

g) Present the s-step ahead forecast xpression from a GARCH(1,1) such as $\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2$.

Q.2 Consider the Fama and French two factor model,

$$r_{it} - rf = c + \beta(mkt_t - rf) + \phi_1 SMB_t + \phi_2 HML_t + \varepsilon_t \tag{1}$$

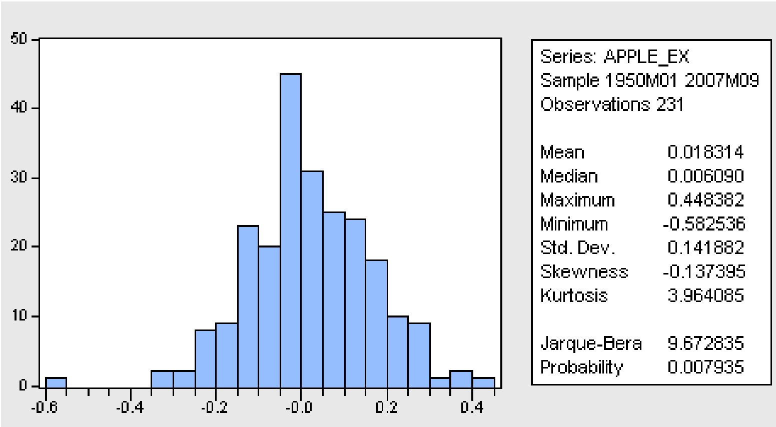
applied to the returns of Apple stocks. Considering a sample of monthly data from 1988:07 - 2007:09 the following least squares estimates where obtained:

Dependent Variable: APPLE_EX
Method: Least Squares
Date: 12/19/11 Time: 10:52
Sample (adjusted): 1988M07 2007M09
Included observations: 231 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.017157	0.008495	2.019733	0.0446
MKT_EX	1.078052	0.239547	4.500375	0.0000
SMB	0.004884	0.002642	1.848407	0.0658
HML	-0.006996	0.003245	-2.155862	0.0321

R-squared	0.209433	Mean dependent var	0.018314
Adjusted R-squared	0.198985	S.D. dependent var	0.141882
S.E. of regression	0.126984	Akaike info criterion	-1.272349
Sum squared resid	3.660353	Schwarz criterion	-1.212740
Log likelihood	150.9563	Hannan-Quinn criter.	-1.248307
F-statistic	20.04524	Durbin-Watson stat	1.914971
Prob(F-statistic)	0.000000		

a) Considering the following output, what can you conclude with regards to normality of Apple stocks excess returns?



b) The following two models (GARCH and GJR) where estimated

Dependent Variable: APPLE_EX
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 12/19/11 Time: 11:01
Sample (adjusted): 1988M07 2007M09
Included observations: 231 after adjustments
Convergence achieved after 48 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(5) + C(6)*RESID(-1)^2 + C(7)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.013557	0.008697	1.558854	0.1190
MKT_EX	1.086313	0.265474	4.091980	0.0000
SMB	0.006939	0.002508	2.766787	0.0057
HML	-0.007718	0.003583	-2.154091	0.0312
Variance Equation				
C	0.001152	0.000818	1.408033	0.1591
RESID(-1)^2	0.083110	0.050932	1.631779	0.1027
GARCH(-1)	0.846633	0.090624	9.342260	0.0000

R-squared	0.205144	Mean dependent var	0.018314
Adjusted R-squared	0.194639	S.D. dependent var	0.141882
S.E. of regression	0.127328	Akaike info criterion	-1.304268
Sum squared resid	3.680211	Schwarz criterion	-1.199952
Log likelihood	157.6429	Hannan-Quinn criter.	-1.262193
F-statistic	9.764392	Durbin-Watson stat	1.921114
Prob(F-statistic)	0.000000		

Dependent Variable: APPLE_EX
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 12/19/11 Time: 11:00
Sample (adjusted): 1988M07 2007M09
Included observations: 231 after adjustments
Convergence achieved after 57 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(5) + C(6)*RESID(-1)^2 + C(7)*RESID(-1)^2*(RESID(-1)<0) + C(8)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.012905	0.008668	1.488871	0.1365
MKT_EX	1.109613	0.263522	4.210698	0.0000
SMB	0.006457	0.002533	2.548969	0.0108
HML	-0.007963	0.003551	-2.242493	0.0249
Variance Equation				
C	0.001091	0.000848	1.287247	0.1980
RESID(-1)^2	0.048374	0.053698	0.900863	0.3677
RESID(-1)^2*(RESID(-1)<0)	0.058689	0.070556	0.831805	0.4055
GARCH(-1)	0.856900	0.091539	9.361078	0.0000

R-squared	0.205382	Mean dependent var	0.018314
Adjusted R-squared	0.194881	S.D. dependent var	0.141882
S.E. of regression	0.127309	Akaike info criterion	-1.299803
Sum squared resid	3.679108	Schwarz criterion	-1.180585
Log likelihood	158.1272	Hannan-Quinn criter.	-1.251718
F-statistic	8.381712	Durbin-Watson stat	1.917400
Prob(F-statistic)	0.000000		

b-i) What restrictions need to be imposed on the parameters each volatility model for it to be valid?

b-ii) Based on the results presented which model would you choose and why?

c) Assuming that some of the exogenous variables may impact volatility as well, the alternative volatility model was estimated:

Dependent Variable: APPLE_EX
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 12/19/11 Time: 11:07
Sample (adjusted): 1988M07 2007M09
Included observations: 231 after adjustments
Convergence achieved after 23 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(5) + C(6)*RESID(-1)^2 + C(7)*RESID(-2)^2 + C(8)*RESID(-3)^2 + C(9)*RESID(-4)^2 + C(10)*HML

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.017422	0.008075	2.157447	0.0310
MKT_EX	1.086565	0.214085	5.075381	0.0000
SMB	0.004306	0.002265	1.901059	0.0573
HML	-0.004929	0.003046	-1.617936	0.1057
	Variance Equation			
C	0.007955	0.002101	3.785678	0.0002
RESID(-1)^2	0.072880	0.065350	1.115218	0.2648
RESID(-2)^2	0.026180	0.068478	0.382321	0.7022
RESID(-3)^2	0.131750	0.098342	1.339716	0.1803
RESID(-4)^2	0.268150	0.120049	2.233672	0.0255
HML	0.000957	0.000523	1.828955	0.0674

R-squared	0.206571	Mean dependent var	0.018314
Adjusted R-squared	0.196085	S.D. dependent var	0.141882
S.E. of regression	0.127214	Akaike info criterion	-1.305466
Sum squared resid	3.673604	Schwarz criterion	-1.156443
Log likelihood	160.7813	Hannan-Quinn criter.	-1.245360
F-statistic	6.566660	Durbin-Watson stat	1.915436
Prob(F-statistic)	0.000000		

c-i) What type of volatility model was estimated.

c-ii) Compute the unconditional variance.

Q.3 Consider that for the first difference of the variable SP_t, the following model was estimated:

Table 2.1: Model for DSP_t

Dependent Variable: DSP
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 12/14/11 Time: 14:06
Sample (adjusted): 3 1120
Included observations: 1118 after adjustments
Convergence achieved after 87 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-1)^2*(RESID(-1)<0) + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.011804	0.046149	-0.255773	0.7981
DSP(-1)	-0.152614	0.040097	-3.806111	0.0001
	Variance Equation			
C	0.539719	0.049707	10.85803	0.0000
RESID(-1)^2	0.408318	0.040682	10.03677	0.0000
RESID(-1)^2*(RESID(-1)<0)	-0.258921	0.048458	-5.343245	0.0000
GARCH(-1)	0.573640	0.029680	19.32763	0.0000

R-squared	-0.010230	Mean dependent var	0.040250
Adjusted R-squared	-0.011136	S.D. dependent var	1.752141
S.E. of regression	1.761870	Akaike info criterion	3.718426
Sum squared resid	3464.270	Schwarz criterion	3.745363
Log likelihood	-2072.600	Hannan-Quinn criter.	3.728609
Durbin-Watson stat	1.783040		

(a) Indicate what model is estimated in Table 2.1 and write down its functional form.

(b) What are the parameter restrictions that have to be observed for a model, as the one that was estimated in Table 2.1, to be valid? Do the results presented satisfy those restrictions.

(c) What is the main difference between the model estimated in Table 2.1 and a pure GARCH(1,1) model.

(d) Consider the following Figures of the graph of the standardized residuals, the autocorrelations associated with the square of the standardized residuals and the histogram of the standardized residuals.

Figure 2.1: Graph of Standardized Residuals

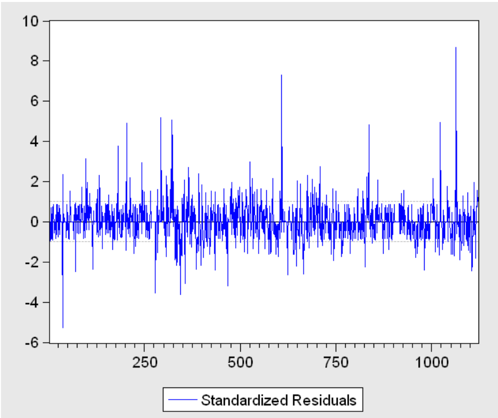
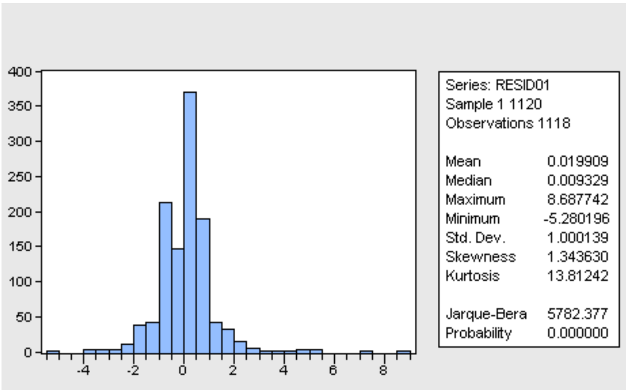


Figure 2.2:Correlogram of Standardized Residuals Squared

Date: 12/14/11 Time: 14:24
Sample: 1 1120
Included observations: 1118

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.016	-0.016	0.2812	0.596
		2	-0.014	-0.014	0.4922	0.782
		3	0.013	0.013	0.6801	0.878
		4	-0.006	-0.006	0.7241	0.948
		5	-0.015	-0.015	0.9930	0.963
		6	-0.015	-0.016	1.2620	0.974
		7	-0.032	-0.033	2.4340	0.932
		8	-0.019	-0.020	2.8372	0.944
		9	0.036	0.034	4.2795	0.892
		10	0.012	0.013	4.4352	0.926
		11	-0.022	-0.022	5.0063	0.931
		12	-0.013	-0.016	5.2101	0.951
		13	0.019	0.017	5.6296	0.959
		14	0.032	0.032	6.7669	0.943
		15	-0.004	-0.002	6.7816	0.963

Figure 2.3:Histogram of Standardized Residual



What do these plots tell you about whether or not the model estimated in Table 2.1 appears to fit volatility well.

(e) What do these plots tell you about the validity of the conditional Normality?

(f) Given the following output and the output in Table 2.1 indicate whether the model provided in Table 2.1 is more suitable by testing the joint significance of the additional regressors.

Dependent Variable: DSP
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 12/14/11 Time: 16:16
Sample (adjusted): 3 1120
Included observations: 1118 after adjustments
Convergence achieved after 59 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-1)^2*(RESID(-1)<0) + C(6)*RESID(-2)^2 + C(7)*RESID(-3)^2 + C(8)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.062782	0.038157	-1.645381	0.0999
DSP(-1)	-0.140252	0.020860	-6.723359	0.0000
	Variance Equation			
C	0.095339	0.033057	2.884079	0.0039
RESID(-1)^2	0.095322	0.032885	2.898641	0.0037
RESID(-1)^2*(RESID(-1)<0)	0.028688	0.019307	1.485898	0.1373
RESID(-2)^2	0.416507	0.050980	8.169975	0.0000
RESID(-3)^2	0.453752	0.032579	-13.92760	0.0000
GARCH(-1)	0.912969	0.030993	29.45744	0.0000

R-squared	-0.010497	Mean dependent var	0.040250
Adjusted R-squared	-0.011403	S.D. dependent var	1.752141
S.E. of regression	1.762103	Akaike info criterion	3.710124
Sum squared resid	3465.186	Schwarz criterion	3.746040
Log likelihood	-2065.959	Hannan-Quinn criter.	3.723700
Durbin-Watson stat	1.802219		

(g) Assuming that the last three volatility estimates where σ^2_{T-2} =1.63, σ^2_{T-1} =3.13 and σ^2_T =3.04, and that $\widehat{u}^2_T = 0.7$, compute forecasts for σ^2_{T+1}, σ^2_{T+2} and σ^2_{T+3}.How would you evaluate the quality of these forecasts?