NOVA School of Business and Economics
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Time Series - Exercise Sheet

1. Rewrite the following model $p_t = 0.7p_{t-1}+1.2p_{t-2}+0.8p_{t-3}+1.6p_{t-4}+u_t$. using the lag operator, L.

2. Consider the following dynamic models,

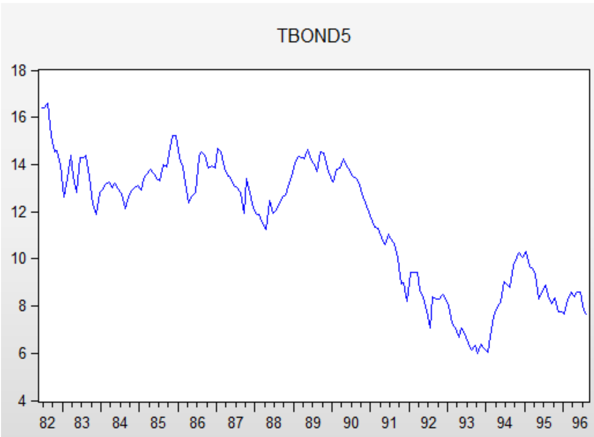
$$\begin{aligned} i) \quad & \widehat{y}_t = 0.4y_{t-1} + 0.3y_{t-2} - 0.5y_{t-3} + 1.1y_{t-4} - 0.7y_{t-5} \\ ii) \quad & \widehat{y}_t = 0.2y_{t-3} + \varepsilon_t + 0.3\varepsilon_{t-1} - 0.2\varepsilon_{t-2} \\ iii) \quad & \widehat{y}_t = \varepsilon_t + 0.1\varepsilon_{t-1} + 0.3\varepsilon_{t-2} + 0.55\varepsilon_{t-3} \\ iv) \quad & \widehat{y}_t = 0.16y_{t-4} \end{aligned}$$

   a) Identify each model.

   b) When can the models in ii) and iii) be considered invertible.

3. Consider the monthly data from June 1982 to August 1996 of the 5 year Tbond given in Figure 1:

Figure 1



a) Given the following correlograms:

Figure 2: Levels

Sample: 1982M06 1996M08
Included observations: 171

	AC	PAC	Q-Stat	Prob
1	0.965	0.965	161.97	0.000
2	0.928	-0.042	312.69	0.000
3	0.895	0.039	453.74	0.000
4	0.867	0.057	587.05	0.000
5	0.842	0.012	713.33	0.000
6	0.821	0.068	834.26	0.000
7	0.801	-0.002	950.01	0.000
8	0.779	-0.028	1060.1	0.000
9	0.757	-0.001	1164.6	0.000
10	0.724	-0.164	1260.9	0.000
11	0.696	0.071	1350.5	0.000
12	0.673	0.028	1434.8	0.000
13	0.647	-0.077	1513.2	0.000
14	0.625	0.055	1586.7	0.000
15	0.603	-0.022	1655.7	0.000
16	0.582	-0.004	1720.2	0.000
17	0.566	0.101	1781.8	0.000
18	0.554	0.025	1841.1	0.000
19	0.540	0.007	1897.9	0.000
20	0.529	0.027	1952.8	0.000

Figure 3:First Differences

Included observations: 170

	AC	PAC	Q-Stat	Prob
1	0.050	0.050	0.4380	0.508
2	-0.068	-0.070	1.2378	0.539
3	-0.002	0.005	1.2384	0.744
4	-0.006	-0.012	1.2458	0.871
5	-0.128	-0.128	4.1542	0.527
6	0.016	0.030	4.2024	0.649
7	0.139	0.121	7.6544	0.364
8	-0.074	-0.089	8.6352	0.374
9	0.159	0.193	13.245	0.152
10	-0.004	-0.056	13.248	0.210
11	-0.066	-0.038	14.054	0.230
12	-0.058	-0.017	14.684	0.259
13	-0.061	-0.102	15.380	0.284
14	-0.066	-0.033	16.189	0.302
15	0.061	0.078	16.902	0.325
16	0.045	-0.049	17.278	0.368
17	-0.034	0.012	17.502	0.421
18	-0.044	-0.083	17.879	0.464
19	-0.017	-0.018	17.936	0.527
20	-0.001	0.052	17.936	0.592

Why are the correlograms of the levels and first differences so different? What can you say about the persistence of the two series and of the eventual time series models you would suggest in both cases?

b) Given the following models for the first differences ($\Delta TBOND5_t$), indicate:

i) What type of model were estimated in each case;

ii) Which one would you choose to model the first differences of TBOND5? Why?

Model A

Dependent Variable: DTBOND5
Method: Least Squares
Date: 11/02/14 Time: 19:24
Sample (adjusted): 1982M08 1996M08
Included observations: 169 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.049568	0.041355	-1.198597	0.2324
DTBOND5(-1)	0.050343	0.077302	0.651246	0.5158

R-squared	0.002533	Mean dependent var	-0.052130
Adjusted R-squared	-0.003440	S.D. dependent var	0.534263
S.E. of regression	0.535181	Akaike info criterion	1.599341
Sum squared resid	47.83196	Schwarz criterion	1.636381
Log likelihood	-133.1443	Hannan-Quinn criter.	1.614373
F-statistic	0.424121	Durbin-Watson stat	1.991852
Prob(F-statistic)	0.515783		

Model B

Dependent Variable: DTBOND5
Method: Least Squares
Date: 11/02/14 Time: 19:26
Sample (adjusted): 1983M04 1996M08
Included observations: 161 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.033492	0.039968	-0.837969	0.4033
DTBOND5(-9)	0.163724	0.073853	2.216878	0.0281

R-squared	0.029982	Mean dependent var	-0.042298
Adjusted R-squared	0.023882	S.D. dependent var	0.510757
S.E. of regression	0.504621	Akaike info criterion	1.482328
Sum squared resid	40.48820	Schwarz criterion	1.520606
Log likelihood	-117.3274	Hannan-Quinn criter.	1.497870
F-statistic	4.914548	Durbin-Watson stat	1.827970
Prob(F-statistic)	0.028050		

Model C

Dependent Variable: DTBOND5
Method: Least Squares
Date: 11/02/14 Time: 19:27
Sample (adjusted): 1983M04 1996M08
Included observations: 161 after adjustments
Convergence achieved after 31 iterations
MA Backcast: 1982M07 1983M03

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.075732	0.071787	-1.054959	0.2931
DTBOND5(-9)	-0.656563	0.062774	-10.45923	0.0000
MA(9)	0.919890	0.018583	49.50105	0.0000

R-squared	0.123500	Mean dependent var	-0.042298
Adjusted R-squared	0.112405	S.D. dependent var	0.510757
S.E. of regression	0.481196	Akaike info criterion	1.393373
Sum squared resid	36.58482	Schwarz criterion	1.450791
Log likelihood	-109.1665	Hannan-Quinn criter.	1.416687
F-statistic	11.13116	Durbin-Watson stat	1.827414
Prob(F-statistic)	0.000030		

Model D

Dependent Variable: DTBOND5
Method: Least Squares
Date: 11/02/14 Time: 19:28
Sample (adjusted): 1982M07 1996M08
Included observations: 170 after adjustments
Convergence achieved after 8 iterations
MA Backcast: 1981M10 1982M06

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.053064	0.051524	-1.029891	0.3046
MA(7)	0.232092	0.072722	3.191497	0.0017
MA(8)	-0.135442	0.072146	-1.877324	0.0622
MA(9)	0.217229	0.072650	2.990067	0.0032

R-squared	0.079319	Mean dependent var	-0.051824
Adjusted R-squared	0.062680	S.D. dependent var	0.532695
S.E. of regression	0.515730	Akaike info criterion	1.536782
Sum squared resid	44.15229	Schwarz criterion	1.610566
Log likelihood	-126.6265	Hannan-Quinn criter.	1.566723
F-statistic	4.767118	Durbin-Watson stat	1.817344
Prob(F-statistic)	0.003252		

c) Considering one-step a head forecasts for the next 10 months, the following statistics were obtained for each model:

Model A

Forecast: DTBOND5F
Actual: DTBOND5
Forecast sample: 1995M11 1996M08
Included observations: 10

Root Mean Squared Error	0.395327
Mean Absolute Error	0.311792
Mean Abs. Percent Error	112.0626
Theil Inequality Coefficient	0.869137
Bias Proportion	0.003809
Variance Proportion	0.891765
Covariance Proportion	0.104426

Model B

Forecast: DTBOND5F
Actual: DTBOND5
Forecast sample: 1995M11 1996M08
Included observations: 10

Root Mean Squared Error	0.422131
Mean Absolute Error	0.323572
Mean Abs. Percent Error	134.6953
Theil Inequality Coefficient	0.826935
Bias Proportion	0.000174
Variance Proportion	0.571207
Covariance Proportion	0.428619

Model C

Forecast: DTBOND5F
Actual: DTBOND5
Forecast sample: 1995M11 1996M08
Included observations: 10

Root Mean Squared Error	0.438385
Mean Absolute Error	0.369255
Mean Abs. Percent Error	344.3515
Theil Inequality Coefficient	0.683007
Bias Proportion	0.010637
Variance Proportion	0.177982
Covariance Proportion	0.811381

Model D

Forecast: DTBOND5F
Actual: DTBOND5
Forecast sample: 1995M11 1996M08
Included observations: 10

Root Mean Squared Error	0.456318
Mean Absolute Error	0.369547
Mean Abs. Percent Error	213.9112
Theil Inequality Coefficient	0.794132
Bias Proportion	0.008843
Variance Proportion	0.339499
Covariance Proportion	0.651659

Which model would you choose based on the analysis of the RMSE and the MAE?

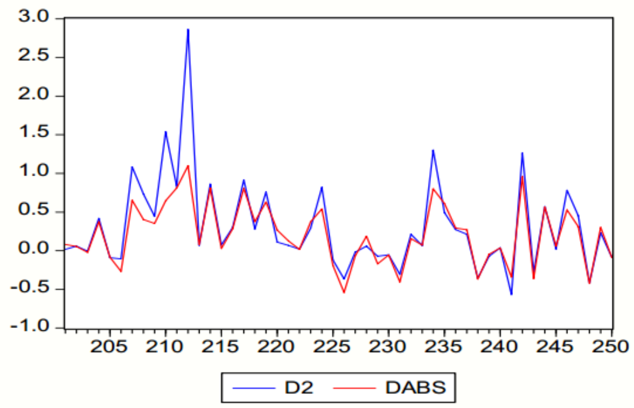
d) Considering models C and D, and assuming that you forecast for the 10 periods (i.e. November 1995 to August 1996) fixing your information in October 1995, what will the forecast for August 1996 from these two models look like.

e) Note that in the forecast exercise of question c) the sample was split into two parts: One for estimation (1982m6 - 1995m10) and the rest for forecasting (1995m11 - 1996m8), why is this important?

4. To statistically compare the forecasting accuracy of the AR(9) and the MA(9) models, we may compute Diebold-Mariano (DM) statistics using the squared error and absolute error loss functions. The DM statistics are based on the following loss differentials

$$
\begin{aligned}
d_{sq,t} &= \left(\widehat{\varepsilon}_t^{MA(9)}\right)^2 - \left(\widehat{\varepsilon}_t^{AR(9)}\right)^2 \\
d_{abs,t} &= \left|\widehat{\varepsilon}_t^{MA(9)}\right| - \left|\widehat{\varepsilon}_t^{AR(9)}\right|
\end{aligned}
$$

computed using the 1-step ahead forecast errors from the AR(9) and MA(9) models, respectively. A time plot of these loss differentials are shown below



In general both loss differentials are positive indicating that the MA(9) model produces a larger forecast error than the AR(9) model. The DM statistic

$$DM = \frac{\overline{d}}{se(\overline{d})}$$

may be computed by regressing the loss differential on a constant and choosing the NW correction to the standard error.

Dependent Variable: D2
Method: Least Squares
Date: 05/24/05 Time: 09:53
Sample: 201 250
Included observations: 50
Newey-West HAC Standard Errors & Covariance (lag truncation=3)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.312600	0.103399	3.023237	0.0040

or

Dependent Variable: DABS
Method: Least Squares
Date: 05/24/05 Time: 10:10
Sample: 201 250
Included observations: 50
Newey-West HAC Standard Errors & Covariance (lag truncation=3)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.211032	0.066393	3.178508	0.0026

The DM statistic has an asymptotic standard normal distribution. Using both the squared and absolute value loss functions we reject the null hypothesis that the AR(9) and MA(9) models have equally forecasting accuracy. Since the t-statistics are positive we conclude that the AR(9) model is more accurate than the MA(9) model.