II A

a) [1.75]

a.1. [0.35] $\max_{\{K_2\}} V_1 = -K_2 + \frac{Q_2}{1+r^*} = -K_2 + \frac{20\sqrt{K_2}}{1+r^*}$

$$\frac{\partial V_1}{\partial K_2} = 0 \Leftrightarrow -1 + \frac{20}{1+r^*} \cdot \frac{1}{2\sqrt{K_2}} = 0 \Leftrightarrow \frac{10}{\sqrt{K_2}} = 1+r^*$$

$$r^* = 0.25 \Rightarrow \frac{10}{\sqrt{K_2}} = 1.25 \Leftrightarrow K_2 = 64 = I_1, \text{ Since } \delta = 1$$

a.2. [0.15] $\Omega = Q_1 + NPV = 120 + 64 = 184$

$$NPV = V_1 = -K_2 + \frac{20\sqrt{K_2}}{1+r^*} = -64 + \frac{20\sqrt{64}}{1+0.25} = 64$$

a.3 [0.35] $\max_{\{C_1, C_2\}} U = C_1 C_2 \text{ s.t. } C_1 + \frac{C_2}{1+r^*} = \Omega$

At the optimum, $C_1 = \frac{\Omega}{2} = \frac{184}{2} = 92$

$C_2 = (1+r^*) C_1 = 1.25 \times 92 = 115$

a.4 [0.3] $TB_1 = Q_1 - C_1 - I_1 = 120 - 92 - 64 = -36$

$TB_2 = Q_2 - C_2 - I_2 = 20\sqrt{64} - 115 - 0 = 160 - 115 = 45$

a.5 [0.3] $NFIA_1 = r^* b_0 = 0.25 \times 0 = 0$

$NFIA_2 = r^* b_1 = 0.25 \times (-36) = -9$

$b_1 = (1+r^*) b_0 + TB_1 = 1.25 \times 0 + (-36) = -36$

a.6 [0.3] $CA_1 = TB_1 + NFIA_1 + NUT_1 = -36 + 0 + 0 = -36$

$CA_2 = TB_2 + NFIA_2 + NUT_2 = 45 - 9 + 0 = 36$

b) 2 [illegible]

.1. In an open economy, the investment decision does not depend on $Q_1$
Therefore, investment will remain optimal: $K_2 = 64 = I_1$

2 $\Omega = Q_1 + NPV = 75 + 64 = 139$

$\max_{C_1, C_2} U = C_1 C_2$ s.t. $C_1 + \frac{C_2}{1+r^*} = \Omega$ At the optimum, $C_1 = \frac{\Omega}{2} = \frac{139}{2} = 69.5$

$C_2 = (1+r^*) C_1 = 1.25 \times 69.5 = 86.875$

3. $TB_1 = Q_1 - C_1 - I_1 = 75 - 69.5 - 64 = -58.5$

$TB_2 = Q_2 - C_2 - I_2 = 20\sqrt{64} - 86.875 - 0 = 160 - 86.875 = 73.125$

.4 Students were required to mention and briefly explain at least two of the following topics:

→ Separation of consumption and investment decisions
→ Smoothing the consumption path
→ Maximize wealth
→ Reduce persistency of impact

(*) Any answer that was in line with the exercise and was well explained was accepted, even if the topics differ from the ones mentioned

c) [1.5]

**C.1.** [0.25] In a closed economy, $O_1 = C_1 + I_1 \Leftrightarrow I_1 = O_1 - C_1 \overset{\delta=1}{\Leftrightarrow} K_2 = O_1 - C_1$

$$O_2 = C_2 + I_2 \Leftrightarrow O_2 = C_2 + 0 \Leftrightarrow C_2 = O_2$$

$$O_2 = 20\sqrt{K_2} = 20\sqrt{O_1 - C_1} = 20\sqrt{75 - C_1}$$

$\therefore$ PPF: $C_2 = 20\sqrt{75 - C_1}$

**C.2.** [0.25] $\max_{\{C_1, C_2\}} U = C_1 C_2$ s.t. $C_2 = 20\sqrt{75 - C_1} \Leftrightarrow$

$$\Leftrightarrow \max_{\{C_1\}} C_1 \cdot 20\sqrt{75 - C_1}$$

$$\frac{\partial U}{\partial C_1} = 0 \Leftrightarrow 20\sqrt{75 - C_1} + C_1 \cdot 20 \cdot \frac{-1}{2\sqrt{75 - C_1}} = 0 \Leftrightarrow 20\sqrt{75 - C_1} = C_1 \cdot \frac{10}{\sqrt{75 - C_1}} \Leftrightarrow$$

$$\Leftrightarrow 20(75 - C_1) = 10C_1 \Leftrightarrow 30C_1 = 1500 \Leftrightarrow C_1 = 50$$

$$C_2 = 20\sqrt{75 - 50} = 20\sqrt{25} = 20 \times 5 = 100$$

**C.3.** [0.15] $I_1 = O_1 - C_1 = 75 - 50 = 25$

**C.4.** [0.2] $1 + r^A = MPK \Leftrightarrow r^A = MPK - 1 \Leftrightarrow r^A = \frac{\partial O_2}{\partial K_2} - 1 \Leftrightarrow r^A = \frac{10}{\sqrt{K_2}} - 1$

$$r^A = \frac{10}{\sqrt{25}} - 1 = \frac{10}{5} - 1 = 2 - 1 = 1 = 100\%$$

**C.5.** [0.15] $O_2 = 20\sqrt{25} = 20 \times 5 = 100$

**C.6** [0.5] Open Economy: $U = 69.5 \times 86.875 = 6037.8125$
Closed Economy: $U = 50 \times 100 = 5000$

More utility in open economy case
Thus, better off

Students were required to mention and briefly explain at least of the following topics:

- → Bigger persistency of shock in closed economy
- → open economy has access to lending/borrowing
- → Separation of consumption and investment decisions

(*) Any answer that was in line with the exercise and was well explained was accepted, even if the topics differ from the ones mentioned

NOVA SCHOOL OF BUSINESS & ECONOMICS

**Number:** **Name:**

**II.B.** The small open economy of Schmidtland has two sectors, a **tradable (T)** and a **non-tradable (N)**. The production functions are given as: $Y_T = L_T$ and $Y_N = L_N$. Further assume that each price weights 50% in the consumer price index [the CPI is given by $P = P_T^a P_N^{1-a}$], that $P^* = 1$ and $P_T^* = \frac{1}{4}$, and that the price of foreign currency in terms of domestic currency is $e = 4$.

d) Assuming that firms in Schmidtland maximize profits, find:

(d1) The labour demand of each of the sectors

$\hookrightarrow \max \pi = \max P_i y_i - wL_i = \max P_i z_i L_i - wL_i$

$\hookrightarrow \pi'_{L_i} = 0 \Leftrightarrow P_i z_i = w \Leftrightarrow \frac{w}{P_i} = z_i \quad \begin{cases} \text{For T}: \frac{w}{P_T} = 1 \Leftrightarrow w = P_T \\ \text{For NT}: \frac{w}{P_{NT}} = 1 \Leftrightarrow w = P_{NT} \end{cases}$

(d2) The price of tradables

$P_T = eP_T^* = 4 \cdot 1/4 = 1$

(d3) The nominal wage rate

$w = P_T = 1$

(d4) The price of non-tradables

$P_{NT} = w = 1$

(d5) The consumer price index

$P = P_T^{1/2} P_{NT}^{1/2} = \sqrt{1} \cdot \sqrt{1} = 1$

(d6) The real exchange rate

$\theta = \frac{eP^*}{P} = \frac{4 \cdot 1}{1} = 4$

e) Consider now that there was a productivity shock in the tradable sector of Schmidtland, such that now $Y_T = 4L_T$. Considering fixed exchange rates, find:

(e1) The price of tradables and the nominal wage rate

$P_T = eP_T^* = 4 \cdot 1/4 = 1 \; (=)$ Note that now $w = 4P_T$ and $w = P_{NT}$

$w = 4P_T = 4 \cdot 1 = 4 \; (\uparrow)$

(e2) The price of non-tradables

$P_{NT} = w = 4 \; (\uparrow)$

(e3) The consumer price index

$P = P_T^{1/2} P_{NT}^{1/2} = \sqrt{1} \cdot \sqrt{4} = \sqrt{4} = 2 \; (\uparrow)$

(e4) The real exchange rate

$\theta = \frac{eP^*}{P} = \frac{4 \cdot 1}{2} = 2 \; (\downarrow)$

e5 Absolute PPP ✗ ($\theta \neq 1$ at all times)

Relative PPP ✗ ($\theta$ is not constant $\Rightarrow$ makes sense because this is a real shock)

2C $m^D = Y/4i$ $P^* = 1$ $i = R + \pi^e$ $P^* = 1$ $R = 0{,}05$ $e = P/P^*$ $Y_F = 225$ $\mu = 0{,}2$

g)

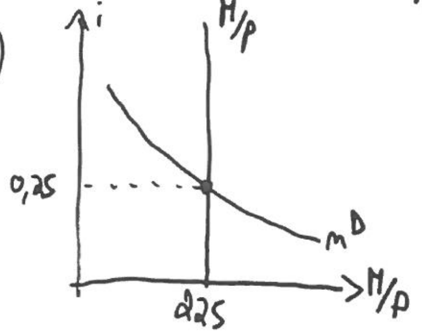
1) $\overline{Y_F} \Rightarrow g = 0.$ $\pi = \mu - g = 0{,}2 - 0 = 0{,}2$ $i = R + \pi^e = 0{,}05 + 0{,}2 = 0{,}25$

2) $m^D = Y/4i = \frac{225}{4 \times 0{,}25} = 225$

3) $\frac{\Delta e}{e} = \pi - \pi^* = 0{,}2 - 0 = 0{,}2$

4) $MV = PY \Leftrightarrow V = \frac{PY}{M} = \frac{Y}{m^D} = \frac{225}{225} = 1$

5)



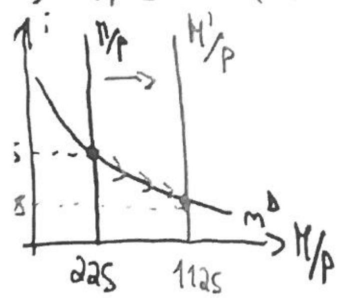

h) Once $M = 450$, then $\bar{e} = 2$. $B_c = 312{,}5$

1) The CB will fix the exchange rate. To do so, it will have to stop growing the money supply. Hence $\mu' = 0$.

$\pi' = \mu' - g = 0 - 0 = 0$ $i' = R + \pi^{e'} = 0{,}05 + 0 = 0{,}05$

2) $m^{D'} = Y/4i' = \frac{225}{4 \times 0{,}05} = 1\,125$ $\bar{P} = \bar{e} P^* = 2 \times 1 = 2$

$\Rightarrow M' = m^{D'} \times \bar{P} = 1\,125 \times 2 = 2\,250$




To fix the exchange rate at $e = 2$, the CB will have to expand the money supply.

3)

| CB | |
|---|---|
| $B_c = 312{,}5$ | $M = 450$ |
| $eB_c^{*} = 137{,}5$ | |

→

| CB | |
|---|---|
| $B_c = 312{,}5$ | $M' = 2\,250$ |
| $eB_c^{*'} = 1937{,}5$ | |

With the announcement of the fixed exchange rate, agents revise their inflation expectations, and demand more domestic currency. The CB expands $M^S$ by purchasing the excess amount of foreign currency in the market.

i) $\frac{\Delta B_c}{B_c} = 0{,}2 \qquad e = 2$

ii) Since the amount of foreign reserves held by the CB is finite, the peg will eventually have to break, once $eB_c^* = 0 \wedge M = B_c$.

Once this happens: $\mu = \frac{\Delta B_c}{B_c} = 0{,}2 \Rightarrow \pi = 0{,}2 \Rightarrow i = 0{,}25 \Rightarrow m^D = 225$

i2) $t=1$: $B_c' = B_c \times 1{,}2 = 312{,}5 \times 1{,}2 = 375$

$eB_c^{*'} = \bar{M} - B_c' = 2\,250 - 375 = 1875$

| CB | |
|---|---|
| $B_c = 375$ | $M = 2\,250$ |
| $eB_c^* = 1875$ | |

This policy is called sterilization, the CB alters the composition of the money supply, while keeping its level fixed.

i3) $t=2$: $B_c' = B_c \times 1{,}2 = 375 \times 1{,}2 = 450$

$eB_c^{*'} = \bar{M} - B_c' = 2\,250 - 450 = 1\,800$

| CB | |
|---|---|
| $B_c = 450$ | $M = 2\,250$ |
| $eB_c^* = 1\,800$ | |

If the attack happens: $eB_c^* = 0 \wedge M = B_c = 450 \Rightarrow \mu = \frac{\Delta B_c}{B_c} = 0{,}2 \Rightarrow i = 0{,}25$

$\Rightarrow m^D = 225 \qquad P = \frac{M}{m^D} = \frac{450}{225} = 2 \qquad e = P/p^* = 2/1 = 2$

With an attack at $t=2$, there would be no price level / exchange rate discontinuity, hence no gains/losses. Since agents are both rational and have perfect foresight, the attack takes place in $t=2$.

| CB | |
|---|---|
| $B_c = 450$ | $M = 450$ |
| $eB_c^* = 0$ | |