NOVA SCHOOL OF BUSINESS & ECONOMICS

Number: **Name:**

II (13)

II.1. Consider a two-period economy with no initial assets. The representative consumer has a lifetime utility function given by: $U = C_1 C_2$. In period 1, there is a pre-determined amount of output: $Q_1 = 40$. As for the second period, there is no exogenous output, but there are investment opportunities, as described by $Q_2 = 10K^{0.5}$, where K depreciates fully after one period. Further assume that this economy is able to borrow and lend in the international markets at the interest rate r*=25%.

a) Find out: (a1) optimal investment; $\overline{\delta = 1} \Rightarrow K_{t+1} = I_t \begin{cases} K_2 = I_1 \\ K_3 = I_2 = 0 \end{cases}$

↳ Goal: max V_1 by setting K_2^*

↳ BC: $C_1 + \frac{C_2}{1+r^*} = Q_1 + K_1(1-\delta) - K_2 + \frac{Q_2 + K_2(1-\delta)}{1+r^*}$

- $\Lambda_{NI} = Q_1 = 40$
- $V_1 = -K_2 + \frac{Q_2}{1+r^*} = -K_2 + \frac{10K_2^{1/2}}{1+r^*}$
- $\max V_1 \Rightarrow V_1'{}_{K_2} = 0 \Leftrightarrow -1 + \frac{1/2 \cdot 10 \cdot K_2^{-1/2}}{1{,}25} = 0 \Leftrightarrow \frac{5}{\sqrt{K_2}} = 1{,}25 \Leftrightarrow \sqrt{K_2} = 4 \Leftrightarrow K_2 = 16$
- Then, $I_1^* = K_2^* = 16$

(a2) NPV;

- $NPV = V_1(16) = -16 + \frac{10 \cdot \sqrt{16}}{1{,}25} = 16$

(a3) life-time wealth;

- $\Lambda_1 = \Lambda_{NI} + V_1(16) = 40 + 16 = 56$ //

(a4) consumption in period 1 and 2;

→ Goal: max U s.t BC ⇒ $\max C_1 C_2$ s.t $C_1 + \frac{C_2}{1{,}25} = 56$

→ $L = C_1 C_2 + \lambda\left[56 - C_1 - \frac{C_2}{1{,}25}\right]$

$L'_{C_1} = 0 \Leftrightarrow C_2 - \lambda = 0 \Leftrightarrow C_2 = \lambda$

$L'_{C_2} = 0 \Leftrightarrow C_1 = \frac{\lambda}{1{,}25} \Leftrightarrow C_1 = \frac{C_2}{1{,}25}$

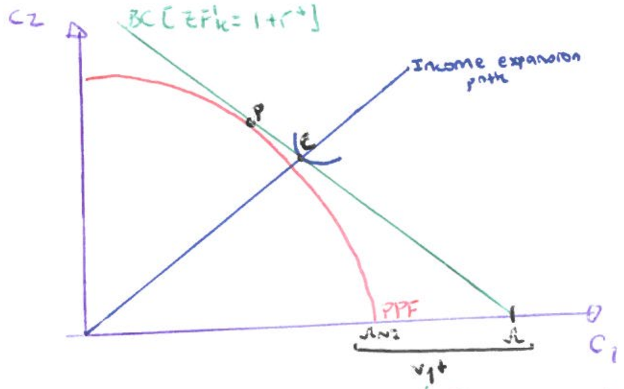
→ Then, $C_1 + \frac{C_2}{1{,}25} = \Lambda \Leftrightarrow \frac{C_2}{1{,}25} + \frac{C_2}{1{,}25} = 56 \Leftrightarrow \frac{2C_2}{1{,}25} = 56 \Leftrightarrow C_2 = 35$

$C_1 = \frac{35}{1{,}25} = 28$

$(C_1^*, C_2^*) = (28, 35)$

Note that $\rho = 0\%$

$\rho < r \Rightarrow C_1 < C_2$ [check out] (a5) represent graphically, identifying the contribution of investment to life-time wealth.



P : production point
[intersection BC x PPF]

C : consumption point
[intersection BC x IEP]

↳ Investment allows to get Ω instead of Ω_{N1}

b) Based on your results in a), find out:

(b1) GNE in period 1:

↳ $GNE_t = A_t = C_t + G_t + I_t = C_t + I_t$

↳ Then, $GNE_1 = C_1 + I_1 = 28 + 16 = 44$

(b2) GNI in period 2:

↳ $GNI_t = Q_t + NFIA_t = Q_t + r^* b^*_{t-1}$

↳ $GNI_2 = Q_2 + r b^*_1 = 10\sqrt{16} + 0{,}25 \cdot (TB_1) = 10 \cdot 4 + 0{,}25 \cdot (Q_1 - C_1 - I_1)$
$= 40 + 0{,}25 \cdot (-4)$
$= 39$

(b3) FA in period 1:

↳ $CA_1 + KA_1 + FA_1 = 0 \Leftrightarrow FA_1 = -CA_1 - KA_1$

↳ $FA_1 = -CA_1 = 4$ [no KA]

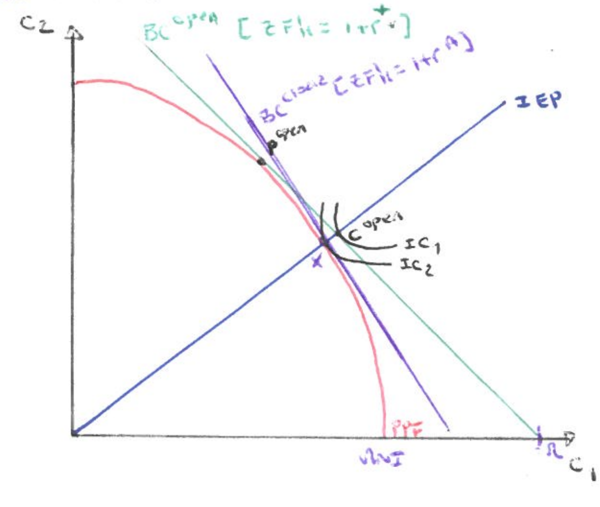
(b4) domestic savings in period 2:

↳ $S_t = I_t + CA_t$

$S_2 = I_2 + CA_2 \Leftrightarrow S_2 = 0 + 4 \Leftrightarrow S_2 = 4$

c) Assume instead that this economy was closed to capital flows. Without calculations, explain how would the resulting equilibrium compare to (a), in terms of: (c1) interest rate; (c2) investment. (c3) Conclude on the benefits of trade openness in this exercise

↳ Goal: max U s.t PPF



Point X is the closed economy equilibrium (intersection IEP x PPF)

Given that the Euler Equation is always true, r^A (autarky interest rate) will adjust to ensure that the BC of the closed economy goes through point X

Therefore, $\begin{cases} r \uparrow \\ I \downarrow \end{cases}$

Note: to conclude on the benefits of trade it would be enough to say that IC_1 yields more utility than

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II.B. Consider an economy with flexible prices where the purchasing power parity and the Fisher principle hold instantaneously. Assume that $P^* = 2$, the real interest rate is 5% and the money demand is given by $m^D = \frac{Y}{4i}$. Full employment output is given by $Y_f = 72$. Initially, the money supply is constant at $M^S = eB_C^* + B = 600 + 120$ and the exchange rate is fixed at e=1.

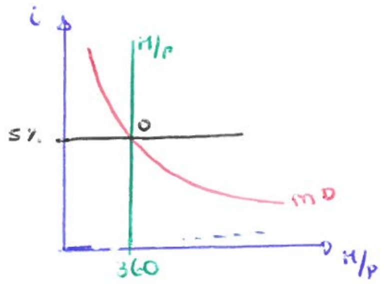
a) Describe in a graph the money market equilibrium, and quantify: the real money demand, the price level and the velocity of money.

↳ $\bar{e} = 1$ and PPP hold. Then, $\bar{e} P^* = \bar{P} \Rightarrow 1 \cdot 2 = \bar{P} \Leftrightarrow \bar{P} = 2$

↳ As prices are fixed, $\pi = 0\%$; y is at FE $\Rightarrow g = 0\%$ } $\mu = 0\%$ [M' is constant]

↳ $i = r + \pi \Leftrightarrow i = r = 5\%$

↳ $M^D = \frac{y}{4i} = \frac{72}{4 \cdot 0{,}05} = 360$; ↳ $\frac{M}{P} = \frac{720}{2} = 360$ } MM in equilibrium



↳ $MV = PY \Leftrightarrow 720V = 2 \cdot 72 \Leftrightarrow V = 0{,}2$

b) Departing from a), assume that the central bank unexpectedly announces a once-and-for all devaluation to E=3. Describe the adjustment, quantifying money demand, price level and the interest rate.

↳ $\bar{e} = 3$ but PPP still holds. Then, $\bar{P} = 3 \cdot 2 = 6$

↳ same π, μ, $g \Rightarrow i = r = 5\%$

↳ Then, $M^D = 360$

↳ MM eq: $\frac{M}{P} = 360 \Leftrightarrow \frac{M}{6} = 360 \Leftrightarrow M = 2160$ (?)

↳ same MM graph (instead of $720/2$ we have $2160/6$)

c) Departing from a), assume that the central bank starts to increase domestic credit by 20% per year. Assuming that the domestic credit expansion is to be fully sterilized and that agents have perfect foresight, find out: c1) the timing of the speculative attack and c2) the reserves that are lost at the time of the attack. c3) Draw the time path of the price level, exchange rate and the interest rate.

↳ (B_{CB}) GR = 20%

↳ The goal is still to keep the peg at $\bar{e} = 1$, that is there will be sterilization

↳ If we attack, $\mu = 20\% \Rightarrow \pi = 20\% \Rightarrow i = 25\%$

Then, $m^D = \frac{72}{4 \cdot 0{,}25} = 72$

↳ PPP holds $\Rightarrow eP^+ = P \Leftrightarrow P = 2e \Leftrightarrow e = \frac{P}{2}$

t = 0

$B_{CB} = 120$

$eB^+_{CB} = 600$

/ no B_{CB} increases yet /

t = 1

$B_{CB} = 120 \cdot 1{,}2 = 144 \quad [\Delta B_{CB} = 24]$

$eB^+_{CB} = 600 - 24 = 576$

what if we attack?

↳ $eB^+_{CB} = 0$

↳ $B_{CB} = M = 144$

↳ $\frac{M}{P} = m^D \Leftrightarrow \frac{144}{P} = 72 \Leftrightarrow \underline{P = 2 \wedge e = 1}$

↳ Then, attacking at t=1 provides no change in e ⇒ optimal to attack at t=1

↳ eB^+_{CB} lost = 576