International Macroeconomics
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Problem Set 3 – Two-Period Model With Investment
Questions

3.1 Consider an economy where the preferences of the representative consumer are given by $U = C_1 C_2$ and the production function is given by $Q_2 = 10 K_2^{1/2}$. Further assume that there are no initial debts or liabilities, and that current output is $Q_1 = 48$. The depreciation rate is equal to 1.

  (a) Assuming that the economy is closed to capital flows:

    (a1) Find the optimal investment for period 1.

    (a2) Find the optimal consumption path.

    (a3) Find the autarky interest rate.

  (b) Suppose that the economy opens to international flows of capital and that the world interest rate is $r^* = 0\%$.

    (b1) Find out the optimal investment.

    (b2) Find the optimal consumption path.

    (b3) Find the trade balance for periods 1 and 2.

    (b4) Represent graphically, comparing with the case in a).

    (b5) Explain why financial openness improves efficiency in the world allocation of capital.

  (c) Suppose now that there is a temporary output shock that drives up current GDP to $Q_1 = 75$. Analyse the impacts in terms of the consumption path when:

    (c1) The economy is closed to capital flows (find the autarky interest rate).

    (c2) The economy is open to capital flows (find the trade balance).

3.2 Consider an economy where the production function is given by $Q_t = 16 K_t^{0.5}$, the current capital stock is $K_1 = 400$, and the depreciation rate is equal to $\delta = 0.15$. Finally, assume that consumers live only two periods, and the preferences of the representative consumer are given by $U = \ln C_1 + \dfrac{\ln C_2}{1+\rho}$, with the rate of time preference equal to 0.25.

  (a) Assuming that the economy is open to capital flows, facing a world interest rate equal to $r^* = 0.25$, find:

    (a1) The optimal investment.

    (a2) The economy' life-time wealth.

    (a3) The optimal levels of current and future consumption.

    (a4) The trade balance, production, NNI and CA in both periods.

  (b) Assume instead that this economy started out with no capital at all ($K_1 = Q_1 = 0$).

    (b1) Would the optimal investment change?

    (b2) What would be the new values of consumption today and in the future?

    (b3) Represent graphically.

3.3 Consider an infinite horizon small open economy, where output is constant and equal to $Q = 100$ and households' welfare is maximized when consumption is constant over time. Assume that initially there are no external assets or liabilities.
   (a) Considering that the world interest rate is equal to $r^* = 25\%$, find out:
      (a1) The households' life-time wealth.
      (a2) The optimal consumption path and the implied trade balance.

   (b) Suppose that production in this economy was related to the existing capital stock, according to $Q_t = 10K_t^{0.5}$, with initial capital being $K_1 = 100$. Further assume that the depreciation rate was $\delta = 0$. Find out:
      (b1) The optimal investment in period 1.
      (b2) The implications for the patterns of consumption and for the TB and CA.

3.4 Consider a one-good economy with no government, where NIIP is initially zero. The representative consumer lives for two periods and has a lifetime utility function given by: $U = C_1C_2$. In period 1, there is a pre-determined amount of output, equal to $Q_1 = 192$. As for the second period, there is no exogenous output, but there are investment opportunities, as described by the following production function, $Q_2 = 16K^{0.5}$, where K depreciates fully after one period.
   (a) Assume that this economy is able to borrow and lend in the international markets at the interest rate r=0%. Find:
      (a1) The optimal investment plan and the corresponding Net Present Value.
      (a2) The optimal consumption path and the implied trade balance.
      (a3) Represent graphically.

   (b) Now assume that the economy was blessed with a productivity surge, so that the production function shifted to $Q_2 = 20K^{0.5}$. Describe the impact of this shock on:
      (b1) The optimal investment.
      (b2) The NPV of the investment opportunities.
      (b3) The consumption in period 1 and 2.
      (b4) The trade balance in period 1 and 2.
      (b5) The current account balance in period 1 and 2.

   (c) Finally, consider the case where the economy was closed to financial trade.
      (c1) If the productivity term increased from A=16 to A=20 as described above, what would be the implications on the optimal values of consumption and of investment?
      (c2) Which of the two cases, (b) or (c1), is more in accordance with the findings of Feldstein and Horioka?

3.5 Consider a two-period economy open to capital flows, with the international interest rate being equal to $1+r_1^* = 1$. Production in period 1 is pre-determined, $Q_1 = 300$, and production in period 2 depends on capital and labour, according to $Q_2 = zK_2^{0.5}N_2^{0.5}$. The labour input is $N_2 = 25$, and capital depreciates fully each year, implying $K_2 = I_1$. In this economy, future productivity can materialize in two different states, A and B, with probability ½: $z^B = 3$ and $z^A = 5$.

(a) Assuming that firms are risk neutral, find:
   (a1) The investment plan that maximizes the expected net present value.
   (a2) The expected lifetime wealth of the representative agent in the good scenario and in the bad scenario?

(b) Assume now that the consumer' optimal choice is such that $C_1 = E[C_2]$. In that case, how much will be:
   (b1) The optimal consumption in period 1?
   (b2) The TB in period 1 and in period 2?
   (b3) Would consumption be smoothed ex post?

(c) Assume now that labour in period 2 was paid according to the corresponding marginal product. Describe the distribution of GDP in terms of dividends and labour income in period 2, $Q_2 = \Pi_2 + W_2 N_2$, in the good scenario, and in the bad scenario.

(d) Finally, consider a foreign economy, with all parameters equal, with the only difference being that future productivity in states A and B are: $z^B = 5$ and $z^A = 3$ Compare the following cross-country investment patters regarding the potential to smooth consumption and, for each case, find the GDP, NFIA, GNI, C, TB and CA for the second period.
   (d1) Households in each country own 100% of the domestic firm.
   (d2) Households in each country own 50% of the domestic firm and 50% of the foreign firm.
   (d3) Households in each country hold 100% of the foreign firm.