Public Economics

Fall 2023

Midterm Exam

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1. You have a total of 80 minutes (1 hour and 20 minutes) to solve the exam.
2. The use of calculators is not allowed.
3. If you need additional space to answer a question, you can use the back *of the same page*.

Read each question carefully. Good luck!

I (6 points)

Discuss the following statements (*max. 10 lines for each*).

a. In a representative democracy, the level of investment in defense chosen by an elected representative will coincide with the median voter's choice.

If there were only one issue (defense) and only two vote-maximizing candidates, they would tend to represent the median voter with respect to the level of investment. However, the result would no longer hold if there are more than two candidates or more than one issue at stake (which is generally the case), if money is a factor in election campaigns, if abstention is possible, if information and knowledge about the issues is asymmetric (among others).

Grading: 1 for one-issue/two-candidate model; 1 for the remaining cases.

b. In an economy with two agents and two goods, if an allocation satisfies no domination, then it will also be envy-free.

False. One counterexample: an economy with two goods with a total of 4 units of x and 4 units of y and an allocation that gives A (2.5,1) and B (1.5,3) satisfies no-domination (no agent receives higher amounts of both). But if both agents see the goods as perfect substitutes (with a 1:1 ratio), A will envy B.

Grading: 0.5 for each definition, 1 for counterexample and conclusion.

c. When there is uncertainty about the costs of reduction, price intervention is the best policy response to an externality that has a very steep marginal damage curve.

If efficiency is the main goal, then with uncertainty about the costs of reduction, price intervention involves a greater deadweight loss than quantity intervention – and quantity intervention would be preferable. It would be more important to control quantity than to allow flexibility in the reduction.

Grading: 0.5 for the conclusion, 1.5 for the justification.

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II (4 points)

Consider an economy with two consumers with utility functions $U_1 = x_1 + y_1$ and $U_2 = 4x_2.y_2$. Assume there are 2 units of x and 2 units of y to distribute among the agents.

a. (2.25 points) Using an Edgeworth box, find the set of Pareto efficient points and find the utility possibility frontier.

MRS for 1 is 1, MRS for 2 is y_2/x_2. Efficient allocations will therefore be such that $x_1=y_1$ and $x_2=y_2$
Then, for all efficient allocations, $U_1=2x_1$ and $U_2 = 4x_2^2$. Therefore, $U_2=(4- U_1)^2$

Grading: 1.25 points for the identification, justification and description of efficient allocations (also using the Edgeworth box); 1 point for the calculation of the UPF.

b. (1.75 points) Find the utilitarian choice for this economy. Will the resulting allocation be egalitarian-equivalent?

We want to maximize U_1+U_2 s.t. $U_2 = U_2=(4- U_1)^2$
Since the UPF is convex, there is a corner solution with $U_1=0$ and $U_2=16$.
The resulting allocation is $x_1=0$, $y_1=0$, $x_2=2$, $y_2=2$ and this is not egalitarian-equivalent: the indifference curve for agent 2 through (2,2) does not intersect the indifference curve for agent 1 going through (0,0) and therefore there is no reference bundle.

Grading: 0.5 for the formulation, 0.5 for the right solution, 0.5 for the justification of egalitarian-equivalence (including the graph) and 0.25 for the conclusion.

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III (5.5 points)

A new airport was just built in Field County. Letting x denote the number of daily flights, revenue is given by $120x$ and the total cost is x^2.
The real estate business is now worried about the effect of the noise on the value of apartments. Letting y the number of apartments sold, revenue for an apartment will be $180y - x.y$ and the total cost is y^2.

a. (1.5 points) Find the numbers of daily flights and apartments that will result from the market.

Airport solves:
$$\max_x \pi_x = 120x - x^2$$
FOC: $\frac{\partial \pi_x}{\partial x} = 0 \Leftrightarrow 120 - 2x = 0 \Leftrightarrow x = 60$

Real estate solves:
$$\max_y \pi_y = 180y - xy - y^2$$
FOC: $\frac{\partial \pi_y}{\partial y} = 0 \Leftrightarrow 180 - x - 2y = 0 \Leftrightarrow y = \frac{180-x}{2} \Leftrightarrow y = 60$

Grading: 0.75 for each maximization problem and respective solution.

b. (1.5 points) Find the efficient numbers of daily flights and apartments.

We need to maximize aggregate total welfare, by simultaneously maximizing the sum of profits:
$$\max_{x,y} \pi_x + \pi_y = 120x - x^2 + 180y - xy - y^2$$
FOC:
$$\begin{cases} \frac{\partial \pi_x}{\partial x} = 0 \\ \frac{\partial \pi_y}{\partial y} = 0 \end{cases} \Leftrightarrow \begin{cases} 120 - 2x - y = 0 \\ 180 - x - 2y = 0 \end{cases} \Leftrightarrow \begin{cases} y = 80 \\ x = 20 \end{cases}$$

Grading: 0.5 for setting the problem, 0.5 for the conditions, 0.5 for solving.

c. (1.25 points) Suppose that the government wants to impose a Pigouvian tax on the airport. What should that tax be?

Pigouvian tax is $t = MEC(x^*, y^*)$
MEC is the unit damage caused by the airport to the real estate, per each daily flight (x):
$$MEC = \frac{\partial \pi_y}{\partial x} = y$$
Thus, the Pigouvian tax is: $t = y(y^* = 80) = 80$.

Grading: 0.5 for understanding how the Pigouvian tax works, 0.75 for finding the MEC and solving.

d. (1.25 points) The Government is considering instead the possibility of mediating a negotiation between the airport and the real estate business. As Government advisor, what would you recommend? (without additional calculations, max 8 lines).

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When there are well defined property rights and costless bargaining, negotiations between the party creating the externality and the party affected by it can bring about the socially optimal. This is the Coase Theorem. In the example, there seems to be only these two identified parties involved, meaning we can apply the theorem. The government just has to assign the property rights to one of the parties – there is either the right to fly, and the real estate pays the airport to reduce its flights, or there is the right not to fly, and the airport pays the real estate per each flight. While Pigouvian taxation could be better for the government as it generates revenue, the Coase Theorem requires less information about the market, and information is usually difficult and expensive for the government to get.

Grading: 0.5 for explaining the Coase Theorem, 0.25 for its application to this example, 0.5 for the recommendation (in comparison with other policy instrument, such as the Pigouvian tax).

IV (4.5 points)

A small town is composed of three distinct citizens with preferences over a pure private good X, money, and a pure public good G, street lighting. The marginal cost of each unit of street lighting is equal to 2. While agent's 1 preferences can be represented by $u_1 = x_1 + 3\ln(G)$, and agent's 2 preferences by $u_2 = x_2 + \ln(G)$, agent 3 derives no utility from the public good.

a. (1.5 points) Show that the socially optimal quantity of street lighting is 2.

To find the socially optimal we use the Samuelson Condition: $\sum MRS_i = MRT$ or $\sum MB_i = MC$.

$MRS_1 = \frac{\frac{\partial u_1}{\partial G}}{\frac{\partial u_1}{\partial x_1}} = \frac{3}{G} = MB_1$

$MRS_2 = \frac{\frac{\partial u_2}{\partial G}}{\frac{\partial u_2}{\partial x_2}} = \frac{1}{G} = MB_2$

$MRS_3 = \frac{\frac{\partial u_3}{\partial G}}{\frac{\partial u_3}{\partial x_3}} = 0 = MB_3$

Thus: $\sum MB_i = MC \Leftrightarrow \frac{3}{G} + \frac{1}{G} + 0 = 2 \Leftrightarrow G^* = 2$.

Grading: 0.25 for stating the Samuelson Condition, 0.25 for each MB, 0.5 for solving.

b. (1.75 points) The government is considering a uniform tax to cover the cost of the public good. If a simple majority is required to decide on the amount of the public good, what will be the amount provided in the economy? Is it efficient? Why or why not?

The uniform tax rate would be: $t = \frac{MC}{3} = \frac{2}{3}$.
At this tax rate, the choices of the agents are as follows:
Agent 1: $\frac{2}{3} = \frac{3}{G_1} \Leftrightarrow G_1 = \frac{9}{2}$
Agent 2: $\frac{2}{3} = \frac{1}{G_2} \Leftrightarrow G_2 = \frac{3}{2}$
Agent 3: $G_3 = 0$
Preferences over the public good are well-behaved and single-peaked, meaning that we can apply the Median Voter Theorem: the outcome of majority voting will be the median voter choice.
The median voter is agent 2, meaning that the outcome with this tax will be $G = \frac{3}{2}$, which is not the efficient level $G^* = 2$.

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The reason why efficiency, in this case, is not reached with majority voting is because the median voter theorem does not account for the intensity of preferences, as the fact that agent one derives much higher utility for the public good is not sufficiently taken into account in majority voting, resulting in a suboptimal choice.

Grading: 0.25 for finding the uniform tax rate, 0.5 for finding the choices the three agent, 0.5 for applying the median voter theorem and finding the outcome of majority voting, 0.5 for explaining that it is not efficient because the intensity of preferences is not accounted.

c. (1.25 point) Suppose that unanimity is required to decide on the amount of the public good and that the taxes must still cover the cost of the public good. What unit taxes should the government charge?

The tax that allows unanimity to be reach and for the public good to be fully funded is Lindahl taxation, where each agent is tax according to the marginal benefit it derives from the consumption of the public good at the socially optimal level:

$t_1 = MB_1(G^* = 2) = \frac{3}{2}$

$t_2 = MB_2(G^* = 2) = \frac{1}{2}$

$t_3 = MB_3(G^* = 2) = 0$

Notice that $t_1 + t_2 + t_3 = 2 = MC$, meaning the public good is, in fact, fully funded.

Grading: 0.25 for stating what Lindahl taxes are, 0.25 for finding each of the unitary taxes, 0.25 for showing that the public good is fully funded.

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