Public Economics

Spring 2023
Midterm Exam

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1. You have a total of 80 minutes (1 hour and 20 minutes) to solve the exam.
2. The use of calculators is not allowed.
3. If you need additional space to answer a question, you can use the back *of the same page*.

Read each question carefully. Good luck!

I (6 points)

Discuss the following statements (*max. 10 lines for each*).

a. The median voter theorem contradicts the Arrow impossibility theorem.

The Arrow impossibility theorem states that, with no restriction of the preference domain, and excluding dictatorship, it is impossible to achieve an aggregation of individual rational preferences into a rational social preference that satisfies respect of unanimity and independence of irrelevant alternatives. The median voter theorem states that, if the preference domain is restricted to single-peaked preferences, the median will achieve that aggregation (and the choice of the median voter will defeat all others in pairwise majority comparisons). Since the assumptions with respect to the preference domain are different, there is no contradiction.

Grading: 0.75 for stating the Arrow Theorem, 0.75 for the median voter theorem, 0.5 for conclusion

b. In an economy with two agents and two goods, an allocation that satisfies no-domination may violate no-envy.

True. One example: an economy with two goods with a total of 4 units of x and 4 units of y and an allocation that gives A (2.5,1) and B (1.5,3) satisfies no-domination (no agent receives higher amounts of both). But if both agents see the goods as perfect substitutes, A will envy B.

Grading: 0.5 for each definition, 1 for counterexample and conclusion.

c. Subsidizing activities that produce positive externalities will generate a Pareto improvement.

In the presence of a positive externality, efficiency is not typically achieved without intervention (there will be underprovision). A subsidy might lead to efficiency, depending on how it is calculated and implemented: a Pigouvian subsidy (equal to the marginal external benefit evaluated at the efficient level) would lead to efficiency. However, moving from inefficiency to efficiency need not be a Pareto improvement: it might be the case that some agents are not made better off (and tax collection would be required to provide a subsidy).

Grading: 0.5 for the efficiency problem, 0.75 for the choice of subsidy, 0.75 for the argument on a Pareto improvement.

NUMBER: NAME:

II (3.75 points)

Consider an economy with two agents with utility functions $U_1 = min\{x_1, y_1\}$ and $U_2 = x_2 + 2y_2$. Assume there are 2 units of x and 2 units of y to distribute among the agents.

a. (2 points) Using an Edgeworth box, find the set of Pareto efficient points and find the utility possibility frontier.

Set of Pareto efficient points: $x_1 = y_1$
UPF: $U_2 = 6 - 3U_1$

Grading: 1 for the set of Pareto efficient points (including 0.5 for the justification in the Edgeworth box), 1 for the UPF.

b. (1.75 points) Find the Rawlsian choice for this economy. Will the resulting allocation be egalitarian-equivalent?

Rawlsian choice: $U_2 = U_1$
Since the UPF is $U_2 = 6 - 3U_1$, we will have $U_2 = U_1 = 3/2$

The resulting allocations will therefore be $x_1 = y_1 = 3/2$ and $x_2 = y_2 = 1/2$
Since the indifference curve for agent 1 going $x_1 = y_1 = 3/2$ does not cross the indifference curve of agent 2 going through $x_2 = y_2 = 1/2$, there is no reference bundle and the allocation is therefore not egalitarian-equivalent.

Grading: 1 for the Rawlsian choice (including 0.5 for the allocation), 0.75 for the conclusion on egalitarian-equivalence (with 0.25 for the notion).

NUMBER: NAME:

III (4.5 points)

Two factories generate pollution through their production processes. This pollution has negative effects on the whole community and the marginal damages are estimated to be 100. However, reducing pollution is costly for the factories (increasing the costs of production). The government estimates that Factory 1 has a total cost of reducing pollution equal to y_1^2, where y_1 is the number of units reduced. In turn, the government estimates that Factory 2 has a total cost of reducing pollution equal to $2y_2^2$, where y_2 is the number of units reduced

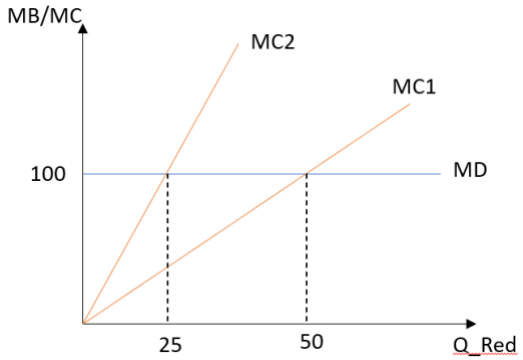
a. (1.35 points) Assuming that the government estimates are correct, find the socially optimal levels of reduction (and represent this graphically).

First, find the MC of each firm by deriving TC with respect to y: $MC_1 = 2y_1$ and $MC_2 = 4y_2$.

Social Optimum is when $MC_1 = MD \Leftrightarrow y_1 = 50$ and $MC_2 = MD \Leftrightarrow y_2 = 25$.

Thus $y = y_1 + y_2 = 75$.

Plot:



Grading: 20% for finding MC, 20% for equilibrium condition, 40% for solving, 20% for plotting.

b. (1.8 points) You are asked to advise the Government on the best way of solving this externality, knowing that there is uncertainty about the costs of reduction. What policy would you recommend? Quantify it and justify your choice. (max. 10 lines for the justification)

When there is uncertainty about the marginal costs of reducing pollution, the best strategy for correcting an externality depends on the slope of the marginal damage curve. If MD is very steep, a quantity regulation approach will generate a lower DWL then a Pigouvian tax approach. If MD is very flat, a Pigouvian tax approach will generate a lower DWL then a quantity regulation approach.

However, in this case, since the MD is simply constant, you can achieve the social optimum without any information on the MCs, that is, you can actually achieve a DWL of 0.

As such, the best way to solve the externality is to impose $t = MD(Q^*) = 100$.

Grading: 25% for discussion based on the slope of the MD curve, 25% for acknowledging and justifying that the tax policy leads to efficiency (DWL of 0), 50% for concluding on the advice and quantifying.

NUMBER: NAME:

c. (1.35 points) A new Government was appointed in the meantime and decided not to ask for your opinion. Instead, the Government will define that pollution is not allowed initially but may happen as a result of negotiation (and written contracts that can be verified) with affected parties. What do you expect to happen in this case? (max. 10 lines, no need for additional calculations)

You can apply the Coase Theorem. The Coase Theorem states that when there are well-defined property rights and costless bargaining, then negotiations between the party creating the externality and the party affected by the externality can bring about the socially optimal market quantity.

Therefore, given that pollution is not allowed, you will have the factories creating the externality paying to the parties affected by the externality a unit payment exactly equal do the MD of 100, and the social optimum will be reached.

However, for the Coase Theorem to work, the assumption of costless bargaining must also be verified. There might be more agents affected by the externality than the ones we think there are, one of the parties may have more bargaining power (namely the firms), or we may have too many parties involved, making aggregating them not costless.

Grading: 40% for defining the Coase Theorem, 30% for explaining the payments, 30% for discussing possible issues.

IV (5.75 points)

Consider a club with three agents who value two goods, where X is a pure private good (money, with a unit price of 1) and G is a nonrival good. Let the marginal cost of the nonrival good be 2 monetary units.

Let x_i denote the amount of the private good consumed by agent i. Agent 1's preferences can be represented by utility function $U_1(x_1, G) = x_1 + \ln(G)$. Agent 2's preferences can be represented by utility function $U_2(x_2, G) = x_2 + 2.\ln(G)$. Agent 3's preferences can be represented by utility function $U_3(x_3, G) = x_3.G$.

The incomes of the agents before the provision of G are m_1=5, m_2=5 and m_3=10.

a. (1 point) Show that agent 1's demand for the nonrival good is $p_1 = \frac{1}{G}$, agent 2's demand for the nonrival good is $p_2 = \frac{2}{G}$, and agent 3's demand for the nonrival good is $p_3 = \frac{5}{G}$

To find the demand functions one solves: $\max U_i \ \ st. m_i = P_x * x_i + P_i * G$

The solution to this, given that $P_x = 1$, is: $MRS_i = P_i$

Applying to all agents:

$$MRS_1 = P_1 \Leftrightarrow \frac{\frac{1}{G}}{1} = P_1 \Leftrightarrow P_1 = \frac{1}{G} \quad ; \quad MRS_2 = P_2 \Leftrightarrow \frac{\frac{2}{G}}{1} = P_1 \Leftrightarrow P_1 = \frac{2}{G}$$

$$MRS_3 = P_3 \Leftrightarrow \frac{x_3}{G} = P_3 \Leftrightarrow \frac{10 - P_3 G}{G} = P_3 \Leftrightarrow P_3 = \frac{5}{G}$$

where we used the budget constraint: $10 = x_3 + P_3 G$

Grading: 25% for setting up the maximization problem, 75% for solving MRS=P for each agent.

b. (1.25 points) Find the socially optimal quantity of the nonrival good.

To find the socially optimal quantity, since the goods is nonrival, you apply the Samuelson Condition:

$$\sum MB_i = MC \Leftrightarrow P_1 + P_2 + P_3 = MC \Leftrightarrow \frac{1}{G} + \frac{2}{G} + \frac{5}{G} = 2 \Leftrightarrow G^* = 4$$

Grading: 30% for stating the Samuelson Condition, 70% for solving.

c. (1.25 points) Suppose that unanimity is required to decide on the amount of the nonrival good and that the taxes must still cover the cost. What unit taxes should the club government charge? Show that they cover the marginal cost of the good.

If unanimity is required, you should set Lindahl Tax Prices where you tax each agent according to the marginal benefit derived from the consumption of the public good, evaluated at the socially optimal level, that is: $\tau_i = MB_i(Q^*)$.

Therefore: $t_1 = \frac{1}{4}$, $t_2 = \frac{2}{4} = \frac{1}{2}$, $t_1 = \frac{5}{4}$.

The taxes cover the cost of the good: $t_1 + t_2 + t_3 = \frac{8}{4} = 2 = MC$.

Grading: 20% for stating the Lindahl Tax Prices, 60% for solving for each agent, 20% for showing it covers the marginal cost.

NUMBER: NAME:

d. (1.25 points) Without additional calculations, comment on the following statement "If each club member were charged the same tax-price, the median voter choice would lead to efficiency". (max. 10 lines)

We can see from c. that for unanimity to and efficiency to be reached, we need to tax agents different tax rates. If we can settle with majority voting, we need to know whether the median voter will desire the socially optimal level, also considering that we will be equally taxing each agent. That tax rate is not the same as prescribed by Lindahl taxation, meaning the median voter outcome will not be the efficient one.

This happens because the median voter theorem does not account for the intensity of preferences. Recall that the social marginal benefits of a public good are the sum of the private marginal benefits that each individual derives from that good. If a small number of individuals derive enormous benefits from the public good, then they should be accounted for in computing total social marginal benefits. This will not necessarily be the case with the median voter, however, because the outcome is determined only by the ranking of voters and not by the intensity of their preferences.

Grading: 40% for discussing the issue of the intensity of preferences, 20% for understanding that the new tax rate will lead to different choices across agents which are not the efficient level, 40% for concluding that efficiency will (most likely) not be reached.

e. (1 point) Another agent would like to join the club. If the goal is to promote efficiency, should she be allowed in? (max. 8 lines)

If the good is nonrival, this means that one individual's consumption of a good does not affect another's opportunity to consume the good. As such, by allowing her to join the club, we can increase the wellbeing of this agent without hurting the wellbeing of the other agents in the economy. Therefore, she should be allowed in to promote efficiency.

However, it might be the case that the good begins suffering from congestion. If that is the case, then we cannot be sure if efficiency will be reached, given that the other agents may suffer a loss – and in that case we are not promoting efficiency.

Also, if another agent joins the club, the social optimum level of the public increases, which brings extra benefits for the agents, but will require a change in taxes for it to remain fully funded.

Grading: 50% for discussing non-rivalry, 25% for discussing congestion, 25% for concluding.