- **Externalities**

A power plant with cost function $c(y)=y^2/2$ faces a market price of p (assume the firm is a price-taker). Each unit of electricity produced generates pollution f such that $f = y$.
A nearby laundry service (that produces x and faces market price q) has a cost function $c(x)=x^2/2 + x.f/2$

a) Find the private optimum.
b) Find the social optimum.
c) What would be the tax that would lead to the level found in b)?
d) If there is the right to no pollution but the laundry service can sell pollution permits to the power plant, what would be the resulting level of pollution (and the unit price for each permit) in the market equilibrium?
e) If instead there is an initial right to pollute but the power plant can sell pollution reduction units to the laundry service, what would be the resulting level of pollution? Compare that with the answer to d)

- **Public Goods**

1. There are two agents (B and J) in the economy. Let y_i (i=B,J) denote units of a private good (money) consumed by agent i and let g denote units of a public good.
Let agent B's utility function be given by $u_B(y_B, g) = 2y_B + g - g^2/2$.
Let agent J's utility function be given by $u_J(y_J, g) = y_J + 2g - g^2$.
The technology (to allow production of public good from private good) is such that the production possibilities are described by $y + g = 1$ (where $y = y_B + y_J$)
Find the efficient level of public good for this economy.

2. Two agents (A and B) want to get fireworks for a party but they are the only ones who actually like fireworks. Their total valuations for a quantity g of fireworks are given by $V_A(g) = 10\ln(g)$ and $V_B(g) = 2\ln(g)$. Each unit of fireworks costs 4 monetary units.
a) Find the socially optimal level of fireworks.
b) If agents choose individually, how much would each agent choose? What type of behavior do you expect to happen in that case?

- **Collective Decision-Making**

1. There are two agents (A and B) in the economy. Let X_i (i=A,B) denote units of a private good (money) consumed by agent i and let G denote units of a public good.
Both agents have Cobb-Douglas preferences. Let agent A's utility function be given by $u_A(X_A, g) = X_A.G$ and let B's utility function be given by $u_B(X_B, G) = X_B.G$.
Each agent initially owns 100 units of money.
The unit price of money is 1. The unit price of the public good is 5.
Find the Lindahl tax-prices for this economy.

2. The local government of city C wants to establish a policy with respect to street cleaning (q). Based on some studies, the local government concluded that there are two different groups of agents in C: agents in group A have high income levels, whereas agents in group B have low incomes. Individual marginal valuations of street cleaning are given by:

$p_A = 0.1 - q/1000$ and $p_B = 0.005 - q/6000$. The marginal cost of street cleaning is c = 9. There are 100 agents in group A and 1000 in group B.
(a) Find the optimal level of street cleaning.
(b) Find the total tax each individual agent will have to pay if a Lindahl system is adopted.
(c) The government official who runs the street cleaning department wants to maximize the budget attributed to that department and sets the tax-prices that must be paid by each of the groups in order to cover the cost of the service. Assuming that the amount of street cleaning is decided by a simple majority, what will be the tax-prices?

3. Consider a community composed of three agents (A; B e C); whose demands for a public good are given by:
$p_A = 10 - g$
$p_B = 5 – 0.5g$
$p_C = 6 - 2g$
Let the marginal cost of the public good be 15.5, and let $t_A = 9$; $t_B = 3$ and $t_C = 3.5$ be the (already defined) individual tax-prices. The decision on the provision of g is, however, made by a government official who knows the demand curves and would like to maximize the budget associated with the provision of the good. Assuming that a simple majority of agents have the power to veto the decision and suspend the provision of the good, determine the amount of g the government official will set.

- **Mixed goods**

  Let the aggregate demand for the public component of a mixed good be given by Q= 10-P. Let the aggregate demand for the private component of a mixed good be given by Q= 10-2P/3. The marginal cost of the mixed good is c=10.

  a) What quantity will be provided with no government intervention?
  b) What is the socially optimal quantity of the mixed good?
  c) Can you identify a subsidy policy that would lead to the quantity you found in b)?